

BIL Counter Evidence to Ed Reports Alignment, Grade 8

GATEWAY TWO: Rigor and Mathematical Practices	
Rigor and Balance	
Each grade's instructional materials reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by helping students develop conceptual understanding, procedural skill and fluency, and application.	
Indicator 2a -- Attention to conceptual understanding: Materials develop conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings.	
Ed Reports Review	BIL Counter Evidence
Indicator 2a -- The instructional materials do not always provide students opportunities to independently demonstrate conceptual understanding throughout the grade-level. During the “Example” sections, the focus is on explaining procedures.	<p>Conceptual problems are intentionally included throughout the program. Each lesson begins with an Exploration where students develop conceptual understanding. In every lesson, the skill examples are directly followed by Self-Assessment for Concepts & Skills exercises which always include at least one conceptual problem. Also, every Concepts, Skills, & Problem Solving set always contains at least one conceptual problem. For example:</p> <p>Self-Assessment for Concepts & Skills</p> <ul style="list-style-type: none"> • 2.2 #5, page 51 • 5.2 #7, page 207 • 7.1 #5, 8, page 277 • 8.3 #17, page 333 <p>Concepts, Skills, & Problem Solving</p> <ul style="list-style-type: none"> • 1.1 #9-10, page 8 • 4.2 #21, page 153 • 4.5 #26, page 172 • 5.2 #5, 6, & 18, 22, pages 209 - 210 • 8.4 #23, 39, & 40, page 342 • 10.1 #22, page 432
Indicator 2c -- Attention to Applications: Materials are designed so that teachers and students spend sufficient time working with engaging applications of the mathematics, without losing focus on the major work of each grade	
Ed Reports Review	BIL Counter Evidence
Indicator 2c -- The instructional materials present opportunities for students to engage in application of grade-level mathematics; however, the problems are scaffolded through teacher-led questions and procedural explanation. The last example of each lesson is titled, “Modeling Real Life,” which provides a real-life problem involving the key standards addressed for each lesson. This section provides a step-by-step solution for the problem; therefore, students do not fully engage in application... Throughout the series, there are examples of routine application problems that require both single and multi-step processes; however, there are limited opportunities to engage in non-routine problems.	<p>In every lesson, each Modeling Real Life example is directly followed by a set of Self-Assessment For Problem Solving exercises that provide students immediate opportunity to independently engage in routine and non-routine application problems. Students have similar opportunities in the Concepts, Skills, & Problem Solving and Connecting Concepts. Examples of non-routine problems:</p> <p>Self-Assessment For Problem Solving</p> <ul style="list-style-type: none"> • 1.2 #18, page 14 • 2.5 #11, page 73 • 3.4 #7, page 126 • 4.2 #11, page 151 • 5.3 #16, page 215 • 7.1 #10, page 278 <p>Concepts, Skills, & Problem Solving</p> <ul style="list-style-type: none"> • 2.3 #21-23, page 61 • 3.3 #23, page 122 • 4.2 #41, page 154 • 10.4 #23, page 452 <p>Connecting Concepts</p> <ul style="list-style-type: none"> • Chapter 5 #3, page 225 • Chapter 8 #2-3, page 361

<p>Indicator 2c -- • Chapter 7, Lesson 3, Problem 15, Dig Deeper, “You and a friend race each other. You give your friend a 50-foot head start. The distance y (in feet) your friend runs after x seconds is represented by the linear function $y=14x+50$. The table shows your distance at various times throughout the race. For what distances will you win the race? Explain. [Table provided.]” (8.F.3, 8.F.2, multi-step, routine)</p> <p>• Chapter 9, Lesson 2, Problem 14 Dig Deeper, “Objects detected by radar are plotted in a coordinate plane where each unit represents 1 mile. The point (0, 0) represents the location of a shipyard. A cargo ship is traveling at a constant speed and in a constant direction parallel to the coastline. At 9 a.m., the radar shows the cargo ship at (0, 15). At 10 a.m., the radar shows the cargo ship at (16, 15). How far is the cargo ship from the shipyard at 4 p.m.? Explain.” (8.G.7, multi-step, routine)</p>	<p>We suggest that these two problems are non-routine as outlined by the justifications below.</p> <ul style="list-style-type: none"> • In 7.3 #15 on page 294, students are given the table for friend and the equation for you. The students need to create a linear equation out of the table by first finding slope. Once both equations are created, students need to solve the system. Then students need to interpret their solution to answer the question. • In 9.2 #14 on page 385, students need to first determine how many miles per hour the ship is traveling to figure out the coordinates of the ship at 4pm. The coordinates are then used in the Pythagorean theorem to answer how far the ship is from the shipyard at 4pm.
<p>Indicator 2d -- Balance: The three aspects of rigor are not always treated together and are not always treated separately. There is a balance of the 3 aspects of rigor within the grade.</p>	
<p>Ed Reports Review</p>	<p>BIL Counter Evidence</p>
<p>Indicator 2d -- The instructional materials for Big Ideas Math: Modeling in Real Life, Grade 8 partially meet expectations that the three aspects of rigor are not always treated together and are not always treated separately.</p>	<p>The Big Ideas Math: Modeling Real Life program consistently across Grades K - 8 strives for a balanced approach to rigor. Each section develops a concept from conceptual understanding (explorations) to procedural fluency (skill examples) to rigorous application (Modeling Real Life examples), engaging students in the mathematics and promoting active learning. Every set of practice problems reflects this balance, giving students the rigorous practice they need to achieve mastery.</p> <p>The Teaching Edition front matter was updated in a recent printing to provide detail on the program philosophy concerning rigor.</p> <ul style="list-style-type: none"> • Front matter, page xxiii
<p>Indicator 2d -- The instructional materials present opportunities in most lessons for students to engage in each aspect of rigor, however, these are often treated together. There is an over-emphasis on procedural skill and fluency.</p>	<p>The following are examples where conceptual understanding is treated by itself or is the focus.</p> <p>Chapter Explorations</p> <ul style="list-style-type: none"> • Chapter 1 Exploration #1-2, page 2 • Chapter 4 Exploration #1 a-h, page 140 • Chapter 6 Exploration #1-2, page 236 • Chapter 8 Exploration #1-2, page 318 <p>The following are examples where application is treated by itself or is the focus.</p> <p>Connecting Concepts and Performance Task</p> <ul style="list-style-type: none"> • Chapter 2, page 89 • Chapter 4, page 185 • Chapter 5, page 225 • Chapter 6, page 263

Mathematical Practice - Content Connections

Materials meaningfully connect the Standards for Mathematical Content and the Standards for Mathematical Practice.

Indicator 2e -- The Standards for Mathematical Practice are identified and used to enrich mathematics content within and throughout each applicable grade.

Ed Reports Review	BIL Counter Evidence
<p>Indicator 2e -- The MPs are identified within some sections in both the Teaching Edition (Exploration and Example sections) and Student Edition (Exploration 1 [within blue boxes], Concept, Skills & Problem Solving section). In the Student Edition, MPs are labeled with “MP” and a shortened version of the MP, such as “Structure, Reasoning, Construct Arguments, Precision, etc.” There is no document that correlates the abbreviated title with the Standards for Mathematical Practice. For example, the label “MP Number Sense” could align to several MPs. Additionally, Big Ideas Math: Modeling in Real Life, Grade 8 added “MP Logic” as a Mathematical Practice. This added practice does not align with the CCSSM Standards for Mathematical Practice.</p>	<p>We have provided a correlation online at <i>bigideasmath.com</i> for students, aligning the MP labels and other headings in the Student Edition with the Standards for Mathematical Practice. Big Ideas Learning will also send the correlation to existing users of our program. The correlation will also be included in future textbook printings. The page is attached here for your reference.</p> <ul style="list-style-type: none"> • <i>Front matter, page vi</i>

Indicator 2f -- Materials carefully attend to the full meaning of each practice standard.

Ed Reports Review	BIL Counter Evidence		
<p>Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP1: Make sense of problems and persevere in solving them. The instructional materials present few opportunities for students to make sense of problems and persevere in solving them.</p>	<p>While examples are stepped out for students, they illustrate opportunities for students to make sense of problems and persevere in solving them when they independently solve the related problems. Students are encouraged to use the methods shown and the Problem-Solving Plan to think through and solve problems. For example:</p> <ul style="list-style-type: none"> • 3.2 Example 3 & Self-Assessment for Prob. Solving #7-8, page 114 • 5.4 Example 3 & Self-Assessment for Prob. Solving #16-17, page 222 • <i>7.3 Example 4 & Self-Assessment for Prob. Solving #6-7, page 292</i> <p>Connecting Concepts pages at the end of each chapter encourage students to make sense of problems and persevere in solving them. For example:</p> <ul style="list-style-type: none"> • Chapter 1, page 31 • Chapter 5, page 225 • Chapter 6, page 263 • <i>Chapter 9, page 415</i> <p>All non-routine problems listed under Indicator 2c also cover MP1.</p> <p>Teaching Edition notes labeled with MP1 give opportunities for the teacher to emphasize these habits to students and for students to use them going forward. For example:</p> <ul style="list-style-type: none"> • <i>2.7 page T-84</i> • 3.3 page T-119 • 4.2 page T-148 <p>The Teaching Edition front matter was updated in a recent printing to emphasize opportunities for in-class problem solving throughout the program.</p> <ul style="list-style-type: none"> • Front matter, page xxiv 		
<p>Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP4: Model with mathematics. The instructional materials present few opportunities for students to model with mathematics.</p>	<p>Modeling with mathematics is covered throughout our program. Every Modeling Real Life example is directly followed by corresponding Self-Assessment for Problem Solving exercises for students to engage in MP4. In addition, every Concepts, Skills, & Problem Solving set contains multiple opportunities for students to model with mathematics in the Modeling Real Life exercises. For example:</p> <table border="0"> <tr> <td> <p>Self-Assessment for Problem Solving:</p> <ul style="list-style-type: none"> • 1.1 #18-20, page 7 • 2.4 #6-7, page 66 • <i>4.1 #13-15, page 144</i> • 7.2 #17-18, page 285 • 7.4 #11-12, page 298 </td><td> <p>Concepts, Skills, & Prob-Solving:</p> <ul style="list-style-type: none"> • 1.1 #33-37, page 9 • 2.4 #15,17,18, page 68 • 4.1 #24,29-31,33, page 146 • <i>7.2 #36-38, page 288</i> • 7.4 #14-18, page 300 </td></tr> </table>	<p>Self-Assessment for Problem Solving:</p> <ul style="list-style-type: none"> • 1.1 #18-20, page 7 • 2.4 #6-7, page 66 • <i>4.1 #13-15, page 144</i> • 7.2 #17-18, page 285 • 7.4 #11-12, page 298 	<p>Concepts, Skills, & Prob-Solving:</p> <ul style="list-style-type: none"> • 1.1 #33-37, page 9 • 2.4 #15,17,18, page 68 • 4.1 #24,29-31,33, page 146 • <i>7.2 #36-38, page 288</i> • 7.4 #14-18, page 300
<p>Self-Assessment for Problem Solving:</p> <ul style="list-style-type: none"> • 1.1 #18-20, page 7 • 2.4 #6-7, page 66 • <i>4.1 #13-15, page 144</i> • 7.2 #17-18, page 285 • 7.4 #11-12, page 298 	<p>Concepts, Skills, & Prob-Solving:</p> <ul style="list-style-type: none"> • 1.1 #33-37, page 9 • 2.4 #15,17,18, page 68 • 4.1 #24,29-31,33, page 146 • <i>7.2 #36-38, page 288</i> • 7.4 #14-18, page 300 		

<p>Indicator 2f --The instructional materials do not present opportunities for students to engage in MP5: Use appropriate tools strategically. The instructional materials present few opportunities for students to choose their own tool, therefore, the full meaning of MP5 is not being attended to. MP5 is only identified seven times throughout the instructional materials and only in four of 10 chapters. The instructional materials provide limited opportunities for students to choose tools strategically, as the materials indicate what tools should be used.</p>	<p>Students have opportunities to choose tools strategically. For example:</p> <ul style="list-style-type: none"> • 3.1 page T-103 • 3.4 page T-123 • 4.2 #31a, page 154 <p>In the Dynamic Student Edition, students have access to the following math tools at all times.</p> <ul style="list-style-type: none"> • Algebra tiles • Desmos geometry tool • Four function calculator • Number line • Probability tools • Simulation • Balance scale • Desmos graphing calculator • Fraction models • Place value • Scientific calculator
<p>Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP7: Look for and make use of structure. The instructional materials often label content MP7 Structure, but the teaching notes and problems do not attend to the full meaning of the MP.</p>	<p>As stated in EdReports Math Grades K - 8 Evidence Guides, "Every instance of an MP being marked does not necessarily have to encompass the full meaning of an MP, but taken together there should be evidence thta the materials carefully attend to the full meaning of each practice standard." Below are several examples across Grade 8 which show opportunities for students to practice the various aspects of MP7.</p> <ul style="list-style-type: none"> • 2.2 Exploration 1, page 49 • 5.4 Exploration 1, page 219 • 5.4 Self-Assessment for Concepts & Skills #7-9, page 221 • 9.4 #23-25, pages 399-400
<p>Indicator 2f -- • Chapter 7, Lesson 1, Laurie's Notes, Exploration 1, "Mathematically proficient students will study the first diagram to discover a relationship between inputs and the outputs. They will recognize that a mapping diagram is similar to a table of value." This is labeled MP7 in Laurie's Notes, but does not ask students to discern patterns or structures to solve problems.</p> <ul style="list-style-type: none"> • Chapter 1, Lesson 3, Laurie's Notes, Example 1, "It may not be necessary to completely solve the equation. Students should notice that the same quantity, $4x$, is being subtracted from different numbers, 3 and 7. They should reason that the two sides of the equation can never be equal, so there is not solution. "How do you know when an equation has no solution?" This example is worked for the student and therefore not solving the problem independently. 	<p>We suggest that the 2 citations listed do show patterns and structure.</p> <ul style="list-style-type: none"> • 7.1 Exploration 1, page T-275: Students have to discover a pattern between inputs and outputs. The red "?" icon below Laurie's Notes on MP7 offers a question to pose for students to discern patterns or structures: "What can you do to the input to get the output?" • 1.3 Example 3, page T-19 (EdReports cited it as Example 1): This Teaching Edition note points out to teachers an important piece of the structure that students will begin to see that will help them to solve equations like these. The intention of the note is for teachers to encourage this MP7 thinking in their students. That way, students can look at the structure of an equation to see if it has no solution instead of solving the equation.
<p>Indicator 2g.i -- Materials prompt students to construct viable arguments and analyze the arguments of others concerning key grade-level mathematics detailed in the content standards.</p>	
<p>Ed Reports Review</p>	<p>BIL Counter Evidence</p>
<p>Indicator 2g.i -- The Student Edition labels Standards of Mathematical Practices with "MP Construct Arguments," however, these activities do not always require students to construct arguments or analyze arguments of others. In the Student Edition "Construct Arguments" was labeled once for students and "Build Arguments" was labeled once for students.</p>	<p>We suggest that when explaining or comparing answers, students must use what they have previously learned to build a logical progression of statements that defends their answers. The ability to critique someone else's reasoning also helps students analyze their own work and formulate good explanations. For example:</p> <ul style="list-style-type: none"> • 1.1 Self-Assessment for Concepts & Skills #17, page 6 • 1.3 #38, page 23 • 3.4 Exploration 1a & c, page 123 • 3.4 #13, page 127 • 4.4 Self-Assessment for Concepts & Skills #5, page 163 • 5.1 #26, page 204 • 5.4 #26, page 224 • 6.4 Self-Assessment for Concepts & Skills #7-9, page 258 • 7.3 #17, page 294 • 8.1 Exploration 2, page 319

Indicator 2g.ii -- Materials assist teachers in engaging students in constructing viable arguments and analyzing the arguments of others concerning key grade-level mathematics detailed in the content standards.	
Ed Reports Review	BIL Counter Evidence
Indicator 2g.ii -- There are some missed opportunities where the materials could assist teachers in engaging students in both constructing viable arguments and analyzing the arguments of others.	<p>The Teaching Edition contains many instances of guidance, along with probing questions the teacher can ask, to engage students in constructing arguments and analyzing the arguments of others. These are often indicated with either a MP3 inline head or a red "?" icon. For example:</p> <p>MP3 inline head</p> <ul style="list-style-type: none">• 2.3 page T-58• 4.3 page T-155• Chapter 6 Exploration page T-236• 6.4 page T-255• 8.6 page T-350• 9.5 page T-403 <p>Red "?" icon</p> <ul style="list-style-type: none">• 1.2 page T-14• 3.1 page T-107• 4.2 page T-149• 5.3 page T-211• 6.3 page T-250• 7.4 page T-295

EXAMPLE 2 Reflecting Figures

The vertices of a triangle are $A(1, 1)$, $B(1, 4)$, and $C(3, 4)$. Draw the figure and its reflection in (a) the x -axis and (b) the y -axis. What are the coordinates of the image?

Another Method

Take the opposite of each y -coordinate. The x -coordinates do not change.

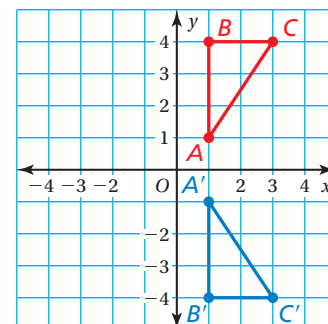
$$A(1, 1) \rightarrow A'(1, -1)$$

$$B(1, 4) \rightarrow B'(1, -4)$$

$$C(3, 4) \rightarrow C'(3, -4) \quad \checkmark$$

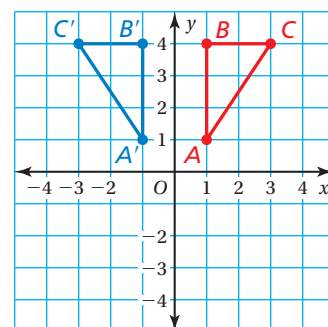
- a. Point A is 1 unit above the x -axis, so plot A' 1 unit below the x -axis. Points B and C are 4 units above the x -axis, so plot B' and C' 4 units below the x -axis.

► The coordinates of the image are $A'(1, -1)$, $B'(1, -4)$, and $C'(3, -4)$.



- b. Points A and B are 1 unit to the right of the y -axis, so plot A' and B' 1 unit to the left of the y -axis. Point C is 3 units to the right of the y -axis, so plot C' 3 units to the left of the y -axis.

► The coordinates of the image are $A'(-1, 1)$, $B'(-1, 4)$, and $C'(-3, 4)$.



Try It

3. The vertices of a rectangle are $A(-4, -3)$, $B(-4, -1)$, $C(-1, -1)$, and $D(-1, -3)$. Draw the figure and its reflection in (a) the x -axis and (b) the y -axis.



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

4. **REFLECTING A FIGURE** The vertices of a triangle are $J(-3, -5)$, $K(-2, 2)$, and $L(1, -4)$. Draw the figure and its reflection in (a) the x -axis and (b) the y -axis.

5. **WHICH ONE DOESN'T BELONG?** Which transformation does *not* belong with the other three? Explain your reasoning.



EXAMPLE 2 Solving a System of Linear Equations

Solve the system using any method.

$$2x + 3y = -3 \quad \text{Equation 1}$$

$$2x = y + 5 \quad \text{Equation 2}$$

Step 1: Both equations have a term of $2x$. So, one solution method is to substitute $y + 5$ for $2x$ in Equation 1 and solve to find the value of y .

$$2x + 3y = -3 \quad \text{Equation 1}$$

$$y + 5 + 3y = -3 \quad \text{Substitute } y + 5 \text{ for } 2x.$$

$$4y + 5 = -3 \quad \text{Combine like terms.}$$

$$4y = -8 \quad \text{Subtract 5 from each side.}$$

$$y = -2 \quad \text{Divide each side by 4.}$$

Step 2: Substitute -2 for y in Equation 2 and solve for x .

$$2x = y + 5 \quad \text{Equation 2}$$

$$2x = -2 + 5 \quad \text{Substitute } -2 \text{ for } y.$$

$$2x = 3 \quad \text{Add.}$$

$$x = 1.5 \quad \text{Divide each side by 2.}$$

▶ The solution is $(1.5, -2)$.

Try It Solve the system. Explain your choice of method.

$$\begin{aligned} 4. \quad y &= -3x + 2 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} 5. \quad 4y &= x \\ x + 4y &= -8 \end{aligned}$$

$$\begin{aligned} 6. \quad 2x + 2y &= 1 \\ -x + 2y &= -3 \end{aligned}$$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. **MP REASONING** Does solving a system of linear equations by graphing give the same solution as solving by substitution? Explain.

SOLVING A SYSTEM OF LINEAR EQUATIONS Solve the system by substitution. Check your solution.

$$\begin{aligned} 8. \quad y &= x - 8 \\ y &= 2x - 14 \end{aligned}$$

$$\begin{aligned} 9. \quad x &= 2y + 2 \\ 2x - 5y &= 1 \end{aligned}$$

$$\begin{aligned} 10. \quad x - 5y &= 1 \\ -2x + 9y &= -1 \end{aligned}$$

CHOOSING A SOLUTION METHOD Solve the system. Explain your choice of method.

$$\begin{aligned} 11. \quad y &= -x + 3 \\ y &= 2x \end{aligned}$$

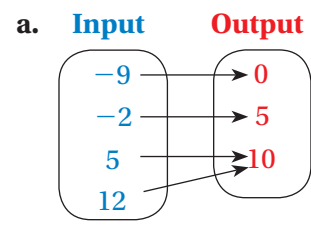
$$\begin{aligned} 12. \quad 0.5x + y &= 2 \\ 0.5x &= 1 + y \end{aligned}$$

$$\begin{aligned} 13. \quad x &= 5y \\ y &= 22 - 2x \end{aligned}$$

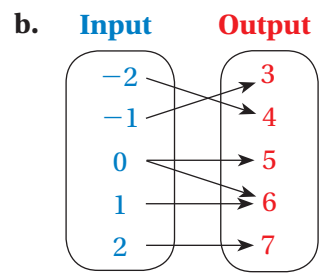
A relation that pairs each input with *exactly one* output is a **function**.

EXAMPLE 2 Determining Whether Relations Are Functions

Determine whether each relation is a function.

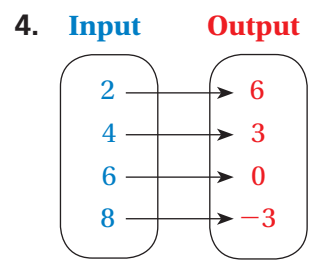
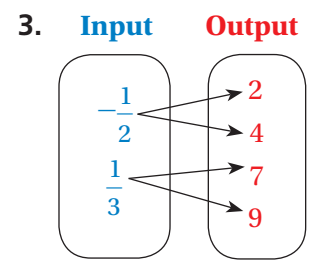


Each input has exactly one output. So, the relation is a function.



The input 0 has two outputs, 5 and 6. So, the relation is *not* a function.

Try It Determine whether the relation is a function.



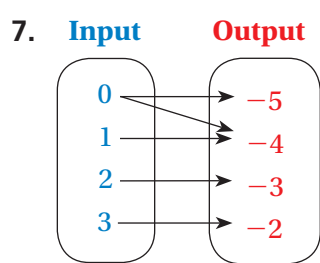
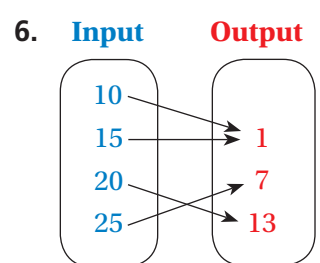
Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria

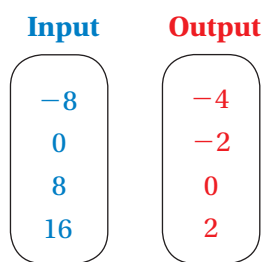
Indicator 2c - In #5 and 8, students demonstrate their conceptual understanding of relations and functions by describing differences, as well as modifying a mapping diagram to make the relation not a function.

5. **MP PRECISION** Describe how relations and functions are different.

IDENTIFYING FUNCTIONS List the ordered pairs shown in the mapping diagram. Then determine whether the relation is a function.



8. **OPEN-ENDED** Copy and complete the mapping diagram at the left to represent a relation that is a function. Then describe how you can modify the mapping diagram so that the relation is *not* a function.



EXAMPLE 3 Simplifying Expressions

a. $\frac{(-4)^9}{(-4)^5} \cdot \frac{(-4)^8}{(-4)^2} = (-4)^{9-5} \cdot (-4)^{8-2}$ Quotient of Powers Property

$= (-4)^4 \cdot (-4)^6$ Simplify.

$= (-4)^{4+6}$ Product of Powers Property

$= (-4)^{10}$ Simplify.

b. $\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} = a^{10-6} \cdot a^{7-4}$ Quotient of Powers Property

$= a^4 \cdot a^3$ Simplify.

$= a^{4+3}$ Product of Powers Property

$= a^7$ Simplify.

You can also simplify the expressions in Example 3 by first multiplying the numerators and multiplying the denominators.

Try It Simplify the expression. Write your answer as a power.

8. $\frac{(-5)^7 \cdot (-5)^6}{(-5)^5 \cdot (-5)^2}$

9. $\frac{d^5 \cdot d^9}{d \cdot d^8}$

10. $\frac{p^3 \cdot p^6}{p^2} \cdot \frac{p^4}{p}$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

SIMPLIFYING EXPRESSIONS Simplify the expression. Write your answer as a power.

11. $\frac{(-3)^9}{(-3)^2}$

12. $\frac{8^6 \cdot 8^2}{8^5}$

13. $\frac{x^{11}}{x^4 \cdot x^6}$

14. $\frac{5^6}{5} \cdot \frac{5^3}{5^2}$

15. $\frac{(-2)^9 \cdot (-2)^4}{(-2)^4 \cdot (-2)^4}$

16. $\frac{b^{10} \cdot b^3}{b^2} \cdot \frac{b^5}{b^3}$

17. **WHICH ONE DOESN'T BELONG?** Which quotient does *not* belong with the other three? Explain your reasoning.

$\frac{(-10)^7}{(-10)^2}$

$\frac{6^3}{6^2}$

$\frac{(-4)^8}{(-3)^4}$

$\frac{5^6}{5^3}$

1.1 Practice



Go to [BigIdeasMath.com](https://www.bigideasmath.com) to get HELP with solving the exercises.

► Review & Refresh

Evaluate the expression.

1. $(3^2 - 8) + 4$

2. $1 + 5 \times 3^2$

3. $4 \times 3 + 10^2$

Identify the terms, coefficients, and constants in the expression.

4. $11q + 2$

5. $h + 9 + g$

6. $6m^2 + 7n$

Write the phrase as an expression.

7. the quotient of 22 and a number a

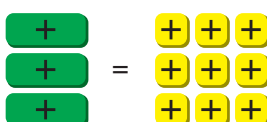
8. the difference of a number t and 9

Indicator 2c - In #9-10, students demonstrate their conceptual understanding of solving equations by using algebra tiles.

► Concepts, Skills, & Problem Solving

USING PROPERTIES OF EQUALITY Which property of equality can you use to solve the equation modeled by the algebra tiles? Solve the equation and explain your method. (See Exploration 1, p. 3.)

9. 

10. 

SOLVING EQUATIONS USING ADDITION OR SUBTRACTION Solve the equation.

Check your solution.

11. $x + 12 = 7$

12. $g - 16 = 8$

13. $-9 + p = 12$

14. $2.5 + y = -3.5$

15. $x - 8\pi = \pi$

16. $4\pi = w - 6\pi$

17. $\frac{5}{6} = \frac{1}{6} + d$

18. $\frac{3}{8} = r + \frac{2}{3}$

19. $n - 1.4 = -6.3$

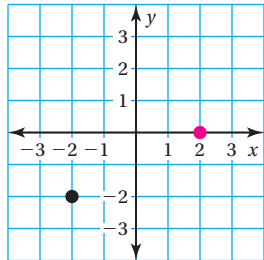
20. **MODELING REAL LIFE** A discounted concert ticket costs \$14.50 less than the original price p . You pay \$53 for a discounted ticket. Write and solve an equation to find the original price.

21. **MP PROBLEM SOLVING** A game of bowling has ten frames. After five frames, your friend's bowling score is 65 and your bowling score is 8 less than your friend's score.
- Write and solve an equation to find your score.
 - By the end of the game, your friend's score doubles and your score increases by 80. Who wins the game? Explain.



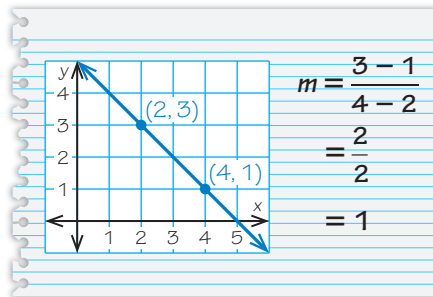
FINDING THE SLOPE OF A LINE Find the slope of the line through the given points.

15. $(4, -1), (-2, -1)$ 16. $(5, -3), (5, 8)$ 17. $(-7, 0), (-7, -6)$
18. $(-3, 1), (-1, 5)$ 19. $(10, 4), (4, 15)$ 20. $(-3, 6), (2, 6)$

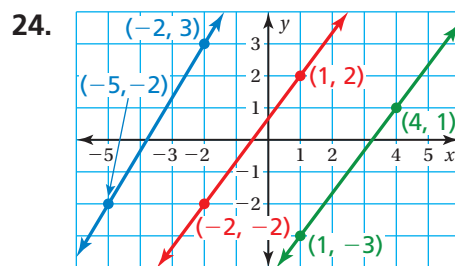
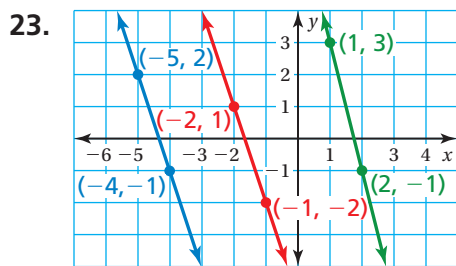


21. **MP REASONING** Draw a line through each point using a slope of $m = \frac{1}{4}$. Do the lines intersect? Explain.

22. **YOU BE THE TEACHER** Your friend finds the slope of the line shown. Is your friend correct? Explain your reasoning.



IDENTIFYING PARALLEL LINES Which lines are parallel? How do you know?



IDENTIFYING PARALLEL LINES Are the given lines parallel? Explain your reasoning.

25. $y = -5, y = 3$ 26. $y = 0, x = 0$ 27. $x = -4, x = 1$

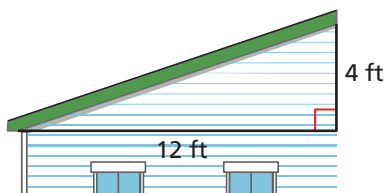
FINDING SLOPE The points in the table lie on a line. Find the slope of the line.

28.

x	1	3	5	7
y	2	10	18	26

29.

x	-3	2	7	12
y	0	2	4	6



30. **MODELING REAL LIFE** Carpenters refer to the slope of a roof as the *pitch* of the roof. Find the pitch of the roof.

18. **YOU BE THE TEACHER** Your friend finds the x -intercept of $-2x + 3y = 12$. Is your friend correct? Explain your reasoning.

$$\begin{aligned} -2x + 3y &= 12 \\ -2(0) + 3y &= 12 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

19. **MODELING REAL LIFE** A charm bracelet costs \$65, plus \$25 for each charm. The equation $-25x + y = 65$ represents the cost y (in dollars) of the bracelet, where x is the number of charms.

- Graph the equation.
- How much does a bracelet with three charms cost?

USING INTERCEPTS TO GRAPH Graph the linear equation using intercepts.

20. $3x - 4y = -12$ 21. $2x + y = 8$ 22. $\frac{1}{3}x - \frac{1}{6}y = -\frac{2}{3}$

23. **MODELING REAL LIFE** Your cousin has \$90 to spend on video games and movies. The equation $30x + 15y = 90$ represents this situation, where x is the number of video games purchased and y is the number of movies purchased. Graph the equation. Interpret the intercepts.



24. **MP PROBLEM SOLVING** A group of friends go scuba diving. They rent a boat for x days and scuba gear for y people, represented by the equation $250x + 50y = 1000$.

- Graph the equation and interpret the intercepts.
- How many friends can go scuba diving if they rent the boat for 1 day? 2 days?
- How much money is spent in total?

25. **DIG DEEPER!** You work at a restaurant as a host and a server. You earn \$9.45 for each hour you work as a host and \$3.78 for each hour you work as a server.

- Write an equation in standard form that models your earnings.
- Graph the equation.

Basic Information	
Number of hours worked as host:	x
Number of hours worked as server:	y
Earnings for this pay period:	\$113.40

26. **MP LOGIC** Does the graph of every linear equation have an x -intercept? Justify your reasoning.

27. **CRITICAL THINKING** For a house call, a veterinarian charges \$70, plus \$40 per hour.

- Write an equation that represents the total fee y (in dollars) the veterinarian charges for a visit lasting x hours.
- Find the x -intercept. Does this value make sense in this context? Explain your reasoning.
- Graph the equation.



5.2 Practice



Go to [BigIdeasMath.com](https://www.BigIdeasMath.com) to get HELP with solving the exercises.

► Review & Refresh

Solve the system by graphing.

1. $y = 2x - 3$

$y = -x + 9$

2. $6x + y = -2$

$y = -3x + 1$

3. $4x + 2y = 2$

$3x = 4 - y$

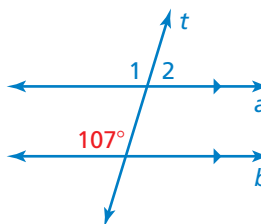
4. Use the figure to find the measure of $\angle 2$.

A. 17°

B. 73°

C. 83°

D. 107°



► Concepts, Skills, & Problem Solving

SOLVING A SYSTEM ALGEBRAICALLY Find the value of each symbol in the system.

(See Exploration 1, p. 205.)

5. $\smile + 1 = \star + \star$ Equation 1

$\smile = 3 + \star$ Equation 2

6. $\odot - \otimes = -2$ Equation 1

$\odot - \otimes = 1 + \otimes$ Equation 2

SOLVING A SYSTEM OF LINEAR EQUATIONS Solve the system by substitution.

Check your solution.

7. $y = x - 4$

$y = 4x - 10$

8. $y = 2x + 5$

$y = 3x - 1$

9. $x = 2y + 7$

$3x - 2y = 3$

10. $4x - 2y = 14$

$y = \frac{1}{2}x - 1$

11. $2x = y - 10$

$2x + 7 = 2y$

12. $8x - \frac{1}{3}y = 0$

$12x + 3 = y$

13. $y - x = 0$

$2x - 5y = 9$

14. $x + 4y = 14$

$3x + 4y = 22$

15. $-2x - 5y = 3$

$3x + 8y = -6$

16. **MODELING REAL LIFE** There are a total of 64 students in a filmmaking club and a yearbook club. The filmmaking club has 14 more students than the yearbook club.

- Write a system of linear equations that represents this situation.
- How many students are in the filmmaking club? the yearbook club?



17. **MODELING REAL LIFE** A drama club earns \$1040 from a production by selling 64 adult tickets and 132 student tickets. An adult ticket costs twice as much as a student ticket.

- Write a system of linear equations that represents this situation.
- What is the cost of each ticket?

18. **OPEN-ENDED** Write a system of linear equations that has the ordered pair (1, 6) as its solution.

CHOOSING A SOLUTION METHOD Solve the system. Explain your choice of method.

19. $y - x = 4$
 $x + y = 6$

20. $0.5x + y = 4$
 $0.5x - y = -1$

21. $y = 2x + 5$
 $y = -3x$

22. **CRITICAL THINKING** A system consists of two different proportional relationships. What is the solution of the system? Justify your answer.

23. **GEOMETRY** The measure of the obtuse angle in the isosceles triangle is two and a half times the measure of one of the acute angles. Write and solve a system of linear equations to find the measure of each angle.



24. **MP NUMBER SENSE** The sum of the digits of a two-digit number is 8. When the digits are reversed, the number increases by 36. Find the original number.

25. **DIG DEEPER!** A hospital employs a total of 77 nurses and doctors. The ratio of nurses to doctors is 9 : 2. How many nurses are employed at the hospital? How many doctors are employed at the hospital?



26. **MP REPEATED REASONING** A DJ has a total of 1075 dance, rock, and country songs on her system. The dance selection is three times the rock selection. The country selection has 105 more songs than the rock selection. How many songs on the system are dance? rock? country?

23. **MP NUMBER SENSE** Without evaluating, order 5^0 , 5^4 , and 5^{-5} from least to greatest. Explain your reasoning.

SIMPLIFYING EXPRESSIONS Simplify. Write the expression using only positive exponents.

24. $6y^{-4}$ 25. $8^{-2} \cdot a^7$ 26. $\frac{9c^3}{c^{-4}}$ 27. $\frac{5b^{-2}}{b^{-3}}$
28. $\frac{8x^3}{2x^9}$ 29. $3d^{-4} \cdot 4d^4$ 30. $m^{-2} \cdot n^3$ 31. $\frac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}}$

32. **OPEN-ENDED** Write two different powers with negative exponents that have the same value. Justify your answer.

MP REASONING In Exercises 33–36, use the table.

33. How many millimeters are in a decimeter?
34. How many micrometers are in a centimeter?
35. How many nanometers are in a millimeter?
36. How many micrometers are in a meter?

Unit of Length	Length (meter)
Decimeter	10^{-1}
Centimeter	10^{-2}
Millimeter	10^{-3}
Micrometer	10^{-6}
Nanometer	10^{-9}

37. **MODELING REAL LIFE** A bacterium is 100 micrometers long. A virus is 1000 times smaller than the bacterium.

- a. Using the table above, find the length of the virus in meters.
b. Is the answer to part (a) *less than*, *greater than*, or *equal to* 1 micrometer?

38. **DIG DEEPER!** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown.

- a. One cubic millimeter of blood contains about 10^4 white blood cells. How many white blood cells are in the donation? ($1 \text{ mm}^3 = 10^{-3} \text{ mL}$)
b. One cubic millimeter of blood contains about 5×10^6 red blood cells. How many red blood cells are in the donation?
c. Compare your answers for parts (a) and (b).

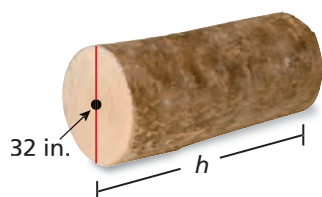
39. **MP PRECISION** Describe how to rewrite a power with a positive exponent as a fraction with a power in the denominator. Use the definition of negative exponents to justify your reasoning.

40. **MP REASONING** The definition of a negative exponent states that $a^{-n} = \frac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.



FINDING A MISSING DIMENSION Find the missing dimension of the cylinder.
Round your answer to the nearest whole number.

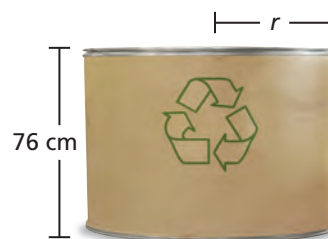
18. Volume = $10,000\pi \text{ in.}^3$



19. Volume = 3785 cm^3



20. Volume = $600,000 \text{ cm}^3$



21. **MODELING REAL LIFE** A cylindrical hazardous waste container has a diameter of 1.5 feet and a height of 1.6 feet. About how many gallons of hazardous waste can the container hold? ($1 \text{ ft}^3 \approx 7.5 \text{ gal}$)

22. **CRITICAL THINKING** How does the volume of a cylinder change when its diameter is halved? Explain.



Round Hay Bale

23. **MP PROBLEM SOLVING** A traditional “square” bale of hay is actually in the shape of a rectangular prism. Its dimensions are 2 feet by 2 feet by 4 feet. How many square bales contain the same amount of hay as one large “round” bale?

24. **MODELING REAL LIFE** A tank on a road roller is filled with water to make the roller heavy. The tank is a cylinder that has a height of 6 feet and a radius of 2 feet. About how many pounds of water can the tank hold? (One cubic foot of water weighs about 62.5 pounds.)



25. **MP REASONING** A cylinder has a surface area of 1850 square meters and a radius of 9 meters. Estimate the volume of the cylinder to the nearest whole number.

26. **DIG DEEPER!** Water flows at 2 feet per second through a cylindrical pipe with a diameter of 8 inches. A cylindrical tank with a diameter of 15 feet and a height of 6 feet collects the water.

- What is the volume (in cubic inches) of water flowing out of the pipe every second?
- What is the height (in inches) of the water in the tank after 5 minutes?
- How many minutes will it take to fill 75% of the tank?

27. **PROJECT** You want to make and sell three different sizes of cylindrical candles. You buy 1 cubic foot of candle wax for \$20 to make 8 candles of each size.
- Design the candles. What are the dimensions of each size of candle?
 - You want to make a profit of \$100. Decide on a price for each size of candle. Explain how you set your prices.



EXAMPLE 4 Modeling Real Life

Find the number x of miles you need to run on Friday so that the mean number of miles run per day is 1.5.

Day	Miles
Monday	2
Tuesday	0
Wednesday	1.5
Thursday	0
Friday	x

Understand the problem.

You are given the number of miles you run each day from Monday through Thursday. You are asked how many miles you need to run on Friday so that your daily average for the five days is 1.5 miles.

Make a plan.

Write and solve an equation using the definition of mean.

Solve and check.

$$\frac{2 + 0 + 1.5 + 0 + x}{5} = 1.5$$

$\frac{\text{sum of the data}}{\text{number of values}} = \text{mean}$

$$\frac{3.5 + x}{5} = 1.5$$

Combine like terms.

$$5 \cdot \frac{3.5 + x}{5} = 5 \cdot 1.5$$

Multiplication Property of Equality

$$3.5 + x = 7.5$$

Simplify.

$$\underline{- 3.5} \qquad \underline{- 3.5}$$

Subtraction Property of Equality

$$x = 4$$

Simplify.

Check You run $2 + 1.5 + 4 = 7.5$ miles in 5 days. So, the mean number of miles run per day is $\frac{7.5}{5} = 1.5$. ✓

So, you need to run 4 miles on Friday.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

Day	Action Figures
Monday	55
Tuesday	45
Wednesday	53
Thursday	44
Friday	x

17. Find the number x of action figures that a small business needs to produce on Friday so that the mean number of action figures produced per day is 50.

18. **DIG DEEPER!** A hard drive is 80% full and has 12,000 MB of free space. One minute of video uses 60 MB of storage. How many minutes of video should be deleted so that the hard drive is 75% full?

19. A teacher spends \$354 on costumes and microphones for six cast members in a play. Each cast member receives a costume that costs \$38 and a microphone that costs $\$c$. What did the teacher spend on each microphone? Justify your answer.

EXAMPLE 5 **Modeling Real Life**



A wildlife refuge is mapped on a coordinate plane, where each grid line represents 1 mile. The refuge has vertices $J(0, 0)$, $K(1, 3)$, and $L(4, 0)$. An expansion of the refuge can be represented by a dilation with a scale factor of 1.5. How much does the area of the wildlife refuge increase?

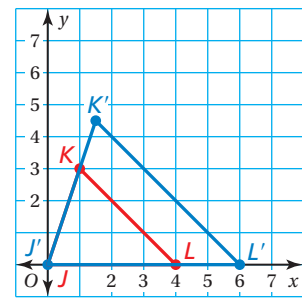
Multiply each x - and y -coordinate by the scale factor 1.5. Then find the area of each figure.

$$(x, y) \longrightarrow (1.5x, 1.5y)$$

$$J(0, 0) \longrightarrow J'(1.5 \cdot 0, 1.5 \cdot 0) \longrightarrow J'(0, 0)$$

$$K(1, 3) \longrightarrow K'(1.5 \cdot 1, 1.5 \cdot 3) \longrightarrow K'(1.5, 4.5)$$

$$L(4, 0) \longrightarrow L'(1.5 \cdot 4, 1.5 \cdot 0) \longrightarrow L'(6, 0)$$



The original figure is a triangle with a base of 4 miles and a height of 3 miles. The image has a base of 6 miles and a height of 4.5 miles. Use the formula for the area of a triangle to find the areas of the original figure and the image.

Original Figure

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(4)(3)$$

$$= 6$$

Write the formula.

Substitute for b and h .

Simplify.

Image

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(6)(4.5)$$

$$= 13.5$$

So, the area of the wildlife refuge increases $13.5 - 6 = 7.5$ square miles.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

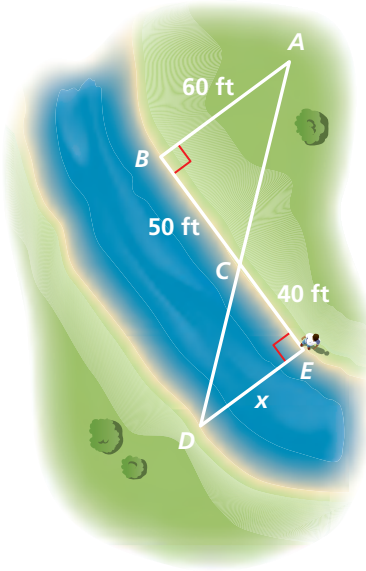


10. A photograph is dilated to fit in a frame, so that its area after the dilation is 9 times greater than the area of the original photograph. What is the scale factor of the dilation? Explain.

11. **DIG DEEPER!** The location of a water treatment plant is mapped using a coordinate plane, where each unit represents 1 foot. The plant has vertices $(0, 0)$, $(0, 180)$, $(240, 180)$, and $(240, 0)$. You dilate the figure with a scale factor of $\frac{1}{3}$. What are the coordinates of the image? What do you need to change so that the image accurately represents the location of the plant? Explain your reasoning.

Indirect measurement uses similar figures to find a missing measure when it is difficult to find directly.

EXAMPLE 3 Modeling Real Life



You plan to cross a river and want to know how far it is to the other side. You take measurements on your side of the river and make the drawing shown. What is the distance x across the river?

Notice that $\angle B$ and $\angle E$ are right angles, so they are congruent. $\angle ACB$ and $\angle DCE$ are vertical angles, so they are congruent. Because two angles in $\triangle ABC$ are congruent to two angles in $\triangle DEC$, the third angles are also congruent and the triangles are similar.

Ratios of corresponding side lengths in similar triangles are equivalent. So, the ratios $x : 60$ and $40 : 50$ are equivalent. Write and solve a proportion to find x .

$$\frac{x}{60} = \frac{40}{50}$$

Write a proportion.

$$60 \cdot \frac{x}{60} = 60 \cdot \frac{40}{50}$$

Multiplication Property of Equality

$$x = 48$$

Simplify.

So, the distance across the river is 48 feet.

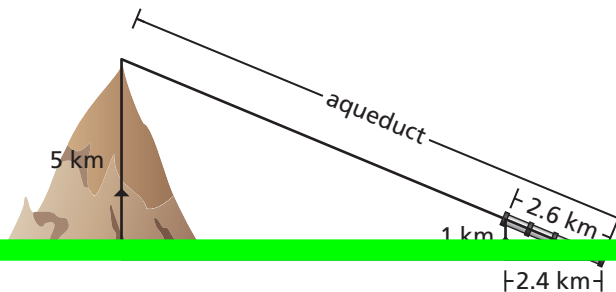


Self-Assessment for Problem Solving

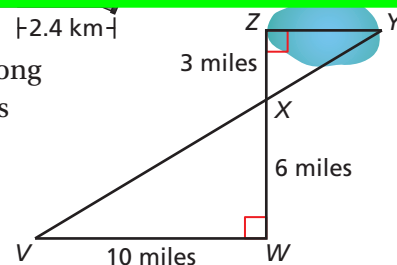
Solve each exercise. Then rate your understanding of the success criteria in your journal.



6. **DIG DEEPER!** Engineers plan to construct an aqueduct to transport water from the top of a ridge to farmland. A portion of the project is complete. Find the length of the entire aqueduct.

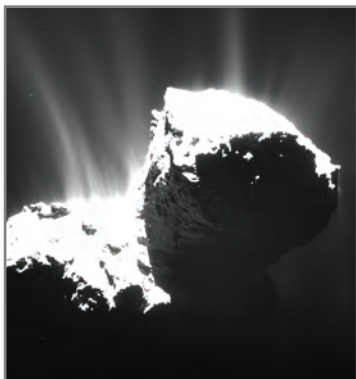


7. You want to go on a swamp tour. How long does it take a swamp vehicle that travels at 3.2 miles per hour to travel across the swamp, from point Z to point Y? Justify your answer.



Indicator 2c - #7 is non-routine because students use similar triangles to find the width of the swamp, ZY, and then use that to answer a bigger question, how long the swamp tour will last.

EXAMPLE 4 Modeling Real Life



© ESA/Rosetta/MPS for OSIRIS TeamMPS/
UPD/LAM/IAA/SSO/INTA/UPM/DASP/IDA

The table shows the distance y (in miles) of a space probe from a comet x minutes after it begins its approach. The points in the table lie on a line. Find and interpret the slope of the line.

x	1	4	7	10
y	8	6	4	2

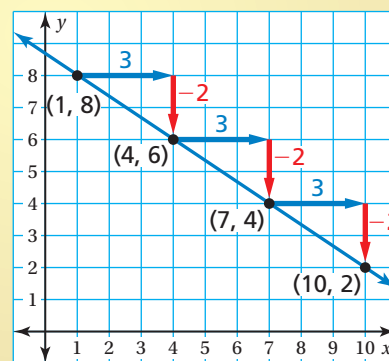
Choose any two points from the table and use the slope formula.

Use the points $(x_1, y_1) = (1, 8)$ and $(x_2, y_2) = (4, 6)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 8}{4 - 1} \\ &= \frac{-2}{3}, \text{ or } -\frac{2}{3} \end{aligned}$$

▶ The slope is $-\frac{2}{3}$. So, the distance between the probe and the comet decreases 2 miles every 3 minutes, or $\frac{2}{3}$ mile every minute.

Check



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

x	y
1	0.5
2	1
3	1.5
4	2



- The table shows the lengths y (in inches) of your hair x months after your last haircut. The points in the table lie on a line. Find and interpret the slope of the line. After how many months is your hair 4 inches long?
- A customer pays an initial fee and a daily fee to rent a snowmobile. The total payment for 3 days is 92 dollars. The total payment for 5 days is 120 dollars. What is the daily fee? Justify your answer.
- You in-line skate from an elevation of 720 feet to an elevation of 750 feet in 30 minutes. Your friend in-line skates from an elevation of 600 feet to an elevation of 690 feet in one hour. Compare your rates of change in elevation.

EXAMPLE 4 Modeling Real Life



You buy 8 hostas and 15 daylilies for \$193. Your friend buys 3 hostas and 12 daylilies for \$117. Find the cost of each daylily.

Use a verbal model to write a system of linear equations. Let x represent the cost of each hosta and let y represent the cost of each daylily.

Number of hostas	•	Cost of each hosta, x	+	Number of daylilies	•	Cost of each daylily, y	=	Total cost
------------------	---	-------------------------	---	---------------------	---	---------------------------	---	------------

The system is:

$$\begin{aligned} 8x + 15y &= 193 && \text{Equation 1 (You)} \\ 3x + 12y &= 117 && \text{Equation 2 (Your friend)} \end{aligned}$$



Step 1: One way to find the cost of each daylily is to eliminate the x -terms and solve for y . Multiply Equation 1 by 3 and Equation 2 by 8.

$$\begin{aligned} 8x + 15y &= 193 && \xrightarrow{\text{Multiply by 3.}} && 24x + 45y = 579 && \text{Revised Equation 1} \\ 3x + 12y &= 117 && \xrightarrow{\text{Multiply by 8.}} && 24x + 96y = 936 && \text{Revised Equation 2} \end{aligned}$$

Step 2: Subtract the revised equations.

$$\begin{aligned} 24x + 45y &= 579 && \text{Revised Equation 1} \\ 24x + 96y &= 936 && \text{Revised Equation 2} \\ \hline -51y &= -357 && \text{Subtract the equations.} \end{aligned}$$

Step 3: Solving the equation $-51y = -357$ gives $y = 7$.

So, each daylily costs \$7.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

- 15.** A fitness instructor purchases exercise bikes and treadmills for two gyms. For the first gym, 2 exercise bikes and 3 treadmills cost \$2200. For the second gym, 3 exercise bikes and 4 treadmills cost \$3000. How much does a treadmill cost?



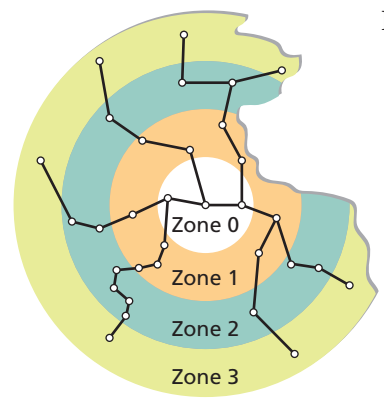
- 16. DIG DEEPER!** At your school, cooking club members raise \$5 per member for a charity and woodshop club members raise \$10 per member for a different charity. The cooking club has three times as many members as the woodshop club. The difference of the number of members in the two clubs is 12 members. How much does each club raise?

EXAMPLE 3 Modeling Real Life

Input, Zone	Output, Price
0	\$2.00
1	\$3.50
2	\$5.00
3	\$6.50

The mapping diagram represents the prices of one-way subway tickets to different zones of a city.

- a. Is the price of a subway ticket a function of the zone number?
Each input has exactly one output.
- So, the price of a subway ticket is a function of the zone number.
- b. Describe the relationship between the price and the zone number.



Input, Zone	Output, Price
0	\$2.00
1	\$3.50
2	\$5.00
3	\$6.50

+1 +1.50
+1 +1.50
+1 +1.50

As each input increases by 1, the output increases by \$1.50.

So, the price of a one-way subway ticket increases by \$1.50 for each additional zone traveled.

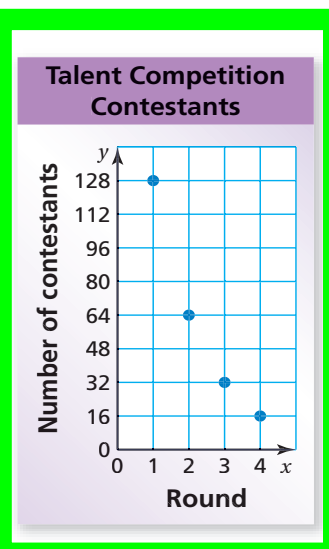


Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. The mapping diagram represents the costs of reserving a hotel room for different numbers of nights.
- a. Is the cost a function of the number of nights reserved?
- b. Describe the relationship between the

Input, Nights	Output, Cost
1	\$85
2	\$170
3	\$255
4	\$340



10. **DIG DEEPER!** The graph represents the number of contestants in each round of a talent competition.
- a. Is the number of contestants a function of the round number?
- b. Predict the number of contestants in the talent competition during Round 7. Explain your reasoning.

ROTATING A FIGURE The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image.

13. $A(2, -2), B(4, -1), C(4, -3), D(2, -4)$
90° counterclockwise about the origin

14. $F(1, 2), G(3, 5), H(3, 2)$
180° about the origin

15. $J(-4, 1), K(-2, 1), L(-4, -3)$
90° clockwise about the origin

16. $P(-3, 4), Q(-1, 4), R(-2, 1), S(-4, 1)$
270° clockwise about the origin

17. $W(-6, -2), X(-2, -2), Y(-2, -6), Z(-5, -6)$
270° counterclockwise about the origin

18. $A(1, -1), B(5, -6), C(1, -6)$
90° counterclockwise about the origin

19. **YOU BE THE TEACHER** The vertices of a triangle are $A(4, 4)$, $B(1, -2)$, and $C(-3, 0)$. Your friend finds the coordinates of the image after a rotation 90° clockwise about the origin. Is your friend correct? Explain your reasoning.

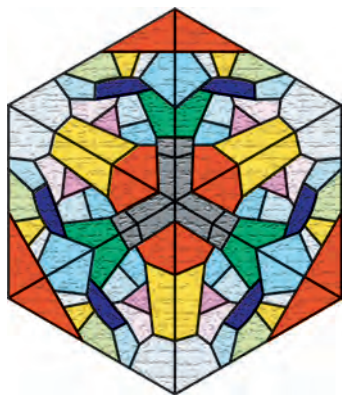
$A(4, 4) \longrightarrow A'(-4, 4)$
 $B(1, -2) \longrightarrow B'(2, 1)$
 $C(-3, 0) \longrightarrow C'(0, -3)$

Indicator 2c - #21-23 are non-routine because students are introduced to the term rotational symmetry and use what they know about rotations to determine whether the figures have rotation symmetry.

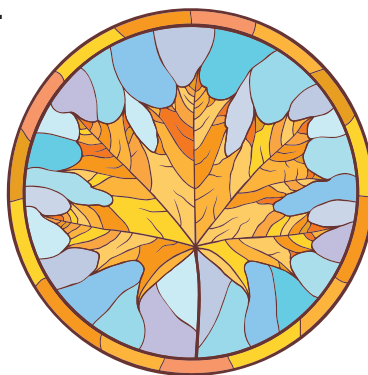
PROBLEM SOLVING A game show contestant spins the wheel shown. The arrow remains in a fixed position as the wheel rotates. The wheel stops spinning, resulting in a rotation 270° clockwise about the center of the wheel. What is the result?

MP PATTERNS A figure has *rotational symmetry* if a rotation of 180° or less produces an image that fits exactly on the original figure. Determine whether the figure has rotational symmetry. Explain your reasoning.

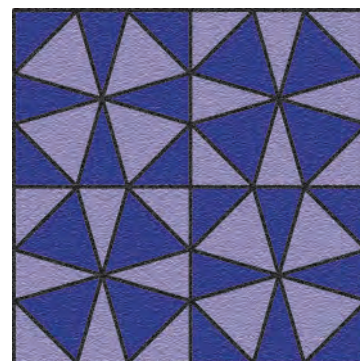
21.



22.



23.

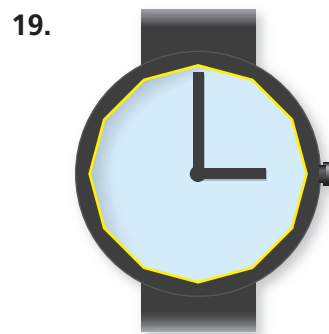
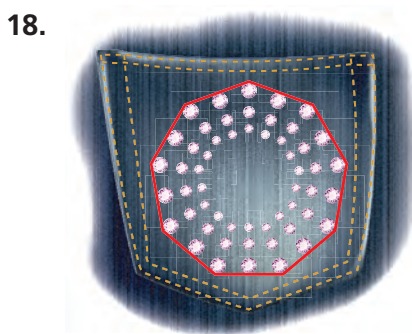
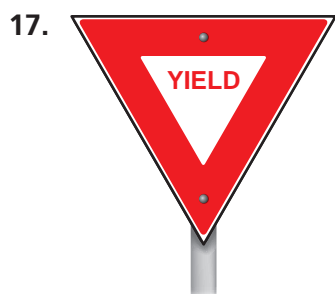


USING MORE THAN ONE TRANSFORMATION The vertices of a figure are given. Find the coordinates of the image after the transformations given.

24. $R(-7, -5), S(-1, -2), T(-1, -5)$
Rotate 90° counterclockwise about the origin. Then translate 3 units left and 8 units up.

25. $J(-4, 4), K(-3, 4), L(-1, 1), M(-4, 1)$
Reflect in the x -axis, and then rotate 180° about the origin.

FINDING A MEASURE Find the measure of each interior angle of the regular polygon.



20. **YOU BE THE TEACHER** Your friend finds the measure of each interior angle of a regular 20-gon. Is your friend correct? Explain your reasoning.



21. **MODELING REAL LIFE** A fire hydrant bolt is in the shape of a regular pentagon.

- What is the measure of each interior angle?
- RESEARCH** Why are fire hydrants made this way?

$$\begin{aligned} S &= (n - 2) \cdot 180^\circ \\ &= (20 - 2) \cdot 180^\circ \\ &= 18 \cdot 180^\circ \\ &= 3240^\circ \\ 3240^\circ \div 18 &= 180^\circ \\ \text{The measure of each} \\ \text{interior angle is } 180^\circ. \end{aligned}$$

22. **MP PROBLEM SOLVING** The interior angles of a regular polygon each measure 165° . How many sides does the polygon have?

23. **MP STRUCTURE** A molecule can be represented by a polygon with interior angles that each measure 120° . What polygon represents the molecule? Does the polygon have to be regular? Justify your answers.



24. **MP PROBLEM SOLVING** The border of a Susan B. Anthony dollar is in the shape of a regular polygon.

- How many sides does the polygon have?
- What is the measure of each interior angle of the border? Round your answer to the nearest degree.

25. **MP REASONING** The center of the stained glass window is in the shape of a regular polygon. What are the measures of the interior angles of the green triangle?



26. **GEOMETRY** Draw a pentagon that has two right interior angles, two 45° interior angles, and one 270° interior angle.
27. **DIG DEEPER!** The floor of a gazebo is in the shape of a heptagon, a seven-sided polygon. Four of the interior angles measure 135° . The other interior angles have equal measures. Find their measures.

31. **PROJECT** The guidelines for a wheelchair ramp suggest that the ratio of the rise to the run be no greater than 1 : 12.

- MP CHOOSE TOOLS** Find a wheelchair ramp in your school or neighborhood. Measure its slope. Does the ramp follow the guidelines?
- Design a wheelchair ramp that provides access to a building with a front door that is 2.5 feet above the sidewalk. Illustrate your design.



USING AN EQUATION Use an equation to find the value of k so that the line that passes through the given points has the given slope.

32. $(1, 3), (5, k); m = 2$

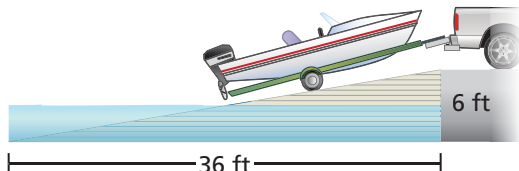
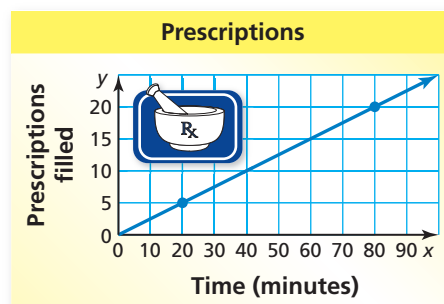
33. $(-2, k), (2, 0); m = -1$

34. $(-4, k), (6, -7); m = -\frac{1}{5}$

35. $(4, -4), (k, -1); m = \frac{3}{4}$

36. **MODELING REAL LIFE** The graph shows the numbers of prescriptions filled over time by a pharmacy.

- Find the slope of the line.
- Explain the meaning of the slope as a rate of change.



37. **CRITICAL THINKING** Which is steeper: the boat ramp, or a road with a 12% grade? Explain. (Note: Road grade is the vertical increase divided by the horizontal distance.)

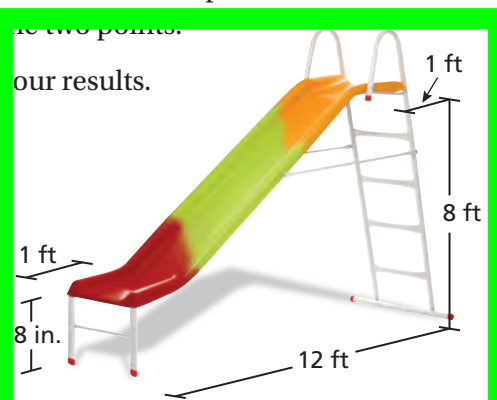
38. **MP REASONING** Do the points $A(-2, -1)$, $B(1, 5)$, and $C(4, 11)$ lie on the same line? Without using a graph, how do you know?

39. **MP PROBLEM SOLVING** A small business earns a profit of \$6500 in January and \$17,500 in May. What is the rate of change in profit for this time period? Justify your answer.

40. **MP STRUCTURE** Choose two points in the coordinate plane. Use the slope formula to find the slope of the line that passes through the two points. Then find the slope using the formula $\frac{y_1 - y_2}{x_1 - x_2}$. Compare your results.

41. **DIG DEEPER!** The top and the bottom of the slide are level with the ground, which has a slope of 0.

- What is the slope of the main portion of the slide?
- Describe the change in the slope when the bottom of the slide is only 12 inches above the ground. Explain your reasoning.



21. **MODELING REAL LIFE** A hemisphere-shaped mole has a diameter of 5.7 millimeters and a surface area of about 51 square millimeters. The radius of the mole doubles. Estimate the new surface area of the mole.



22. **MP REASONING** The volume of a 1968 Ford Mustang GT engine is 390 cubic inches. Which scale model of the Mustang has the greater engine volume, a 1 : 18 scale model or a 1 : 24 scale model? How much greater is it?

23. **DIG DEEPER!** You have a small marble statue of Wolfgang Mozart. It is 10 inches tall and weighs 16 pounds. The original marble statue is 7 feet tall.

- Estimate the weight of the original statue. Explain your reasoning.
- If the original statue were 20 feet tall, how much would it weigh?



Wolfgang Mozart

24. **MP REPEATED REASONING** The nesting dolls are similar. The largest doll is 7 inches tall. Each of the other dolls is 1 inch shorter than the next larger doll. Make a table that compares the surface areas and the volumes of the seven dolls.



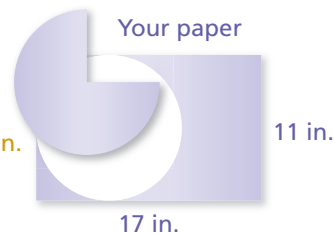
25. **MP PRECISION** You and a friend make paper cones to collect beach glass. You cut out the largest possible three-fourths circle from each piece of paper.



- Are the cones similar? Explain your reasoning.
- Your friend says that because your sheet of paper is twice as large, your cone will hold exactly twice the volume of beach glass. Is this true? Explain your reasoning.



Friend's paper



Your paper

5 Connecting Concepts

► Using the Problem-Solving Plan

1. An animal shelter has a total of 65 cats and dogs. The ratio of cats to dogs is 6 : 7. Find the number of cats and the number of dogs in the shelter.

Understand the problem.

You know the total number of cats and dogs in an animal shelter, and the ratio of cats to dogs. You are asked to find the number of cats and the number of dogs in the shelter.

Make a plan.

Write a system of equations. Use the total number of cats and dogs to write an equation relating the number x of cats and the number y of dogs. Use the ratio of cats to dogs to write a second equation. Then solve the system.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The measure of $\angle 1$ is 15 degrees less than the measure of $\angle 2$. Find the measure of each angle formed by the intersecting lines. Justify your answer.

Indicator 2c - #3 is non-routine because students must use the ratio of the perimeters of the parks to find the ratio of the areas. Then, students need to create a system and solve to find the areas of Park A and Park B.



3. A landscaper plants grass seed over the entire area of two parks that are similar in shape. The ratio of the perimeter of Park A to the perimeter of Park B is 2 : 1. The parks have a combined area of 9000 square feet. How many square feet does the landscaper cover with grass seed at Park A? Park B? Justify your answer.

Performance Task



Mixing Alloys

At the beginning of this chapter, you watched a STEAM Video called "Gold Alloys." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



8

Connecting Concepts

► Using the Problem-Solving Plan

- Atoms are made of protons, neutrons, and electrons. The table shows the numbers of protons and the masses of several atoms. Use a line of best fit to estimate the mass (in grams) of an atom that has 29 protons.

Protons, x	Mass (gram), y
1	1.67×10^{-24}
5	1.79×10^{-23}
53	2.11×10^{-22}
20	6.65×10^{-23}
14	4.66×10^{-23}
3	1.15×10^{-23}
40	1.51×10^{-22}
16	5.32×10^{-23}

Understand the problem.

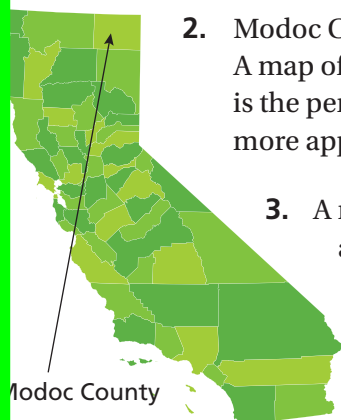
You know the numbers of protons and the masses of several atoms. You are asked to use the line of best fit to estimate the mass of an atom that has 29 protons.

Make a plan.

Use a graphing calculator to find an equation of the line of best fit. Then evaluate the equation when $x = 29$.

Solve and check.

Use the plan to solve the problem. Then check your solution.



- Modoc County, California, is 74.9 miles long and 56.2 miles wide. A map of the county is drawn using a scale factor of 2.11×10^{-6} . What is the perimeter of the county on the map? Express your answer using more appropriate units.

- A research company estimates that in the United States, about 8.37×10^7 adult males and 6.59×10^7 adult females watch NFL football, while 3.13×10^7 adult males and 5.41×10^7 adult females do *not* watch NFL football. Organize the results in a two-way table. Include the marginal frequencies.

Performance Task

Elements in the Universe



At the beginning of this chapter, you watched a STEAM Video called "Carbon Atoms." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



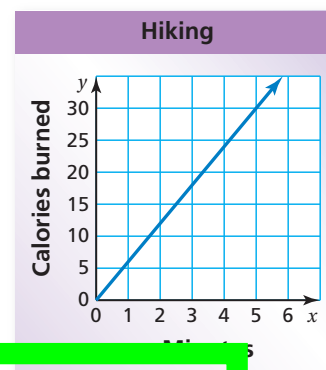
13. **INTERPRETING A LINEAR FUNCTION** The table shows the percent y (in decimal form) of battery power remaining x hours after you turn on a laptop computer.

Hours, x	0	2	4
Power Remaining, y	1.0	0.6	0.2

- Write and graph a linear function that relates y to x .
- Interpret the slope, the x -intercept, and the y -intercept.
- After how many hours is the battery power at 75%?

14. **MODELING REAL LIFE** The number y of calories burned after x minutes of kayaking is represented by the linear function $y = 4.5x$. The graph shows the number of calories burned by hiking.

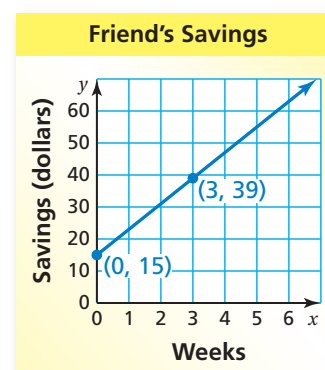
- Which activity burns more calories per minute?
- You perform each activity for 45 minutes. How many total calories do you burn? Justify your answer.



15. **DIG DEEPER!** You and a friend race each other. You give your friend a 50-foot head start. The distance y (in feet) your friend runs after x seconds is represented by the linear function $y = 14x + 50$. The table shows your distance at various times throughout the race. For what distances will you win the race? Explain.

Time (seconds), x	2	4	6	8
Distance (feet), y	38	76	114	152

16. **MP REASONING** You and your friend are saving money to buy bicycles that cost \$175 each. You have \$45 to start and save an additional \$5 each week. The graph shows the amount y (in dollars) that your friend has after x weeks. Who can buy a bicycle first? Justify your answer.



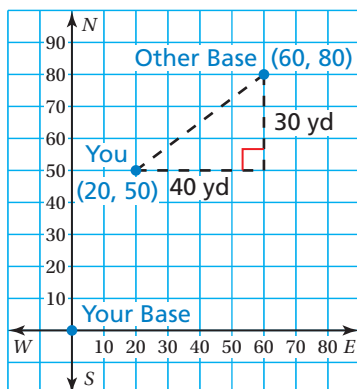
17. **CRITICAL THINKING** Is every linear equation a linear function? Explain your reasoning.



18. **MP PROBLEM SOLVING** The heat index is calculated using the relative humidity and the temperature. For every 1 degree increase in the temperature from 94°F to 97°F at 75% relative humidity, the heat index rises 4°F. On a summer day, the relative humidity is 75%, the temperature is 94°F, and the heat index is 124°F. Estimate the heat index when the relative humidity is 75% and the temperature is 100°F. Use a function to justify your answer.

EXAMPLE 5 Modeling Real Life

You play capture the flag. You are 50 yards north and 20 yards east of your team's base. The other team's base is 80 yards north and 60 yards east of your base. How far are you from the other team's base?



Step 1: Draw the situation in a coordinate plane. Let the origin represent your team's base. From the descriptions, you are at (20, 50) and the other team's base is at (60, 80).

Step 2: Draw a right triangle with a hypotenuse that represents the distance between you and the other team's base. The lengths of the legs are 30 yards and 40 yards.

Step 3: Use the Pythagorean Theorem to find the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

Write the Pythagorean Theorem.

$$30^2 + 40^2 = c^2$$

Substitute 30 for a and 40 for b .

$$900 + 1600 = c^2$$

Evaluate the powers.

$$2500 = c^2$$

Add.

$$50 = c$$

Take the positive square root of each side.

So, you are 50 yards from the other team's base.

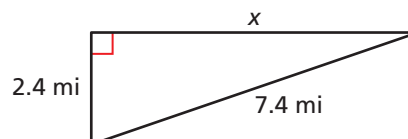


Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



13. A zookeeper knows that an escaped red panda is hiding somewhere in the triangular region shown. What is the area (in square miles) that the zookeeper needs to search? Explain.



14. **DIG DEEPER!** Objects detected by radar are plotted in a coordinate plane where each unit represents 1 mile. The point (0, 0) represents the location of a shipyard. A cargo ship is traveling at a constant speed and in a constant direction parallel to the coastline. At 9 A.M., the radar shows the cargo ship at (0, 15). At 10 A.M., the radar shows the cargo ship at (16, 15). How far is the cargo ship from the shipyard at 4 P.M.? Explain.

Professional Development

Rigorous by Design

4.7 Writing Equations in Point-Slope Form

Learning Target: Write equations of lines in point-slope form.

Success Criteria:

- I can use a point on a line and the slope to write an equation of the line.
- I can use any two points to write an equation of a line.
- I can write equations in point-slope form to solve real-life problems.

EXPLORATION 1 **Deriving an Equation**

Work with a partner. Let (x_1, y_1) represent a specific point on a line. Let (x, y) represent any other point on the line.

Math Practice

Conceptual Understanding

Explorations help students reach a deeper level of conceptual understanding.

Procedural Fluency

Lessons follow a gradual release model and give teachers opportunities for flexible instruction, providing opportunities for all levels of learners to attain procedural fluency. Self-Assessments provide students the opportunity to assess their understanding of the success criteria, taking ownership of their learning.

EXAMPLE 2 Writing an Equation Using Two Points

Write an equation in slope-intercept form of the line that passes through the given points.

Find the slope. Use the points $(2, 4)$ and $(5, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{5 - 2} = \frac{-6}{3} = -2$$

x	y
-1	10
2	4
5	-2

Then use the slope $m = -2$ and the point $(2, 4)$ to write an equation of the line.

$$y - y_1 = m(x - x_1) \quad \text{Write the point-slope form.}$$

$$y - 4 = -2(x - 2) \quad \text{Substitute } -2 \text{ for } m, 2 \text{ for } x_1, \text{ and } 4 \text{ for } y_1.$$

$$y - 4 = -2x + 4 \quad \text{Distributive Property}$$

$$y = -2x + 8 \quad \text{Write in slope-intercept form.}$$

Try It Write an equation in slope-intercept form of the line that passes through the given points.

4. $(-2, 1), (3, -4)$

5.

x	-5	-3	-1
y	5	3	1

EXAMPLE 3 Modeling Real Life

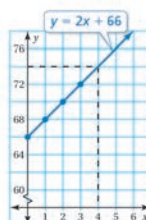


A tropical storm becomes a hurricane when wind speeds are at least 74 miles per hour.

The wind speed y (in miles per hour) of a tropical storm is $y = 2x + 66$, where x is the number of hours after the storm enters the Gulf of Mexico. When does the storm become a hurricane?

Use a graph to find the time it takes for the storm to become a hurricane. Make a table of values. Plot the ordered pairs and draw a line through the points.

x	$y = 2x + 66$	y	(x, y)
0	$y = 2(0) + 66$	66	(0, 66)
1	$y = 2(1) + 66$	68	(1, 68)
2	$y = 2(2) + 66$	70	(2, 70)
3	$y = 2(3) + 66$	72	(3, 72)



Another Method Use the equation $y = 2x + 66$ to find x when $y = 74$.

$$74 = 2x + 66$$

$$8 = 2x$$

$$4 = x \quad \checkmark$$

From the graph, you can see that $y = 74$ when $x = 4$.

So, the storm becomes a hurricane 4 hours after it enters the Gulf of Mexico.

Application

Modeling Real Life examples bring problem solving into the classroom, promoting application of concepts and skills and reaching higher levels of DOK.

Getting Ready for Chapter

Indicator 2d - In this Chapter Exploration, one aspect of rigor (conceptual understanding) is emphasized. The focus is on using algebra tiles to introduce the conceptual understanding for solving equations.

Chapter Exploration

1. Work with a partner. Use algebra tiles to model and solve each equation.

a. $x + 3 = -3$

$\text{+} = +1$ $\text{−} = -1$ $\text{+} = x$

$\text{+} \text{+} \text{+} \text{+} = \text{−} \text{−} \text{−}$

Model the equation $x + 3 = -3$.

$\text{+} \text{+} \text{+} \text{+} = \text{−} \text{−} \text{−}$

Add three -1 tiles to each side.

$\text{+} = \text{−} \text{−} \text{−}$

Remove the zero pairs from the left side.

$\text{+} =$

Write the solution of the equation.

b. $-3 = x - 2$

$\text{−} \text{−} \text{−} = \text{+} \text{−} \text{−}$

Model the equation $-3 = x - 2$.

$\text{−} \text{−} \text{−} = \text{+} \text{−} \text{−}$

Add two $+1$ tiles to each side.

$\text{−} = \text{+}$

Remove the zero pairs from the each side.

$= \text{+}$

Write the solution of the equation.

c. $x - 4 = 1$

d. $x + 5 = -2$

e. $-7 = x + 4$

f. $x + 6 = 7$

g. $-5 + x = -3$

h. $-4 = x - 4$

2. **WRITE GUIDELINES** Work with a partner. Use your models in Exercise 1 to summarize the *algebraic steps* that you can use to solve an equation.

Vocabulary

The following vocabulary term is defined in this chapter. Think about what the term might mean and record your thoughts.

literal equation

Getting Ready for Chapter

4

Chapter Exploration

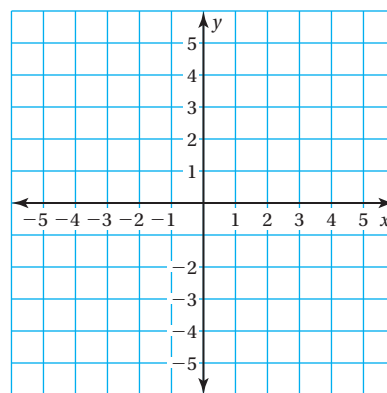
1. Work with a partner.

- a. Use the equation $y = \frac{1}{2}x + 1$ to complete the table. (Choose any two x -values and find the y -values.)

	Solutions	
x		
$y = \frac{1}{2}x + 1$		

- b. Write the two ordered pairs given by the table. These are called *solutions* of the equation.

- c. **PRECISION** Plot the two solutions. Draw a line *exactly* through the points.



- d. Find a different point on the line. Check that this point is a solution of the equation $y = \frac{1}{2}x + 1$.

- e. **LOGIC** Do you think it is true that *any* point on the line is a solution of the equation $y = \frac{1}{2}x + 1$? Explain.

- f. Choose five additional x -values for the table below. (Choose both positive and negative x -values.) Plot the five corresponding solutions. Does each point lie on the line?

	Solutions				
x					
$y = \frac{1}{2}x + 1$					

- g. **LOGIC** Do you think it is true that any solution of the equation $y = \frac{1}{2}x + 1$ is a point on the line? Explain.

- h. Why do you think $y = ax + b$ is called a *linear equation*?

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

linear equation

slope

y -intercept

solution of a linear equation

x -intercept

Getting Ready for Chapter

6

Chapter Exploration

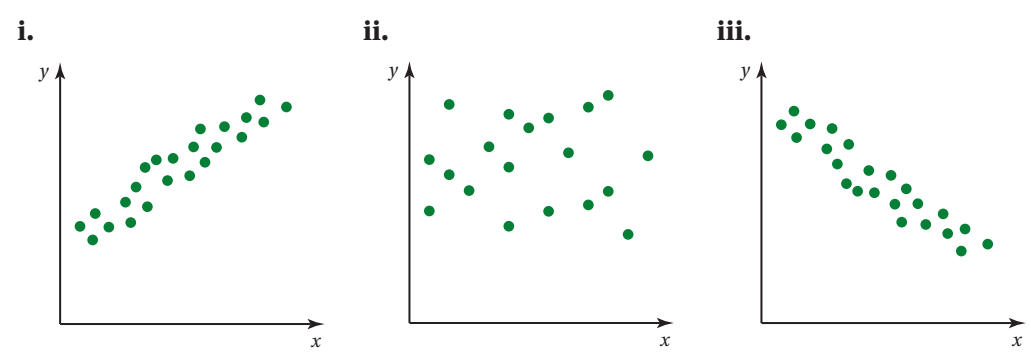
1. Work with a partner. The table shows the number of absences and the final grade for each student in a sample.

- a. Write the ordered pairs from the table. Then plot them in a coordinate plane.
- b. Describe the relationship between absences and final grade.
- c. **MODELING** A student has been absent 6 days. Use the data to predict the student's final grade. Explain how you found your answer.

Absences	Final Grade
0	95
3	88
2	90
5	83
7	79
9	70
4	85
1	94
10	65
8	75

2. Work with a partner. Match the data sets with the most appropriate scatter plot. Explain your reasoning.

- a. month of birth and birth weight for infants at a day care
- b. quiz score and test score of each student in a class
- c. age and value of laptop computers



Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

- scatter plot
- line of fit
- two-way table
- joint frequency

Getting Ready for Chapter



Chapter Exploration

1. Work with a partner. Write each distance as a whole number. Which numbers do you know how to write in words? For instance, in words, 10^2 is equal to *one hundred*.

- a. 10^{27} meters:
diameter of
the observable
universe



- b. 10^{21} meters:
diameter of
the Milky Way
galaxy



- c. 10^{16} meters:
diameter of
the solar
system



- d. 10^7 meters:
diameter of
Earth



- e. 10^4 meters:
diameter of
Halley's Comet



- f. 10^3 meters:
diameter of
a meteor crater



2. Work with a partner. Write the numbers of wives, sacks, cats, and kits as powers.

*As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?*

Nursery Rhyme, 1730



Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

power

base of a power

exponent of a power

scientific notation

2

Connecting Concepts

Using the Problem-Solving Plan

1. A scale drawing of a helipad uses a scale of 1 ft : 20 ft. The scale drawing has an area of 6.25 square feet. What is the area of the actual helipad?

Understand the problem.

You know the scale of the drawing and the area of the helipad in the drawing. You are asked to find the area of the actual helipad.

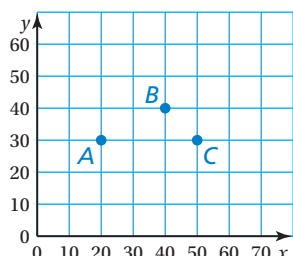


Make a plan.

A scale drawing is similar to the actual object. So, use the scale 1 ft : 20 ft and the ratio $6.25 \text{ ft}^2 : A \text{ ft}^2$ to write and solve a proportion that represents the area A of the actual helipad.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The locations of three cargo ships are shown in the coordinate plane. Each ship travels at the same speed in the same direction. After 1 hour, the x - and y -coordinates of Ship A increase 80%. Use a translation to describe the change in the locations of the ships. Then find the new coordinates of each ship.



3. All circles are similar. A circle with a radius of 2 inches is dilated, resulting in a circle with a circumference of 22π inches. What is the scale factor? Justify your answer.

Performance Task



Master Puppeteer

At the beginning of this chapter, you watched a STEAM Video called "Shadow Puppets." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



4 Connecting Concepts

Indicator 2d - In the exercises and the Performance Task, one aspect of rigor (application) is emphasized. Students use their learning from the chapter and previous chapters to complete the exercises.

▶ Using the Problem-Solving Plan

1. Every item in a retail store is on sale for 40% off. Write and graph an equation that represents the sale price y of an item that has an original price of x dollars.

Understand the problem.

You know the percent discount of items in a retail store. You are asked to write and graph an equation that represents the sale price of an item that has an original price of x dollars.



Make a plan.

Selling an item for 40% off is the same as selling an item for 60% of its original price. Use this information to write and graph an equation that represents the situation.

Solve and check.

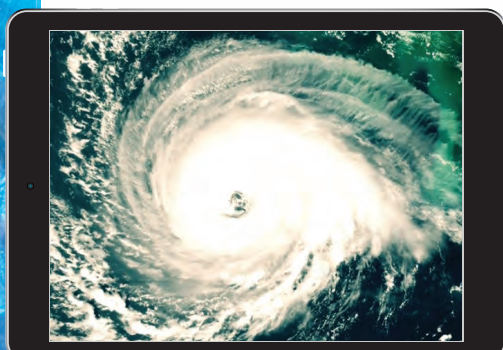
Use the plan to solve the problem. Then check your solution.

2. Two supplementary angles have angle measures of x° and y° . Write and graph an equation that represents the relationship between the measures of the angles.
3. A mechanic charges a diagnostic fee plus an hourly rate. The table shows the numbers of hours worked and the total costs for three customers. A fourth customer pays \$285. Find the number of hours that the mechanic worked for the fourth customer.

Hours, x	1	3	5
Cost, y (dollars)	90	210	330

Performance Task

Anatomy of a Hurricane



At the beginning of this chapter, you watched a STEAM Video called "Hurricane!" You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



5 Connecting Concepts

► Using the Problem-Solving Plan

1. An animal shelter has a total of 65 cats and dogs. The ratio of cats to dogs is 6 : 7. Find the number of cats and the number of dogs in the shelter.

Understand the problem.

You know the total number of cats and dogs in an animal shelter, and the ratio of cats to dogs. You are asked to find the number of cats and the number of dogs in the shelter.

Make a plan.

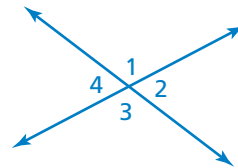
Write a system of equations. Use the total number of cats and dogs to write an equation relating the number x of cats and the number y of dogs. Use the ratio of cats to dogs to write a second equation. Then solve the system.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The measure of $\angle 1$ is 15 degrees less than two times the measure of $\angle 2$. Find the measure of each of the four angles formed by the intersecting lines. Justify your answer.



3. A landscaper plants grass seed over the entire area of two parks that are similar in shape. The ratio of the perimeter of Park A to the perimeter of Park B is 2 : 1. The parks have a combined area of 9000 square feet. How many square feet does the landscaper cover with grass seed at Park A? Park B? Justify your answer.

Performance Task

Mixing Alloys

At the beginning of this chapter, you watched a STEAM Video called "Gold Alloys." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



6 Connecting Concepts

► Using the Problem-Solving Plan

1. You randomly survey middle school students about whether they prefer action, comedy, or animation movies. The two-way table shows the results. Estimate the probability that a randomly selected middle school student prefers action movies.

		Grade		
		6	7	8
Genre	Action	12	18	10
	Comedy	8	6	3
	Animation	9	11	14

Understand the problem.

You know the results of a survey about movie preference. You are asked to estimate the probability that a randomly selected middle school student prefers action movies.

Make a plan.

Find the marginal frequencies for the data. Then use the marginal frequencies to find the probability that a randomly selected middle school student prefers action movies.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. An equation of the line of best fit for a data set is $y = -0.68x + 2.35$. Describe what happens to the slope and the y -intercept of the line when each y -value in the data set increases by 7.
3. On a school field trip, there must be 1 adult chaperone for every 16 students. There are 8 adults who are willing to be a chaperone for the trip, but only the number of chaperones that are necessary will attend. In a class of 124 students, 80 attend the trip. Make a two-way table that represents the data.



Performance Task

Cost vs. Fuel Economy

At the beginning of this chapter, you watched a STEAM Video called "Fuel Economy." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



Indicator 2e - The front matter provides a correlation aligning the MP labels and other headings in the Student Edition with the Standards for Mathematical Practice.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.



- Multiple representations are presented to help students move from concrete to representative and into abstract thinking.
- *Modeling Real Life Examples* and **PROBLEM-SOLVING** exercises encourage students to use problem-solving strategies, such as drawing a diagram, making a table, and solving a simpler problem. They also use a formal problem-solving plan: understand the problem, make a plan, and solve and check.

2 Reason abstractly and quantitatively.

- Visual problem-solving models help students create a coherent representation of the problem.
- *Explorations* allow students to investigate concepts to understand the **REASONING** behind the rules.
- Questions ask students to explain and justify their **REASONING**.
- Questions encourage students to apply **NUMBER SENSE** and formulate consistent and appropriate **REASONING**.

3 Construct viable arguments and critique the reasoning of others.

- *Explorations* help students make conjectures, use **LOGIC**, and **CONSTRUCT ARGUMENTS** to support their conjectures.
- Exercises, such as **YOU BE THE TEACHER**; **DIFFERENT WORDS, SAME QUESTION**; and **WHICH ONE DOESN'T BELONG?**, provide students the opportunity to critique the reasoning of others.

4 Model with mathematics.

- Real-life situations are translated into diagrams, tables, equations, and graphs to help students analyze relations and to draw conclusions.
- Real-life problems are provided to help students apply the mathematics they are learning to everyday life.
- **MODELING REAL LIFE** examples and exercises help students see that math is used across content areas, other disciplines, and in their own experiences.

5 Use appropriate tools strategically.

- *Graphic Organizers* support the thought process of what, when, and how to solve problems.
- A variety of tools, such as number lines and graph paper, manipulatives, and digital tools, are available as students **CHOOSE TOOLS** and begin **USING TOOLS** to solve problems.

6 Attend to precision.

- **PRECISION** exercises encourage students to formulate consistent and appropriate reasoning.
- Cooperative learning opportunities support precise communication.

7 Look for and make use of structure.

- *Learning Targets* and *Success Criteria* at the start of each chapter and section help students understand what they are going to learn.
- *Explorations* provide students the opportunity to see **PATTERNS** and **STRUCTURE** in mathematics.
- Real-life problems help students use the **STRUCTURE** of mathematics to break down and solve more difficult problems.

8 Look for and express regularity in repeated reasoning.

- Opportunities are provided to help students make generalizations through **REPEATED REASONING**.
- Students are continually encouraged to check for reasonableness in their solutions.

vi

The colored words above are used throughout the program to indicate exercises that correlate to the Standards for Mathematical Practice.

EXAMPLE 3 Modeling Real Life

An airplane leaves Miami and travels around the Bermuda Triangle as shown in the diagram. What is the measure of the interior angle at Miami?

Understand the problem.

You are given expressions representing the interior angle measures of the Bermuda Triangle. You are asked to find the measure of the interior angle at Miami.

Make a plan.

Use what you know about interior angle measures of triangles to write and solve an equation for x .

Solve and check.

$$x + (2x - 44.8) + 62.8 = 180$$

$$3x + 18 = 180$$

$$3x = 162$$

$$x = 54$$

Write an equation.

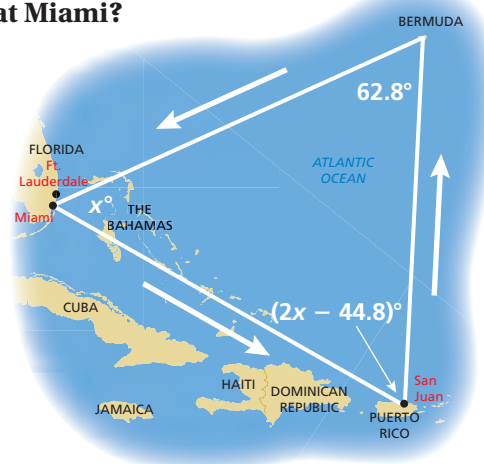
Combine like terms.

Subtract 18 from each side.

Divide each side by 3.

Check

$$\begin{aligned} x + (2x - 44.8) + 62.8 &= 180 \\ 54 + [2(54) - 44.8] + 62.8 &\stackrel{?}{=} 180 \\ 54 + 63.2 + 62.8 &\stackrel{?}{=} 180 \\ 180 &= 180 \checkmark \end{aligned}$$



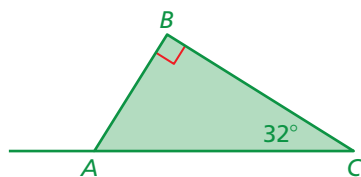
So, the measure of the interior angle at Miami is 54° .



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

- The *Historic Triangle* in Virginia connects Jamestown, Williamsburg, and Yorktown. The interior angle at Williamsburg is 120° . The interior angle at Jamestown is twice the measure of the interior angle at Yorktown. Find the measures of the interior angles at Jamestown and Yorktown. Explain your reasoning.



- A helicopter travels from point C to point A to perform a medical supply drop. The helicopter then needs to land at point B. How many degrees should the helicopter turn at point A to travel towards point B? Justify your answer.

EXAMPLE 3 Modeling Real Life

You and your friend plant an urban garden. You pay \$15.00 for 6 tomato plants and 6 pepper plants. Your friend pays \$22.50 for 9 tomato plants and 9 pepper plants. How much does each plant cost?

Understand the problem.

You are given the total costs of two different combinations of tomato plants and pepper plants. You are asked to find the cost of each plant.

Make a plan.

Use a verbal model to write a system of linear equations. Let x represent the cost of each tomato plant and let y represent the cost of each pepper plant. Then solve the system.

Solve and check.

Number of tomato plants	•	Cost of each tomato plant, x	+	Number of pepper plants	•	Cost of each pepper plant, y	=	Total cost
-------------------------	---	--------------------------------	---	-------------------------	---	--------------------------------	---	------------

The system is: $6x + 6y = 15$ Equation 1 (You)

$9x + 9y = 22.5$ Equation 2 (Your friend)



One way to solve is to use elimination. Multiply Equation 1 by 1.5 and subtract the equations.

$6x + 6y = 15$	Multiply by 1.5.	$9x + 9y = 22.5$	Revised Equation 1
$9x + 9y = 22.5$		$9x + 9y = 22.5$	Equation 2
		$0 = 0$	Subtract.

The equation $0 = 0$ is always true. The system has infinitely many solutions.

So, there is not enough information to find the cost of each plant.

Look Back Revised Equation 1 shows that the two equations in the system are equivalent. So, the system has infinitely many solutions. ✓



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

- Your friend wants to sell painted rocks. He spends \$10.00 on startup costs, and each painted rock costs him \$0.75 to make. A store offers to pay your friend's startup costs and buy his painted rocks for \$0.75 each. How many painted rocks does your friend need to sell to make a profit?
- DIG DEEPER!** The difference in age of two orangutans is 6 years. In 4 years, is it possible for the older orangutan to be twice as old as the younger orangutan? three times as old? Justify your answers.



EXAMPLE 4 Modeling Real Life

The cost y (in dollars) of buying x cubic yards of mulch from Company A, including a one-time shipping fee, is represented by the linear function $y = 29x + 30$. The table shows the cost, including a one-time shipping fee, of buying mulch from Company B. Which company charges less per cubic yard of mulch?

Mulch (cubic yards), x	Cost (dollars), y
1	48.50
2	82.00

Understand the problem.

Make a plan.

Solve and check.

Check Reasonableness

For Company B, use the points (2, 82) and (3, 115.50) to find the slope.

$$\begin{aligned}\text{slope} &= \frac{115.50 - 82.00}{3 - 2} \\ &= 33.5 \quad \checkmark\end{aligned}$$

You are given functions for two different companies. Company A charges more per cubic yard of mulch than Company B.

The table shows a cost per cubic yard of mulch for Company B. The slope of the graph of each function is the cost per cubic yard of mulch.

Company A

$$y = 29x + 30$$

The slope is 29.

Company B charges \$29.00 per cubic yard of mulch.

So, Company B charges less per cubic yard of mulch.

Indicator 2f - In Example 4, the Problem-Solving Plan is shown to help students make sense of problems and persevere in solving them. Students then use the Problem-Solving Plan to help them solve #6-7.

MP1 Make sense of problems and persevere in solving them.

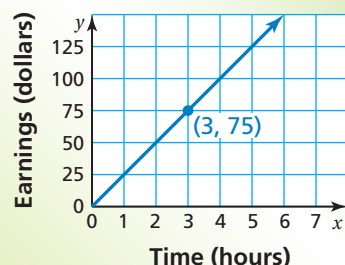
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.... Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

Earnings of Manager B



- Manager A earns \$15 per hour and receives a \$50 bonus. The graph shows the earnings of Manager B. (a) Which manager has a greater hourly wage? (b) After how many hours does Manager B earn more money than Manager A?
- Each month, you start with 2 gigabytes of data and use 0.08 gigabyte per day. The table shows the amount y (in gigabytes) of data that your friend has left x days after the start of each month. Who runs out of data first? Justify your answer.

Day, x	Data (gigabytes), y
0	3
7	2.3
14	1.6

1

Connecting Concepts

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Using the Problem-Solving Plan

- The battery life of a one-year-old cell phone is 75% of its original battery life. When the battery is charged to 50% of its capacity, it dies after $4\frac{1}{2}$ hours. Find the original battery life of the phone. Justify your answer.

Understand the problem.

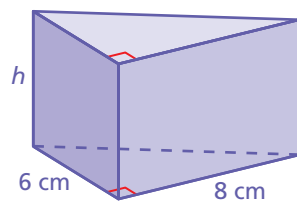
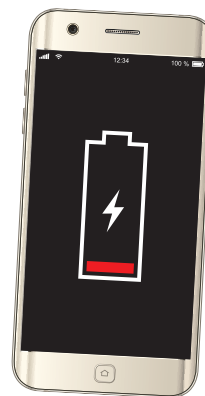
You know how long a cell phone battery lasts when it is charged to 50% of its capacity. You also know that the battery life of the phone is 75% of its original battery life. You are asked to find the original battery life of the phone.

Make a plan.

First, find the battery life of the one-year-old cell phone. Then use this information to write and solve an equation for the original battery life of the phone.

Solve and check.

Use the plan to solve the problem. Then check your solution.



- The triangular prism shown has a volume of 132 cubic centimeters. Find the height of the prism. Justify your answer.

Performance Task



Target Heart Rates

At the beginning of this chapter, you watched a STEAM Video called "Training for a Half Marathon." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



5 Connecting Concepts

► Using the Problem-Solving Plan

1. An animal shelter has a total of 65 cats and dogs. The ratio of cats to dogs is 6 : 7. Find the number of cats and the number of dogs in the shelter.

Understand the problem.

You know the total number of cats and dogs in an animal shelter, and the ratio of cats to dogs. You are asked to find the number of cats and the number of dogs in the shelter.

Make a plan.

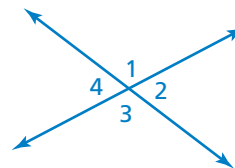
Write a system of equations. Use the total number of cats and dogs to write an equation relating the number x of cats and the number y of dogs. Use the ratio of cats to dogs to write a second equation. Then solve the system.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The measure of $\angle 1$ is 15 degrees less than two times the measure of $\angle 2$. Find the measure of each of the four angles formed by the intersecting lines. Justify your answer.



3. A landscaper plants grass seed over the entire area of two parks that are similar in shape. The ratio of the perimeter of Park A to the perimeter of Park B is 2 : 1. The parks have a combined area of 9000 square feet. How many square feet does the landscaper cover with grass seed at Park A? Park B? Justify your answer.

Performance Task



Mixing Alloys

At the beginning of this chapter, you watched a STEAM Video called "Gold Alloys." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



6 Connecting Concepts

▶ Using the Problem-Solving Plan

1. You randomly survey middle school students about whether they prefer action, comedy, or animation movies. The two-way table shows the results. Estimate the probability that a randomly selected middle school student prefers action movies.

		Grade		
		6	7	8
Genre	Action	12	18	10
	Comedy	8	6	3
	Animation	9	11	14

Understand the problem.

You know the results of a survey about movie preference. You are asked to estimate the probability that a randomly selected middle school student prefers action movies.

Make a plan.

Find the marginal frequencies for the data. Then use the marginal frequencies to find the probability that a randomly selected middle school student prefers action movies.

Solve and check.

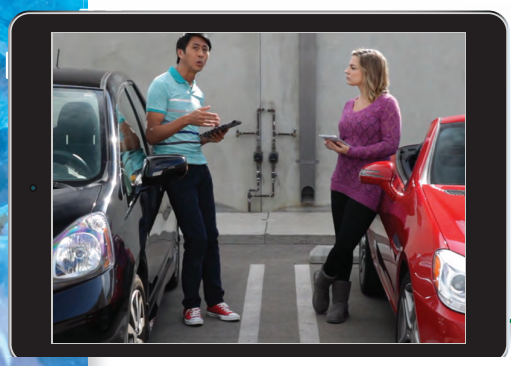
Use the plan to solve the problem. Then check your solution.

2. An equation of the line of best fit for a data set is $y = -0.68x + 2.35$. Describe what happens to the slope and the y -intercept of the line when each y -value in the data set increases by 7.
3. On a school field trip, there must be 1 adult chaperone for every 16 students. There are 8 adults who are willing to be a chaperone for the trip, but only the number of chaperones that are necessary will attend. In a class of 124 students, 80 attend the trip. Make a two-way table that represents the data.



Performance Task

Cost vs. Fuel Economy



At the beginning of this chapter, you watched a STEAM Video called "Fuel Economy." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.

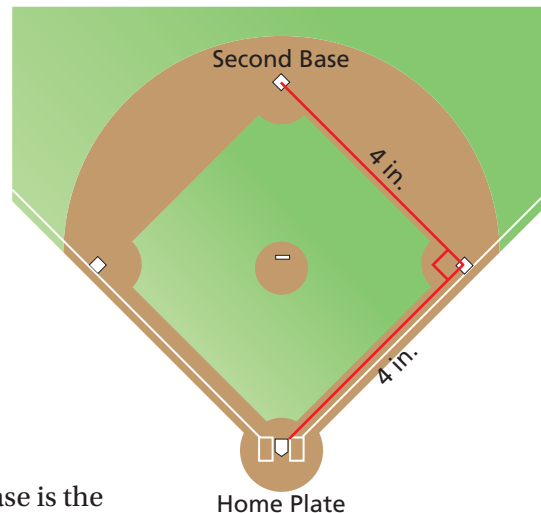


9

Connecting Concepts

Using the Problem-Solving Plan

- The scale drawing of a baseball field has a scale factor of $\frac{1}{270}$. Approximate the distance from home plate to second base on the actual baseball field to the nearest tenth.



Understand the problem.

You know several measurements and the scale factor in a scale drawing of a baseball field. You are asked to approximate the distance from home plate to second base on the actual baseball field.

Make a plan.

The distance from home plate to second base is the hypotenuse of a right triangle. Approximate the distance in the scale drawing. Use the scale factor to approximate the distance on the actual field.

Solve and check.

Use the plan to solve the problem.

Indicator 2f - In each exercise, students have to use their knowledge of the current chapter and previous concepts to solve them. This helps students make sense of problems and persevere in solving them. For instance, #2 requires the skills of the Pythagorean Theorem and surface area.

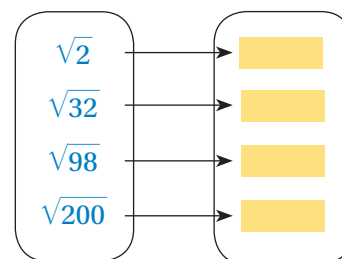


- You cut the wood cube shown into two identical triangular prisms. Approximate the surface area of each triangular prism to the nearest tenth. Justify your answer.

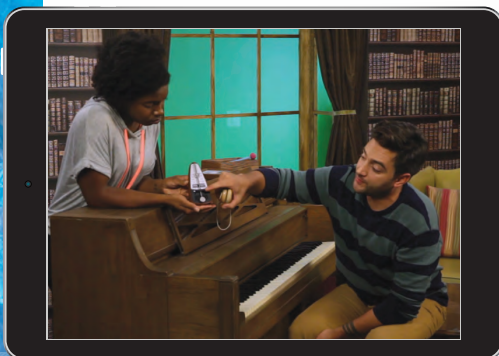
- Complete the mapping diagram representing the relationship between the lengths of the hypotenuse and the legs of an isosceles right triangle. Is the relationship linear? Explain.

Hypotenuse

Legs



Performance Task



Identify and Correct the Error!

At the beginning of this chapter, you watched a STEAM Video called "Metronome Design." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



Laurie's Notes

Scaffolding Instruction

- In the exploration, students used patterns to investigate how changes in dimensions of similar figures affect the perimeter and area. They will now use these relationships to solve problems.
- Emerging:** Students may struggle to extend the patterns beyond the figures in the exploration. They will benefit from guided instruction.
- Proficient:** Students recognize the patterns and can extend them. They should review the Key Ideas before proceeding to Example 2 and the Self-Assessment exercises.

Key Idea

? "How do you identify similar triangles?" *Corresponding side lengths are proportional and corresponding angles are equal.*

- Write some side lengths on the two triangles.
- "Find the two perimeters and show that they are proportional to the corresponding sides." *12 units and 24 units;*

EXAMPLE 1

? "What is the ratio of the corresponding sides?" *4 : 6 or 2 : 3* The ratio of the perimeters is also 2 : 3.

? **MP1 Make Sense of Problems and Persevere in Solving Them:** "Why do you not need to know both dimensions of one of the rectangles to find the ratio of the perimeters?" *The rectangles are similar, so only one pair of corresponding sides is necessary.*

Try It

- Select several students to explain their reasoning.

ELL Support

After demonstrating Example 1, have students work in pairs to discuss and complete Try It Exercise 1. Provide guiding questions: What is the relationship between the heights of similar figures and their perimeters? Can you simplify your answer? Expect students to perform according to their language levels.

Beginner: Write out the process.

Intermediate: Discuss the process using simple sentences.

Advanced: Use detailed sentences and help guide discussion.

Key Idea

- Representation:** Draw two triangles whose corresponding sides appear to have a ratio of 1 : 2.



? "If the corresponding sides have a ratio of 1 : 2, then what will the ratio of the areas be?" *1 : 4* You can use pattern blocks to show this relationship.

- MP2 Reason Abstractly and Quantitatively:** There are 4 copies of the smaller triangle inside the larger triangle. In other words, the larger triangle has an area 4 times greater than the area of the smaller triangle.

Formative Assessment Tip

Accountable Language Stems

Students are often asked to discuss their ideas and any discrepancies, but they may not understand what you are asking them to do. *Accountable Language Stems* help students resist the urge to say anything that comes to mind, while challenging them to check the quality of claims and arguments in a respectful manner. You can provide specific sentence stems for a particular problem or post.

Indicator 2f - This note appears in the Teaching Edition to point out that students have to make sense of the similar figures to be able to answer the questions in Example 1. This way, students only need to know one pair of corresponding sides to find the ratio of the perimeters.

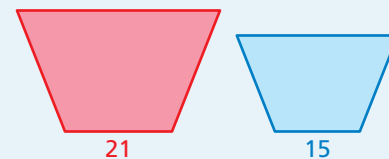
____ said, because ____."

- "I can see that ____; however, I disagree with (or can't see) ____."
- "I'm not sure I understood you when you said _____. Could you say more about that?"
- "I want to know more about ____."

Accountable Language Stems help student learn to "talk" with their partners or neighbors. They also support class discussions.

Extra Example 1

Find the value of the ratio (red to blue) of the perimeters of the similar trapezoids.



$\frac{7}{5}$

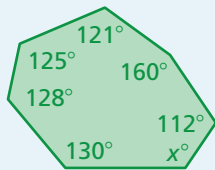
Try It

1. $\frac{3}{5}$

Laurie's Notes

Extra Example 2

Find the value of x .



124

Try It

3. 105
4. 75

Self-Assessment for Concepts & Skills

5. *Sample answer:* Subtract 2 from the number of sides the polygon has and multiply that value by 180° .
6. 360°
7. 70
8. 155

EXAMPLE 2

- **Connection:** This example integrates equation solving with finding a missing angle.
- ? "How many sides does the polygon have?" 7
- ? "How do you find the sum of the interior angle measures of a 7-gon?"
Solve $S = (7 - 2) \cdot 180^\circ$.
- Once the sum is known, write and solve the equation as shown. Caution students to be careful with their arithmetic.

Try It

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students should check with their neighbors to make sure they are setting up the equation correctly. Each problem has two parts: determining the sum of the interior angle measures and then writing the equation to solve for the missing angle.

angle measures 90° .

✓ Self-Assessment for Concepts & Skills

- **Think-Pair-Share:** Students should read each exercise independently and then work in pairs to solve the exercises. Have each pair compare their answers with another pair and discuss any discrepancies.
- **Fist of Five:** Ask students to indicate their understanding of the first and second success criterion. Then select students to explain each one.

ELL Support

Proceed as described in Laurie's Notes. Have each pair display their answers for Exercises 6–8 on a whiteboard for your review.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at BigIdeasMath.com.

Laurie's Notes

Scaffolding Instruction

- Students explored slopes of lines. They will now use more formal definitions to find slopes using graphs, tables, and formulas.
- Emerging:** Students understand that some lines are steeper than others, but they may struggle with the fact that the slope of a line is constant and how to find it. They need guided instruction for the Key Ideas and examples.
- Proficient:** Students understand that the slope of a line is determined by the ratio of the vertical change to the horizontal change between any two points. They also understand that parallel lines have the same slope. These students should review the Key Ideas and the Summary before proceeding to the Self-Assessment exercises.

Key Idea

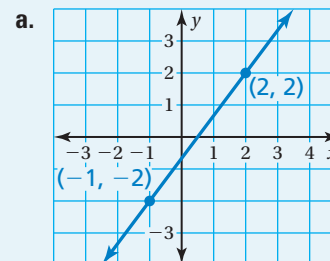
- Write the Key Idea. Define **slope** of a line.
- Tell students that it is traditional to use m to represent slope. They will also see this in future mathematics courses. Students may ask why m is used. There does not seem to be agreement about this among mathematicians. In fact, some countries use different letters.
- Note the use of color in the definition and on the graph. The *change in y* and the *vertical change arrow* are both red. The *change in x* and the *horizontal change arrow* are both blue.
- Explain that it is common to refer to the triangle formed by the line and the change in x and change in y arrows as a *slope triangle*.
- Discuss the difference in positive and negative slopes.
- Remind students that graphs are read from left to right.
- Explain to students that in addition to finding the **rise** and the **run** graphically, they can also subtract coordinates to find the rise and the run.
- Go over the Reading note, so students understand how to read variables with subscripts.

EXAMPLE 1

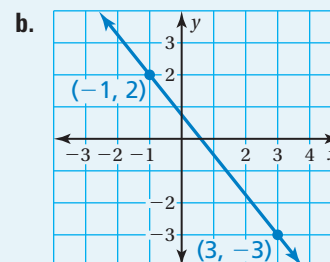
- MP1 Make Sense of Problems and Persevere in Solving Them:** Drawing the arrow diagrams will help students visualize the *slope triangle*.
- Students often ask if they can move in the y -direction first, followed by the x -direction. The answer is yes. Demonstrate this on either graph.
 - In part (a), start at $(-3, -1)$ and move up 5 units in the y -direction and then to the right 6 units in the x -direction. You will end at $(3, 4)$.
 - In part (b), start at $(-1, 1)$ and move down 3 units in the y -direction and then to the right 2 units in the x -direction. You will end at $(1, -2)$.
- Discuss:** When finding the slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2) . The labeling of the ordered pairs is arbitrary.

Extra Example 1

Describe the slope of each line. Then find each slope.



The line rises from left to right, so the slope is positive; $\frac{4}{3}$



The line falls from left to right, so the slope is negative; $-\frac{5}{4}$

In-Class Problem Solving

2 Connecting Concepts

Using the Problem-Solving Plan

1. A scale drawing of a helipad uses a scale of 1 ft : 20 ft. The scale drawing has an area of 6.25 square feet. What is the area of the actual helipad?

Understand the problem.

You know the scale of the drawing and the area of the helipad in the drawing. You are asked to find the area of the actual helipad.

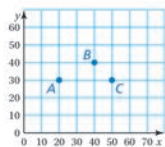


Make a plan.

A scale drawing is similar to the actual object. So, use the scale 1 ft : 20 ft and the ratio $6.25 \text{ ft}^2 : A \text{ ft}^2$ to write and solve a proportion that represents the area A of the actual helipad.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The locations of three cargo ships are shown in the coordinate plane. Each ship travels at the same speed in the same direction. After 1 hour, the x- and y-coordinates of Ship A increase 80%. Use a translation to describe the change in the locations of the ships. Then find the new coordinates of each ship.



3. All circles are similar. A circle with a radius of 2 inches is dilated, resulting in a circle with a circumference of 22π inches. What is the scale factor? Justify your answer.

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Connecting Concepts pages combine previously learned skills with concepts from the current chapter, so students practice problem solving for high-stakes assessments. Students use the Problem-Solving Plan along with a variety of problem-solving strategies.

Self-Assessment for Problem Solving gives teachers the opportunity for continual formative assessment and allows students to communicate mathematically in every lesson.

Performance Task

Master Puppeteer

At the beginning of this chapter, you watched a STEAM Video called "Shadow Puppets." You are now ready to complete the performance task related to this video, available at BigIdeasMath.com. Be sure to use the problem-solving plan as you work through the performance task.



Self-Assessment for Problem Solving

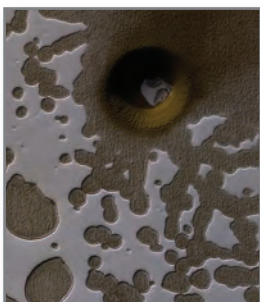
Solve each exercise. Then rate your understanding of the success criteria in your journal.



10. A photograph is dilated to fit in a frame, so that its area after the dilation is 9 times greater than the area of the original photograph. What is the scale factor of the dilation? Explain.
11. **DIG DEEPER!** The location of a water treatment plant is mapped using a coordinate plane, where each unit represents 1 foot. The plant has vertices $(0, 0)$, $(0, 180)$, $(240, 180)$, and $(240, 0)$. You dilate the figure with a scale factor of $\frac{1}{3}$. What are the coordinates of the image? What do you need to change so that the image accurately represents the location of the plant? Explain your reasoning.

EXAMPLE 4

Modeling Real Life



The temperature in a crater on Mars is 0°C at 1 P.M. The temperature decreases 8°C every hour. When will the temperature be -50°C ?

To determine when the temperature will be -50°C , find how long it will take the temperature to decrease by 50°C . Write and solve an equation to find the time.

Verbal Model

Change in
temperature
($^{\circ}\text{C}$)

=

Hourly change
in temperature
($^{\circ}\text{C}$ per hour)

•

Time
(hours)

Variable Let t be the time for the temperature to decrease 50°C .

Equation

$$\begin{array}{ccccccc} & -50 & = & -8 & \cdot & t \\ & \uparrow & & \uparrow & & \\ -50 & = & -8t & & & \end{array}$$

Write the equation.

$$\frac{-50}{-8} = \frac{-8t}{-8}$$

Division Property of Equality

$$6.25 = t$$

Simplify.

The changes in temperature are negative because the temperatures are decreasing.

The temperature will be -50°C at 6.25 hours after 1 P.M., or 6 hours and 15 minutes after 1 P.M.

▶ So, the temperature will be -50°C at 7:15 P.M.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



18. A shipwreck is 300 meters away from a diving station. An undersea explorer travels away from the station at a speed of 2 meters per second. The explorer is x meters away from the station and will reach the shipwreck in 100 seconds. What is the value of x ?
19. You conduct an inventory for a hardware store and count 40 rolls of duct tape. Your manager wants to keep 7 boxes of duct tape in stock. If each box holds 8 rolls of duct tape, how many boxes should you order? Justify your answer.
20. **DIG DEEPER!** Your fitness tracker overestimates the number of steps you take by 5%. The tracker indicates that you took 7350 steps today. Write and solve an equation to find the actual number of steps you took today.

SOLVING EQUATIONS USING MULTIPLICATION OR DIVISION Solve the equation.
Check your solution.

22. $7x = 35$
23. $4 = -0.8n$
24. $6 = -\frac{w}{8}$
25. $\frac{m}{\pi} = 7.3$
26. $-4.3g = 25.8$
27. $\frac{3}{2} = \frac{9}{10}k$
28. $-7.8x = -1.56$
29. $-2 = \frac{6}{7}p$
30. $3\pi d = 12\pi$

31. **YOU BE THE TEACHER** Your friend solves the equation. Is your friend correct? Explain your reasoning.

$-1.5 + k = 8.2$
 $k = 8.2 + (-1.5)$
 $k = 6.7$

32. **MP STRUCTURE** A gym teacher orders 42 tennis balls. The tennis balls come in packs of 3. Which of the following equations represents the number x of packs?

- $x + 3 = 42$
- $3x = 42$
- $\frac{x}{3} = 42$
- $x = \frac{3}{42}$

33. **MODELING REAL LIFE** You clean a community park for 6.5 hours. You earn \$42.25. How much do you earn per hour?
34. **MODELING REAL LIFE** A rocket is scheduled to launch from a command center in 3.75 hours. What time is it now?
35. **MODELING REAL LIFE** After earning interest, the balance of an account is \$420. The new balance is $\frac{7}{6}$ of the original balance. How much interest did it earn?
36. **MODELING REAL LIFE** After a cleanup, algae covers 2 miles of a coastline. The length of the coastline covered after the cleanup is $\frac{1}{3}$ of the previous length. How many miles of the coast did the algae previously cover?

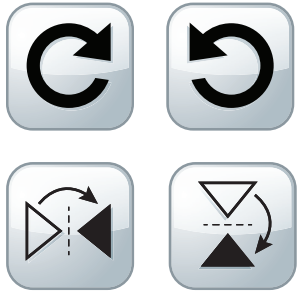


Roller Coasters at Cedar Point	
Coaster	Height (feet)
Top Thrill Dragster	420
Millennium Force	310
Valravn	225
Mantis	?

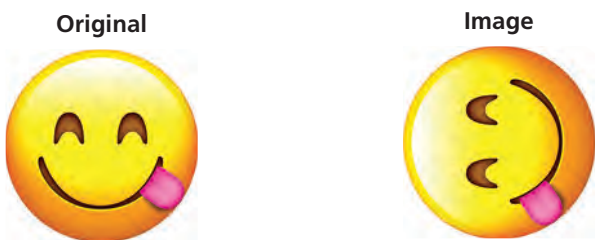
37. **MP PROBLEM SOLVING** Cedar Point, an amusement park, has some of the tallest roller coasters in the United States. The Mantis is 165 feet shorter than the Millennium Force. What is the height of the Mantis?

EXAMPLE 3

Modeling Real Life



You can use the buttons shown at the left to transform objects in a computer program. You can rotate objects 90° in either direction and reflect objects in a horizontal or vertical line. How can you transform the emoji as shown below?



When you rotate the emoji 90° counterclockwise, the tongue is in the wrong place. Reflect the emoji in a horizontal line to move the tongue to the correct location.

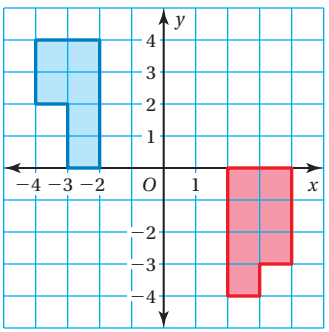


▶ To transform the emoji as shown, you can use a 90° counterclockwise rotation followed by a reflection in a horizontal line.



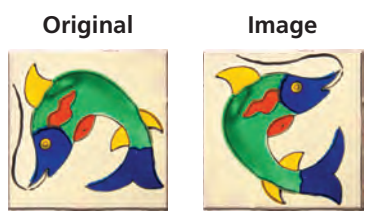
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



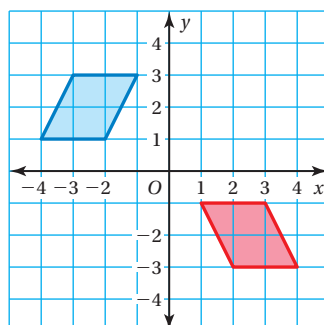
6. In the coordinate plane at the left, each grid line represents 50 feet. Each figure represents a pasture.
- a. Are the figures congruent? Use rigid motions to justify your answer.
 - b. How many feet of fencing do you need to enclose each pasture?

7. A home decorator uses a computer to design a floor tile. How can the decorator transform the tile as shown?



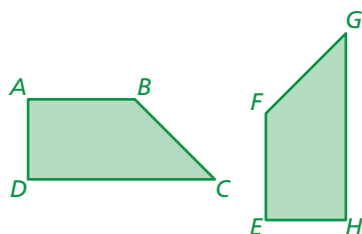
12. **YOU BE THE TEACHER** Your friend describes a sequence of rigid motions between the figures. Is your friend correct? Explain your reasoning.

Reflect the red figure in the x -axis, and then translate it left 5 units.

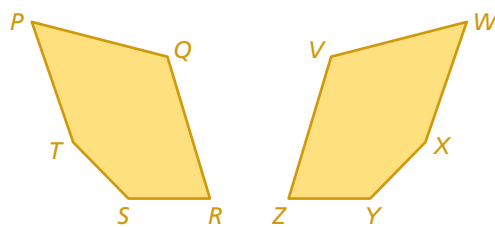


NAMING CORRESPONDING PARTS The figures are congruent. Name the corresponding angles and the corresponding sides.

13.



14.



15. **MODELING REAL LIFE** You use a computer program to transform an emoji. How can you transform the emoji as shown?

Original



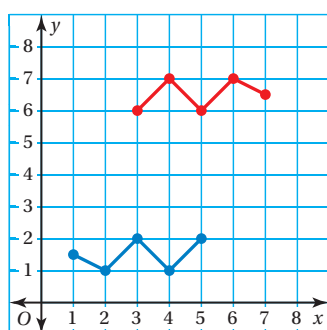
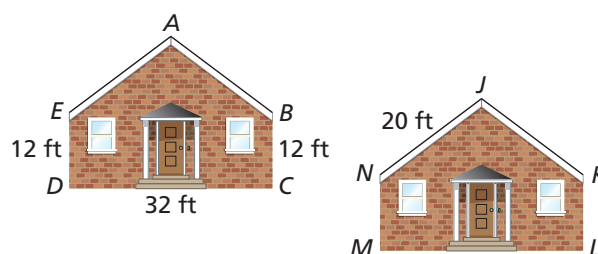
Image



16. **CRITICAL THINKING** Two figures are congruent. Are the areas of the two figures the same? the perimeters? Explain your reasoning.

17. **DIG DEEPER!** The houses are identical.

- What is the length of side LM ?
- Which angle of $JKLMN$ corresponds to $\angle D$?
- Side AB is congruent to side AE . What is the length of side AB ? What is the perimeter of $ABCDE$?



18. **MP REASONING** Two constellations are represented by the figures in the coordinate plane shown. Are the figures congruent? Justify your answer.

EXAMPLE 3 Modeling Real Life



A tropical storm becomes a hurricane when wind speeds are at least 74 miles per hour.

The wind speed y (in miles per hour) of a tropical storm is $y = 2x + 66$, where x is the number of hours after the storm enters the Gulf of Mexico. When does the storm become a hurricane?

Use a graph to find the time it takes for the storm to become a hurricane. Make a table and draw a line through the points.

x	$y = 2x + 66$	y
0	$y = 2(0) + 66$	66
1	$y = 2(1) + 66$	68
2	$y = 2(2) + 66$	70
3	$y = 2(3) + 66$	72

Another Method Use the equation $y = 2x + 66$ to find x when $y = 74$.

$$74 = 2x + 66$$

$$8 = 2x$$

$$4 = x \quad \checkmark$$

From the graph, you can see that

So, the storm becomes a hurricane 4 hours after it enters the Gulf of Mexico.

Indicator 2f - In #13-15, students model with mathematics by being presented real-life situations. In #14, students use multiple representations of linear equations to represent a real-life problem which deepens students' understanding.

MP4 Model with mathematics - Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.... Mathematically proficient students who can apply what they know... are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



- A game show contestant earns y dollars for completing a puzzle in x minutes. This situation is represented by the equation $y = -250x + 5000$. How long did a contestant who earned \$500 take to complete the puzzle? Justify your answer.
- The total cost y (in dollars) to join a cheerleading team and attend x competitions is represented by the equation $y = 10x + 50$.
 - Graph the linear equation.
 - You have \$75 to spend. How many competitions can you attend?
- The seating capacity y for a banquet hall is represented by $y = 8x + 56$, where x is the number of extra tables you need. How many extra tables do you need to double the original seating capacity?



- 24. MODELING REAL LIFE** The equation $y = 2x + 3$ represents the cost y (in dollars) of mailing a package that weighs x pounds.
- Use a graph to estimate how much it costs to mail the package.
 - Use the equation to find exactly how much it costs to mail the package.

SOLVING A LINEAR EQUATION Solve for y . Then graph the linear equation.

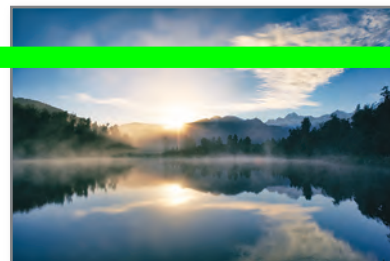
25. $y - 3x = 1$

26. $5x + 2y = 4$

27. $-\frac{1}{3}y + 4x = 3$

28. $x + 0.5y = 1.5$

- 29. MODELING REAL LIFE** The depth y (in inches) of a lake after x years is represented by the equation $y = 0.2x + 42$. How much does the depth of the lake increase in four years? Use a graph to justify your answer.



- 30. MODELING REAL LIFE** The amount y (in dollars) of money in your savings account after x months is represented by the equation $y = 12.5x + 100$.
- Graph the linear equation.
 - How many months will it take you to save a total of \$237.50?



- 31. MP PROBLEM SOLVING** The radius y (in millimeters) of a chemical spill after x days is represented by the equation $y = 6x + 50$.
- Graph the linear equation.
 - The leak is noticed after two weeks. What is the area of the leak when it is noticed? Justify your answer.

- 32. GEOMETRY** The sum S of the interior angle measures of a polygon with n sides is $S = (n - 2) \cdot 180^\circ$.
- Plot four points (n, S) that satisfy the equation. Is the equation a linear equation? Explain your reasoning.
 - Does the value $n = 3.5$ make sense in the context of the problem? Explain your reasoning.

- 33. DIG DEEPER!** One second of video on your cell phone uses the same amount of memory as two pictures. Your cell phone can store 2500 pictures.
- Create a graph that represents the number y of pictures your cell phone can store when you take x seconds of video.
 - How many pictures can your cell phone store in addition to a video that is one minute and thirty seconds long?

EXAMPLE 4 Modeling Real Life



A car produces 20 pounds of carbon dioxide for every gallon of gasoline burned. Write and graph a function that describes the relationship.

Use a verbal model to write a function rule.

Verbal Model Carbon dioxide (pounds) = Pounds per gallon • Gasoline used (gallons)

Variable Let p represent the number of pounds of carbon dioxide, and let g represent the number of gallons of gasoline used.

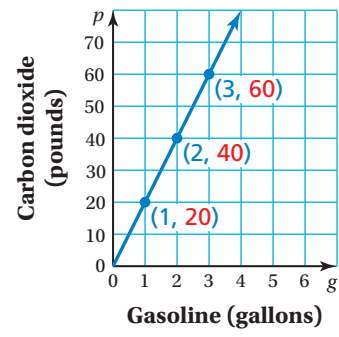
Equation $p = 20 \cdot g$

Make an input-output table that represents the function $p = 20g$.

Input, g	$20g$	Output, p	Ordered Pair, (g, p)
1	$20(1)$	20	$(1, 20)$
2	$20(2)$	40	$(2, 40)$
3	$20(3)$	60	$(3, 60)$

Plot the ordered pairs and draw a line through the points.

Because you cannot burn a negative number of gallons of gasoline, use only positive values of g .



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



- The World Health Organization (WHO) suggests having 23 health-care workers for every 10,000 people. How many health-care workers are needed to meet the WHO suggestion for a population of 250,000 people? Justify your answer using a graph.
- DIG DEEPER!** A truck produces 22 pounds of carbon dioxide for every gallon of diesel fuel burned. The fuel economy of the truck is 18 miles per gallon. Write and graph a function that describes the relationship between carbon dioxide produced and distance traveled.

36. **MP PROBLEM SOLVING** You decide to make and sell bracelets. The cost of your materials is \$84.00. You charge \$3.50 for each bracelet.

- Write a function that represents the profit P for selling b bracelets.
- Which variable is independent? dependent? Explain.
- You will *break even* when the cost of your materials equals your income. How many bracelets must you sell to break even?



37. **MODELING REAL LIFE** A furniture store is having a sale where everything is 40% off.

- Write and graph a function that represents the amount of discount on an item at regular price.
- You buy a bookshelf that has a regular price of \$85. What is the sale price of the bookshelf?

Indicator 2f - In #36-38, students model with mathematics by being presented real-life situations. Students write, evaluate, and/or graph a function for these situations.

38. **MP REASONING** You want to take a two-hour airboat tour. Which is a better deal, Snake Tours or Gator Tours? Use functions to justify your answer.



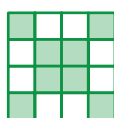
39. **MP REASONING** The graph of a function is a line that passes through the points $(3, 2)$, $(5, 8)$, and $(8, y)$. What is the value of y ?

40. **CRITICAL THINKING** Make a table where the independent variable is the side length of a square and the dependent variable is the *perimeter*. Make a second table where the independent variable is the side length of a square and the dependent variable is the *area*. Graph both functions in the same coordinate plane. Compare the functions.

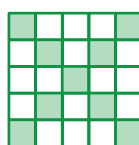
41. **PUZZLE** The blocks that form the diagonals of each square are shaded. Each block has an area of one square unit. Find the “green area” of Square 20. Find the “green area” of Square 21. Explain your reasoning.



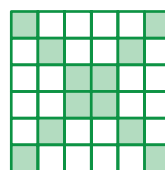
Square 1



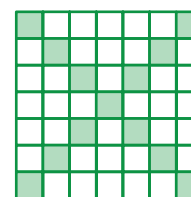
Square 2



Square 3



Square 4



Square 5

EXAMPLE 4 Modeling Real Life

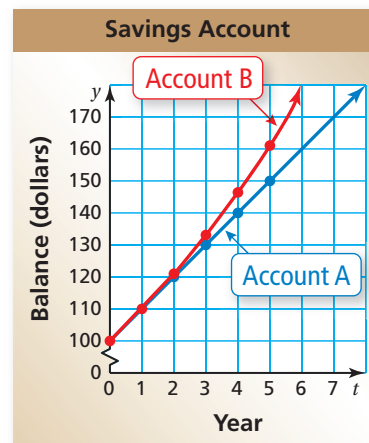
Year, t	Account A Balance	Account B Balance
0	\$100	\$100
1	\$110	\$110
2	\$120	\$121
3	\$130	\$133.10
4	\$140	\$146.41
5	\$150	\$161.05

Two accounts earn different types of interest. The table shows the balances of each account for five years. Graph the data and compare the balances of the accounts over time.

Plot the points in the table for each account.

The points for Account A lie on a line. Draw a line through the points.

The points for Account B do not lie on a line. Draw a curve through the points.



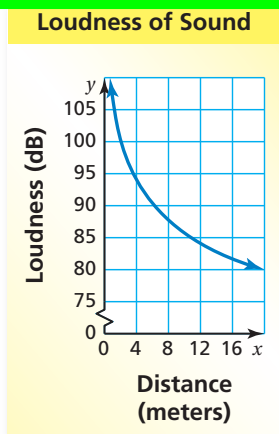
The graphs show that both balances are positive and increasing. The graphs also show that the balance of Account B grows faster.

The balance of Account A has a constant rate of change of \$10. The balance of Account B increases by different amounts each year. So, Account A shows linear growth and Account B shows nonlinear growth.



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

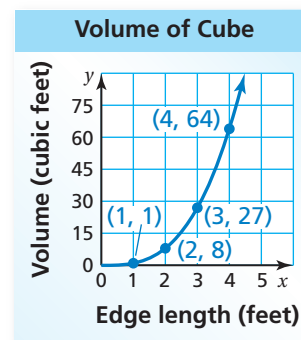


11. The loudness of sound is measured in *decibels* (dB). The graph shows the loudness y of a sound (in decibels) x meters from the source of the sound. Is the relationship between loudness and distance *linear* or *nonlinear*? Approximate the loudness of the sound 12 meters from the source.






12. **DIG DEEPER!** A *video blogger* is someone who records a video diary. A new website currently hosts 90 video bloggers and projects a gain of 10 video bloggers per month. The table below shows the actual numbers of video bloggers. How does the projection differ from the actual change?

Month	0	1	2	3	4	5
Video Bloggers	90	97	110	128	153	190

14. **IDENTIFYING A FUNCTION** The graph shows the volume V (in cubic feet) of a cube with an edge length of x feet. Does the graph represent a *linear* or *nonlinear* function? Explain.



15. **MODELING REAL LIFE** The frequency y (in terahertz) of a light wave is a function of its wavelength x (in nanometers). Is the function relating the wavelength of light to its frequency *linear* or *nonlinear*?

					
Color	Red	Yellow	Green	Blue	Violet
Wavelength, x	660	595	530	465	400
Frequency, y	454	504	566	645	749

16. **DIG DEEPER!** The table shows the cost y (in dollars) of x pounds of sunflower seeds.

Pounds, x	Cost, y
2	2.80
3	?
4	5.60

- What is the missing y -value that makes the table represent a linear function?
- Write a linear function that represents the cost y of x pounds of seeds. Interpret the slope.
- Does the function have a maximum value? Explain your reasoning.

17. **MODELING REAL LIFE** A birch tree is 9 feet tall and grows at a rate of 2 feet per year. The table shows the height h (in feet) of a willow tree after x years.

Years, x	Height, h
0	5
1	11
4	17
9	23

- Does the table represent a *linear* or *nonlinear* function? Explain.
- Which tree is taller after 10 years? Explain.

18. **CRITICAL THINKING** In their first year, Show A has 7 million viewers and Show B has 5 million viewers. Each year, Show A has 90% of the viewers it had in the previous year. Show B loses 200,000 viewers each year.

- Determine whether the function relating the year to the number of viewers is *linear* or *nonlinear* for each show.
- Which show has more viewers in its sixth year?

19. **NUMBER SENSE** The ordered pairs represent a function.

$(0, -1), (1, 0), (2, 3), (3, 8),$ and $(4, 15)$

- Graph the ordered pairs and describe the pattern. Is the function *linear* or *nonlinear*?
- Write an equation that represents the function.



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
8.G.A.5

Learning Target

Find missing angle measures created by the intersections of lines.

Success Criteria

- Identify congruent angles when a transversal intersects parallel lines.
- Find angle measures when a transversal intersects parallel lines.

Laurie's Notes

Preparing to Teach

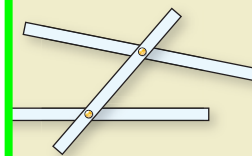
- Students worked with transformations and congruent figures in the previous chapter. Now they will make conjectures about angles created by parallel lines and transversals.
- **MP5 Use Appropriate Tools Strategically:** There are many appropriate tools that mathematically proficient students may use to gain an understanding of a new concept or to solve a problem. Several tools could be useful in the exploration. Students might use technology, transparencies, or paper and pencil. To measure angles it will be helpful for students to have protractors.

Motivate

Indicator 2f - In preparation for Section 3.1, Laurie's Notes mention various tools to use in the study of parallel lines and transversals.

MP5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.... Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

big ideas of this lesson. Cut
e of two strips and punch two
brass fasteners.



Model B

on an overhead. Show students
rom Model A to Model B.
etween the four angles on L_1
es will always be congruent

r on the overhead and
they think are congruent. Use

Exploration 1

- no; If you extend line A and line B , they will intersect.
- The vertical angles are congruent.
- Line A and line B are parallel.
- yes
- When a line intersects two parallel lines, the angles created by the intersection of the line and the first parallel line are congruent to the corresponding angles created by the intersection of the line and the second parallel line.

Exploration 1

- Students should use the geometry software available at BigIdeasMath.com. Lines A and B will be given. If you chose to use different software, students will need to draw the lines so that they are nearly parallel.
- This exploration can also be completed using paper, a pencil, and a protractor. You may want students to begin with parallel lines, measure the angles, and then see what happens when a rotation causes the lines to be nonparallel.
- In part (b), students should recognize several congruent angles. They know vertical angles are congruent and they can use protractors to measure the others. They have not yet studied corresponding angles, **interior angles**, or **exterior angles**. Ask students about some of these angles so that they see that the angles are close in measure, but not exact because the lines are nonparallel.
- Once students have parallel lines in part (c), they will see many more pairs of congruent angles.
- For parts (c) and (d), it might be easier for students to see that the angles are congruent if they use technology or transparencies to translate the angles.



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
8.G.A.5

Learning Target

Use similar triangles to find missing measures.

Success Criteria

- Use angle measures to determine whether triangles are similar.
- Use similar triangles to solve real-life problems.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at BigIdeasMath.com.

ELL Support

Write the word *similar* on the board, say it, and have students repeat. Ask them what it means and have them describe or provide as many synonyms as possible. You could suggest words such as *same*, *alike*, and *related*. Then review the mathematical definition for the phrase *similar triangles*.

Exploration 1

- a-b. no; yes; The triangles are not congruent because they are not the same size. The triangles are similar because the second triangle is a dilation of the first triangle.
- c. Two triangles that have two pairs of congruent angles are similar.

Exploration 2

See Additional Answers.

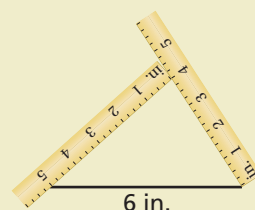
Laurie's Notes

Preparing to Teach

- **MP5 Use Appropriate Tools Strategically:** Similar triangles can be investigated using geometry software or a protractor and a ruler. Mathematically proficient students consider the available tools when solving a mathematics problem.

Motivate

- Ask students to work with partners to construct a triangle with side lengths of 4 inches, 5 inches, and 6 inches. Each pair will need 2 rulers, a protractor, and a piece of paper.
- Without a compass, they will need to work together, using both rulers to locate the third vertex.
- Have students measure the angles in the triangle they constructed.
- Have students hold up their constructions and look at the work of others.
- Discuss the results; namely, that all of the triangles are congruent.



? Ask, "What is the difference between similar triangles and congruent triangles?" **Congruent triangles must have the same size and shape. Similar triangles have the same shape but not necessarily the same size.**

Exploration 1

- Geometry software is available at BigIdeasMath.com. Alternatively, the exploration can be completed using paper, a pencil, and a protractor.
- The length of \overline{BC} is not specified.
- Students can use the measure function to find the measure of angle A in each triangle. Students can also measure sides to see that corresponding sides are proportional.

Exploration 2

- **Teaching Tip:** This is a fun exploration for students to conduct themselves, if time and weather permit. Each pair will need a measuring tape. Take students outside (near a flag pole) to measure their heights and shadows. They also need to measure the shadow of the flag pole.
- ? "Is your shadow shorter at noon or 5 P.M.? Explain." **noon; The Sun is overhead, not at a lower position in the sky.**
- ? "Do adjacent objects of different heights cast the same length shadow? Explain." **No, taller objects cast longer shadows.**
- The triangles are similar because they both have a right angle and the parallel rays of the Sun are at the same angle to the ground.
- Discuss students' methods for solving the problem. They may use their conjectures from Exploration 1 or a dilation to show that the triangles are similar. Then students can set up a proportion to find the height of the flag pole.

31. **PROJECT** The guidelines for a wheelchair ramp suggest that the ratio of the rise to the run be no greater than 1 : 12.

- MP CHOOSE TOOLS** Find a wheelchair ramp in your school or neighborhood. Measure its slope. Does the ramp follow the guidelines?
- Design a wheelchair ramp that provides access to a building with a front door that is 2.5 feet above the sidewalk. Illustrate your design.



USING AN EQUATION Use an equation to find the value of k so that the line that passes through the given points has the given slope.

32. $(1, 3), (5, k); m = 2$

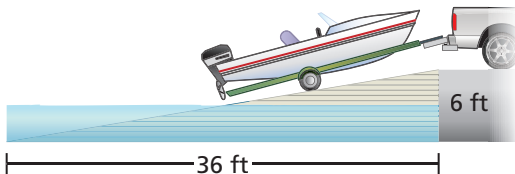
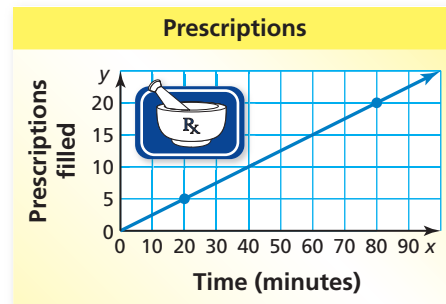
33. $(-2, k), (2, 0); m = -1$

34. $(-4, k), (6, -7); m = -\frac{1}{5}$

35. $(4, -4), (k, -1); m = \frac{3}{4}$

36. **MODELING REAL LIFE** The graph shows the numbers of prescriptions filled over time by a pharmacy.

- Find the slope of the line.
- Explain the meaning of the slope as a rate of change.



37. **CRITICAL THINKING** Which is steeper: the boat ramp, or a road with a 12% grade? Explain. (Note: Road grade is the vertical increase divided by the horizontal distance.)

38. **MP REASONING** Do the points $A(-2, -1)$, $B(1, 5)$, and $C(4, 11)$ lie on the same line? Without using a graph, how do you know?

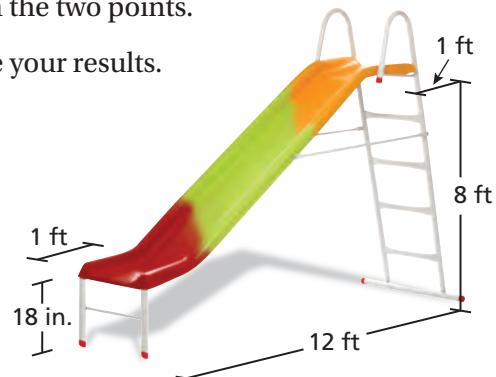
39. **MP PROBLEM SOLVING** A small business earns a profit of \$6500 in January and \$17,500 in May. What is the rate of change in profit for this time period? Justify your answer.

40. **MP STRUCTURE** Choose two points in the coordinate plane. Use the slope formula to find the slope of the line that passes through the two points.

Then find the slope using the formula $\frac{y_1 - y_2}{x_1 - x_2}$. Compare your results.

41. **DIG DEEPER!** The top and the bottom of the slide are level with the ground, which has a slope of 0.

- What is the slope of the main portion of the slide?
- Describe the change in the slope when the bottom of the slide is only 12 inches above the ground. Explain your reasoning.



2.2 Reflections

Learning Target: Reflect figures in the coordinate plane.

- Success Criteria:**
- I can identify a reflection.
 - I can find the coordinates of a figure reflected in an axis.
 - I can use coordinates to reflect a figure in the x - or y -axis.

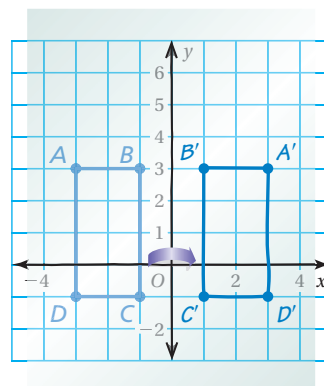
EXPLORATION 1

Reflecting Figures

Work with a partner.

- a. For each figure below, draw the figure in the coordinate plane. Then copy the axes and the figure onto a piece of transparent paper. Flip the transparent paper and align the origin and the axes with the coordinate plane. For each pair of figures, describe the line of symmetry.

- point
- line segment
- line
- triangle
- rectangle



- b. When you reflect figures, what do you notice about sides, angles, and parallel lines?

Math Practice

Look for Structure

How can you show that the image of a figure reflected in the coordinate plane is identical to the original figure?

- c. Describe the relationship between each point below and the point $A(4, 7)$ in terms of reflections.

$$B(-4, 7)$$

$$C(4, -7)$$

$$D(-4, -7)$$

- d. A point with coordinates (x, y) is reflected in the x -axis. What are the coordinates of the image?
- e. Repeat part (d) when the point is reflected in the y -axis.

5.4 Solving Special Systems of Linear Equations

Learning Target: Solve systems with different numbers of solutions.

Success Criteria:

- I can determine the number of solutions of a system.
- I can solve a system of linear equations with any number of solutions.

EXPLORATION 1

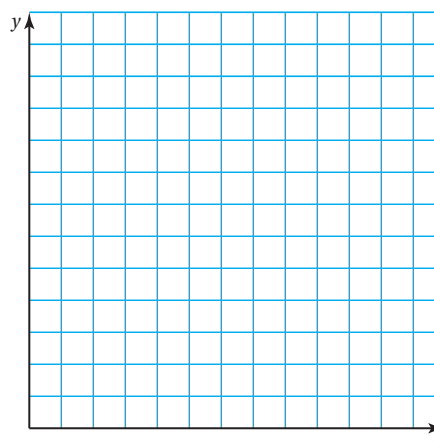
Exploring Solutions of Systems

Indicator 2f - In Exploration 1, students use the structure of the graph to answer parts b and c. Students also use structure of systems of equations to answer part d and the Math Practice note.

MP7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property.... They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects....

Work with a partner. You spend \$50 on a sewing machine to make dog backpacks. Each backpack costs you \$15 for materials.

- a. Represent the cost y (in dollars) to make x backpacks in the coordinate plane.



- b. You charge \$25 per backpack. How many backpacks do you have to sell to *break even*? Use a graph to justify your answer.
- c. Can you break even when you sell each backpack for \$20? \$15? Use graphs to justify your answers.
- d. Explain whether it is possible for a system of linear equations to have the numbers of solutions below.
- no solution
 - exactly one solution
 - exactly two solutions
 - infinitely many solutions



Math Practice

Look for Structure

How can you use slopes and y-intercepts to determine the number of solutions of a system of linear equations?

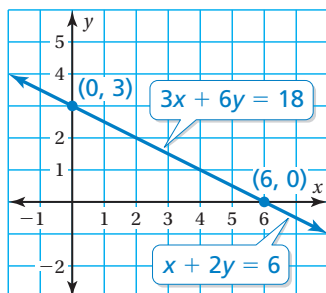
EXAMPLE 2 Solving a System with Infinitely Many Solutions

Solve the system using any method.

$$x + 2y = 6 \quad \text{Equation 1}$$

$$3x + 6y = 18 \quad \text{Equation 2}$$

Method 1: Solve by graphing.



The lines have the same slope, $-\frac{1}{2}$, and the same y-intercept, 3. So, the two equations in the system represent the same line.

▶ All the points on the line are solutions of the system. So, the system has infinitely many solutions.

Method 2: Solve by elimination.

Multiply Equation 1 by 3 and subtract the equations.

$$\begin{array}{rcl} x + 2y = 6 & \xrightarrow{\text{Multiply by 3.}} & 3x + 6y = 18 \quad \text{Revised Equation 1} \\ 3x + 6y = 18 & & 3x + 6y = 18 \quad \text{Equation 2} \\ \hline & & 0 = 0 \quad \text{Subtract.} \end{array}$$

The equation $0 = 0$ is always true. You can also see from Revised Equation 1 that the two equations in the system are equivalent.

▶ All the points on the line are solutions of the system. So, the system has infinitely many solutions.

Try It Solve the system. Explain your choice of method.

4. $x + y = 3$
 $x = y - 3$

5. $2x + y = 5$
 $4x + 2y = 0$

6. $2x - 4y = 10$
 $-12x + 24y = -60$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

MP STRUCTURE Without graphing or solving, determine the number of solutions of the system. Explain your reasoning.

7. $y = 5x - 9$
 $y = 5x + 9$

8. $y = 6x + 2$
 $y = 3x + 1$

9. $y = 8x - 2$
 $y - 8x = -2$

CHOOSING A METHOD Solve the system. Explain your choice of method.

10. $2x + y = 6$
 $x - y = 3$

11. $4y - 4x = 8$
 $y = x + 2$

12. $5x - 4y = 12$
 $7.5x = 6(y - 1)$

13. $-6x = 9$
 $6x - y = 3$

14. $0.5x + 4y = -11$
 $-1.5x - 12y = 33$

15. $x = y + 2$
 $3x = 6(y + 2)$

9.4 Practice



Go to **BigIdeasMath.com** to get
HELP with solving the exercises.

► Review & Refresh

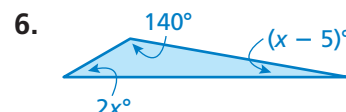
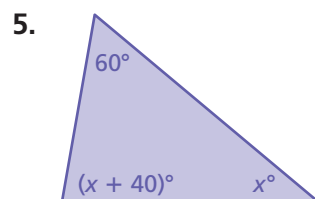
Evaluate the expression.

1. $2 + \sqrt[3]{27}$

2. $1 - \sqrt[3]{8}$

3. $7\sqrt[3]{125} - 12$

Find the measures of the interior angles of the triangle.



► Concepts, Skills, & Problem Solving

WRITING REPEATING DECIMALS AS FRACTIONS Write the repeating decimal as a fraction. (See Exploration 1, p. 395.)

7. $0.777\ldots$

8. $0.858585\ldots$

9. $0.232323\ldots$

WRITING FRACTIONS OR MIXED NUMBERS AS DECIMALS Write the fraction or mixed number as a decimal.

10. $-\frac{3}{20}$

11. $9\frac{1}{12}$

12. $\frac{5}{36}$

13. $6\frac{1}{40}$

14. $\frac{11}{75}$

15. $-2\frac{7}{18}$

16. **MP PRECISION** Your hair is $\frac{5}{16}$ inch long. Write this length as a decimal.

WRITING A REPEATING DECIMAL AS A FRACTION Write the repeating decimal as a fraction or a mixed number.

17. $-0.\overline{5}$

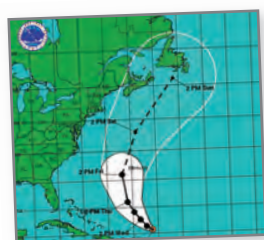
18. $4.\overline{1}$

19. $-0.3\overline{56}$

20. $6.0\overline{89}$

21. $0.18\overline{72}$

22. $11.\overline{510}$



23. **MP STRUCTURE** A forecast cone defines the probable path of a tropical cyclone. The probability that the center of a particular tropical cyclone remains within the forecast cone is $0.\overline{8}$. Write this probability as a fraction.

24. **MP STRUCTURE** Describe how to write a decimal with 12 repeating digits as a fraction.

25. **MP STRUCTURE** An approximation for the value of π is $\frac{22}{7}$. Write this number as a repeating decimal.

26. **MODELING REAL LIFE** The density of iodine is about 6.281 times the density of acetone. The density of acetone is about 785 kilograms per cubic meter. What is the density of iodine? Write your answer as a repeating decimal.

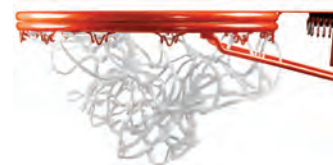
27. **MODELING REAL LIFE** A disinfectant manufacturer suggests that its product kills 99.98% of germs. Write this percent as a repeating decimal and then as a fraction. How many germs would survive when the disinfectant is applied to an object with 18,000 germs?



28. **MODELING REAL LIFE** You and your friend are making pear tarts for a bake sale. Your recipe uses $\frac{7}{6}$ times the weight of the diced pears used in your friend's recipe. Your friend's recipe calls for 0.3 pound of diced pears. How many pounds of pears should you buy to have enough for both recipes?

29. **MP PROBLEM SOLVING** The table shows the principal and interest earned per year for each of three savings accounts with simple annual interest. Which account has the greatest interest rate? Justify your answer.

	Principal	Interest Earned
Account A	\$90.00	\$4.00
Account B	\$120.00	\$5.50
Account C	\$100.00	\$4.80



30. **DIG DEEPER!** The probability that an athlete makes a half-court basketball shot is 22 times the probability that the athlete makes a three-quarter-court shot. The probability that the athlete makes a three-quarter-court shot is 0.009. What is the probability that the athlete makes a half-court shot? Write your answer as a percent.



MP NUMBER SENSE Determine whether the numbers are equal. Justify your answer.

31. $\frac{9}{22}$ and $0.40\overline{9}$

32. $\frac{1}{999}$ and 0

33. $\frac{135}{90}$ and 1.5

ADDING AND SUBTRACTING RATIONAL NUMBERS Add or subtract.

34. $0.40\overline{9} + 0.68\overline{1}$

35. $-0.6\overline{3} + \frac{5}{99}$

36. $\frac{11}{6} - 0.2\overline{7}$

37. $0.0\overline{3} - 0.0\overline{4}$

38. **MP STRUCTURE** Write a repeating decimal that is between $\frac{9}{7}$ and $\frac{10}{7}$. Justify your answer.



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
8.F.A.1

Learning Target

Understand the concept of a function.

Success Criteria

- Represent a relation as a set of ordered pairs.
- Determine whether a relation is a function.
- Use functions to solve real-life problems.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at BigIdeasMath.com.

ELL Support

Discuss the word *relation*.

Explain that a relation describes a connection between concepts, objects, or people. The word is most commonly used to explain that a person is in your family. You may want to demonstrate using the word to describe family relationships. Explain that the inputs and outputs of a function have a special relation. Discuss the meanings of the compound words *input* and *output* by analyzing their prefixes and the root word *put*.

Exploration 1

- The output is the square of the input; output: 64; *Sample answer:* input: 9; yes; The input 8 only has one possible output, but the output 81 has two possible inputs.
- The output is a possible color of the input (fruit); *Sample answer:* input: lime; output: green; yes; The missing input could be any fruit that is green.

Exploration 2

- See Additional Answers.

Laurie's Notes

Preparing to Teach

- **MP2 Reason Abstractly and Quantitatively:** In these explorations, students will complete mapping diagrams and describe relationships between quantities. Mathematically proficient students are able to make sense of quantities and their relationships in problem situations.

Motivate

- **?** "What is a vending machine and what does it do?" Give students time to explain how vending machines operate. Have them discuss the idea of inserting money (**input**) and getting the desired item (**output**).
- Share some history of vending machines.
 - A Greek mathematician invented a machine in 215 B.C. to vend holy water in Egyptian temples.
 - During the early 1880s, the first commercial coin-operated vending machines were introduced in London, England, and dispensed postcards.
 - Vending machines soon offered other things, including stamps. In Philadelphia, a completely coin-operated restaurant called Horn & Hardart was opened in 1902 and served customers until 1962.

Exploration 1

- Tell students that these diagrams are similar to a vending machine because they have an input and an output.
- **MP7 Look for and Make Use of Structure:** Mathematically proficient students will study the first diagram to discover a relationship between the inputs and the outputs. They will recognize that a **mapping diagram** is similar to a table of values.
- **?** If students are struggling, ask, "What can you do to the input to get the output?" As students offer conjectures, ask them if their conjectures work for each pair of values.

- In part (a), students should recognize that the missing output (yes) must make sense for both an apple and the missing input (fruit).
- Ask students to share their answers and reasoning for part (b).

Exploration 2

- Part (a) is very literal. Students state how many outputs there are for each input. In Play A, each input has exactly one output. In Play B, the inputs each have multiple outputs.
- In part (b), describing the ticket prices helps students realize how a mapping diagram can be used to show a relationship between two real-life quantities.
- Play B is more complicated than Play A. Students need to analyze the mapping diagram carefully.
- Ask volunteers to share their descriptions of the ticket prices for each play.
- In part (c), encourage students to use *Paired Verbal Fluency* to discuss their ideas. Circulate and listen to discussions, making note of any ideas that should be shared with the class.

Laurie's Notes

Discuss

? Ask a few questions about equation solving:

- "Does every equation have a solution?" **no**
- "Does every equation have just one solution?" **no**
- "Is it possible for an equation to have two solutions?" **yes** Students may say no, but using the third example below will convince them otherwise. They will study these types of equations in Chapter 9.
- Share some common equations and discuss the number of solutions:
 $x + 2 = 7$ **one solution, 5**
 $x + 2 = x + 7$ **no solution**
 $x^2 = 4$ **two solutions, 2 and -2**
 $x + 2 = x + 2$ **infinitely many solutions**
- Explain that in Examples 3 and 4, students will investigate equations that have no solution or infinitely many solutions. Assure students that they will use the same techniques for solving equations as before.

EXAMPLE 3

- **Teaching Tip:** Instead of telling students when an equation has no solution, work through the example and ask students about the "solution" $3 = -7$.
- Work through the problem as shown and then by first collecting the constant terms on one side of the equation. Show that the solution is the same both ways.

- **MP7 Look for and Make Use of Structure:** It may not be necessary to completely solve the equation. Students should notice that the same quantity, $4x$, is being subtracted from different numbers, 3 and -7 . They should reason that the two sides of the equation can never be equal, so there is no solution.

How do you know when an equation has no solution? **Solve the equation normally and if you end up with a false statement, the equation has no solution.**

Try It

- Have students use *Think-Pair-Share* as they solve these problems.

EXAMPLE 4

- **Teaching Tip:** Instead of telling students when an equation has infinitely many solutions, work through the example and ask students about the "solution" $4 = 4$.
- To check the solution, ask volunteers to choose several values for x .
- Substitute these values into the original equation and show that they all result in true statements.
- Students should notice that in the second step, the expressions on both sides of the equal sign are the same. They should reason that both sides will be equal for any value of x , so there are infinitely many solutions.

Try It

- Have students use *Think-Pair-Share* as they solve these problems.

Extra Example 3

Solve $3x - 5 = 7 + 3x$. **no solution**

Try It

5. no solution
6. no solution

Extra Example 4

Solve $\frac{1}{4}(8x - 12) = 2x - 3$.

infinitely many solutions

ELL Support

After demonstrating Examples 3 and 4, have students work in groups to discuss and complete Try It Exercises 5–8. Provide guiding questions: What properties are useful? How are they applied? Is there one solution, no solution, or infinitely many solutions? Expect students to perform according to their language levels.

Beginner: Write out each step of the process.

Intermediate: Discuss the problem using simple sentences.

Advanced: Use detailed sentences and help guide discussion.

Try It

7. infinitely many solutions
8. infinitely many solutions

EXAMPLE 3 Identifying the Solution of an Equation

What value of k makes the equation $k + 4 \div 0.2 = 5$ true?

- A. -15 B. -5 C. -3 D. 1.5

$$k + 4 \div 0.2 = 5 \quad \text{Write the equation.}$$

$$k + 20 = 5 \quad \text{Divide 4 by 0.2.}$$

$$\begin{array}{r} -20 \\ -20 \end{array} \quad \text{Subtraction Property of Equality}$$

$$k = -15 \quad \text{Simplify.}$$

▶ The correct answer is **A**.

Check

$$\begin{array}{r} k + 4 \div 0.2 = 5 \\ -15 + 4 \div 0.2 \stackrel{?}{=} 5 \\ -15 + 20 \stackrel{?}{=} 5 \\ 5 = 5 \quad \checkmark \end{array}$$

Try It Solve the equation. Check your solution.

8. $p - 8 \div \frac{1}{2} = -3$

9. $q + |-10| = 2$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING Are the equations equivalent? Explain.

10. $x + 3 = 4$ and $x = 1$

11. $-\frac{y}{5} = 2$ and $y = 10$

12. **OPEN-ENDED** Write an equation that you can use the Division Property of Equality to solve.

SOLVING EQUATIONS Solve the equation. Check your solution.

13. $-5 = w - 3$

14. $-\frac{2}{3}n = 8$

15. $p - 9 \div \frac{1}{2} = 6$

16. $q + |3| = -5$

17. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$x - 2 = 4$

$x - 3 = 9$

$x - 5 = 1$

$x - 6 = 0$

21. **MODELING REAL LIFE** Write and solve an equation to find the number of miles you must drive to have the same cost for each of the car rentals.



\$20 plus \$0.50 per mile



\$30 plus \$0.25 per mile

SOLVING AN EQUATION Solve the equation. Check your solution, if possible.

22. $x + 6 = x$ 23. $3x - 1 = 1 - 3x$ 24. $3x + 15 = 3(x + 5)$
25. $4x - 9 = 3.5x - 9$ 26. $\frac{1}{3}(9x + 3) = 3x + 1$ 27. $5x - 7 = 4x - 1$
28. $\frac{1}{2}x + \frac{1}{2}x = x + 1$ 29. $2x + 4 = -(-7x + 6)$ 30. $5.5 - x = -4.5 - x$
31. $-3(2x - 3) = -6x + 9$ 32. $10x - \frac{8}{3} - 4x = 6x$ 33. $6(7x + 7) = 7(6x + 6)$

34. **YOU BE THE TEACHER** Your friend solves the equation shown. Is your friend correct? Explain your reasoning.

35. **OPEN-ENDED** Write an equation with variables on both sides that has no solution. Explain why it has no solution.

$$\begin{aligned} -4(2n - 3) &= 12 - 8n \\ -8n + 12 &= 12 - 8n \\ -8n &= -8n \\ 0 &= 0 \end{aligned}$$

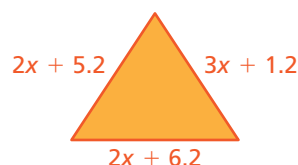
The solution is $n = 0$.

36. **MODELING REAL LIFE** A cable television provider charges \$75 for installation and \$39.96 per month for a basic entertainment package. A satellite television provider offers free installation and charges \$13.32 per month for service for each television. Your neighbor subscribes to the cable provider the same month you subscribe to the satellite provider. After how many months is your neighbor's total cost the same as your total cost when you own three televisions?



37. **MODELING REAL LIFE** A pizza parlor makes 52 pizza crusts the first week of summer and 180 pizza crusts each subsequent week. A diner makes 26 pizza crusts the first week of summer and 90 pizza crusts each subsequent week. In how many weeks will the total number of pizza crusts made by the pizza parlor be twice the total number of pizza crusts made by the diner?

38. **MP PRECISION** Is the triangle an equilateral triangle? Justify your answer.



3.4 Using Similar Triangles

Learning Target: Use similar triangles to find missing measures.

Success Criteria:

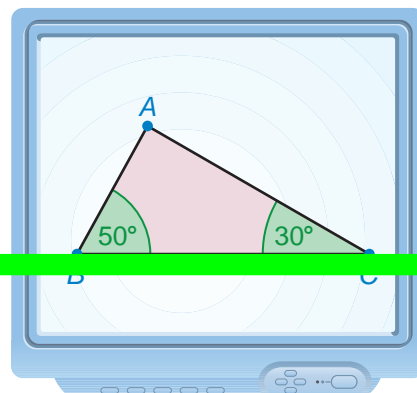
- I can use angle measures to determine whether triangles are similar.
- I can use similar triangles to solve real-life problems.

EXPLORATION 1

Drawing Triangles Given Two Angle Measures

Work with a partner. Use geometry software.

- a. Draw a triangle that has a 50° angle and a 30° angle. Then draw a triangle that is either larger or smaller that has the same two angle measures. Are the triangles congruent? similar? Explain your reasoning.



- b. Choose any two angle measures whose sum is less than 180° . Repeat part (a) using the angle measures you chose.

- c. Compare your results in parts (a) and (b) with other pairs of students. Make a conjecture about two triangles that have two pairs of congruent angles.

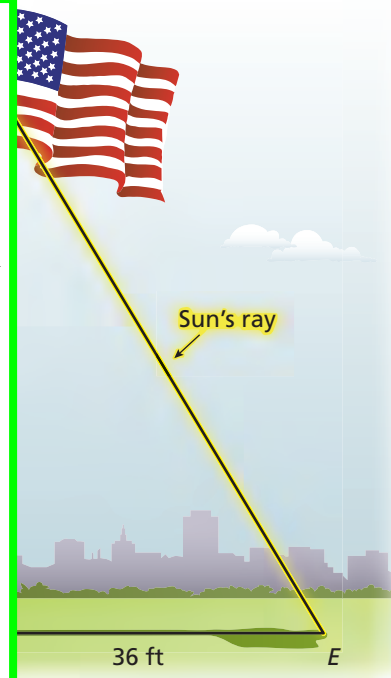
EXPLORATION 2

Using Indirect Measurement

Indicator 2g.i - In Exploration 1, students have to construct arguments about two triangles that have two pairs of congruent angles.

MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.



3.4 Practice



Go to [BigIdeasMath.com](https://www.BigIdeasMath.com) to get HELP with solving the exercises.

► Review & Refresh

Find the measure of each interior angle of the regular polygon.

1. octagon
2. decagon
3. 18-gon

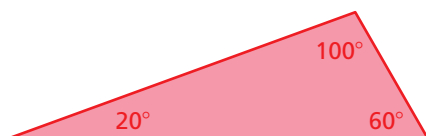
Solve the equation. Check your solution.

4. $3.5 + y = -1$
5. $9x = 54$
6. $-4 = \frac{2}{7}p$

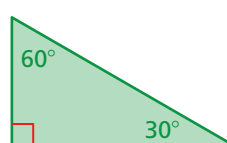
► Concepts, Skills, & Problem Solving

CREATING SIMILAR TRIANGLES Draw a triangle that is either larger or smaller than the one given and has two of the same angle measures. Explain why the new triangle is similar to the original triangle. (See Exploration 1, p. 123.)

7.

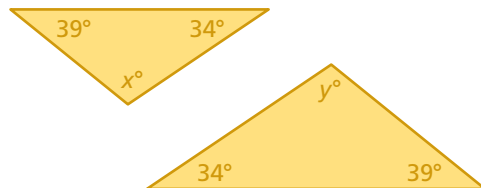


8.

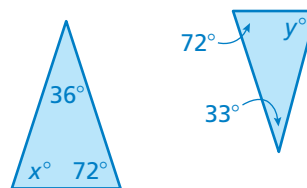


IDENTIFYING SIMILAR TRIANGLES Tell whether the triangles are similar. Explain.

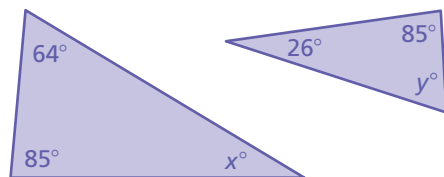
9.



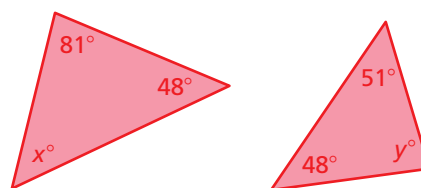
10.



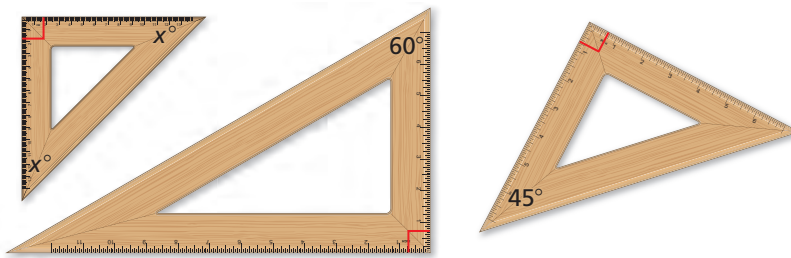
11.



12.



13. **GEOMETRY** Which of the rulers are similar in shape? Explain.



EXAMPLE 2 Graphing a Linear Equation in Slope-Intercept Form

Graph $y = -3x + 3$. Identify the x -intercept.

Step 1: Find the slope and the y -intercept.

$$y = -3x + 3$$

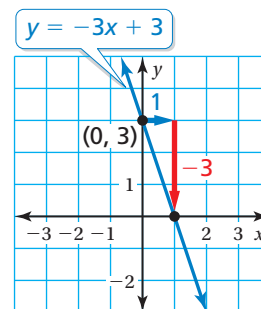
slope \rightarrow -3 \rightarrow y -intercept \rightarrow 3

Step 2: The y -intercept is 3. So, plot $(0, 3)$.

Step 3: Use the slope to find another point and draw the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-3}{1}$$

Plot the point that is **1 unit right** and **3 units down** from $(0, 3)$. Draw a line through the two points.



▶ The line crosses the x -axis at $(1, 0)$. So, the x -intercept is 1.

Try It Graph the linear equation. Identify the x -intercept.

3. $y = x - 4$

4. $y = -\frac{1}{2}x + 1$



Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

5. **IN YOUR OWN WORDS** Consider the graph of the equation $y = mx + b$.

- How does changing the value of m affect the graph of the equation?
- How does changing the value of b affect the graph of the equation?

IDENTIFYING SLOPE AND y -INTERCEPT Find the slope and the y -intercept of the graph of the linear equation.

6. $y = -x + 0.25$

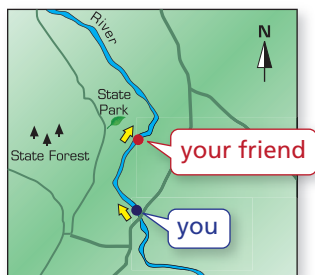
7. $y - 2 = -\frac{3}{4}x$

GRAPHING A LINEAR EQUATION Graph the linear equation. Identify the x -intercept.

8. $y = x - 7$

9. $y = 2x + 8$

19. **MODELING REAL LIFE** You have a total of 42 math and science problems for homework. You have 10 more math problems than science problems. How many problems do you have in each subject? Use a system of linear equations to justify your answer.
20. **MP PROBLEM SOLVING** A generator contains 60 gallons of fuel and uses 2.5 gallons per hour. A more efficient power generator contains 40 gallons of fuel and uses 1.5 gallons per hour. After how many hours do the generators have the same amount of fuel? Which generator runs longer? Justify your answers.

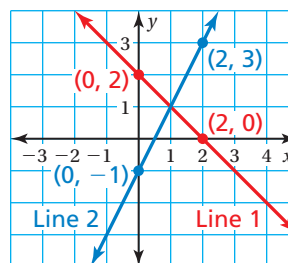


21. **MP PROBLEM SOLVING** You and your friend are in a canoe race. Your friend is a half mile in front of you and paddling 3 miles per hour. You are paddling 3.4 miles per hour.
- You are 8.5 miles from the finish line. How long will it take you to catch up to your friend?
 - You both maintain your paddling rates for the remainder of the race. How far ahead of your friend will you be when you cross the finish line?

OPEN-ENDED Write a system of linear equations that fits the description. Use a graph to justify your answer.

22. The solution of the system is a point on the line $y = -9x + 1$.
23. The solution of the system is $(3, -1)$.

24. **DIG DEEPER!** A graph of a system of two linear equations is shown. Write the system of linear equations represented by the graph. What is the solution of the system?



Month	Friend's Hair (in.)	Cousin's Hair (in.)
March	4	7
August	6.5	9

25. **CRITICAL THINKING** Your friend is trying to grow her hair as long as her cousin's hair. The table shows their hair lengths (in inches) in different months.
- Write a system of linear equations that represents this situation. Let $x = 1$ represent January.
 - Will your friend's hair ever be as long as her cousin's hair? If so, in what month?

26. **MP REASONING** Is it possible for a system of two linear equations to have multiple solutions? Explain your reasoning.

27. **GEOMETRY** The length of a rectangle is 8 feet more than its width. The perimeter of the rectangle is 72 feet. Find the width of the rectangle.

21. **MP REASONING** One equation in a system of linear equations has a slope of -3 . The other equation has a slope of 4 . How many solutions does the system have? Explain.

22. **MP LOGIC** How can you use the slopes and the y -intercepts of equations in a system of linear equations to determine whether the system has *one solution*, *infinitely many solutions*, or *no solution*?

23. **MP PROBLEM SOLVING** You and a friend both work two different jobs. The system of linear equations represents the total earnings (in dollars) for x hours worked at the first job and y hours worked at the second job. Your friend earns twice as much as you.



$$4x + 8y = 64$$

You

$$8x + 16y = 128$$

Your Friend

- One week, both of you work 4 hours at the first job. How many hours do you and your friend work at the second job?
- Both of you work the same number of hours at the second job. Compare the numbers of hours you and your friend work at the first job.

24. **MODELING REAL LIFE** You download a digital album for \$10.00. Then you and your friend each download the same number of individual songs for \$0.99 each. Write a system of linear equations that represents this situation. Will you and your friend spend the same amount of money? Explain.

25. **MODELING REAL LIFE** The table shows the research activities of two students at an observatory. How much does a student pay to use the telescope for one hour? the supercomputer for one hour?

	Telescope Use	Supercomputer Use	Total Cost
Student 1	5 hours	3 hours	\$70.50
Student 2	6 hours	2 hours	\$67.00

26. **MP REASONING** Does the system shown *always*, *sometimes*, or *never* have a solution when $a = b$? $a \geq b$? $a < b$? Explain your reasoning.

$$\begin{aligned} y &= ax + 1 \\ y &= bx + 4 \end{aligned}$$

Group	1	2	3
Number of Lift Tickets	36	24	18
Number of Ski Rentals	18	12	18
Total Cost (dollars)	684	456	432

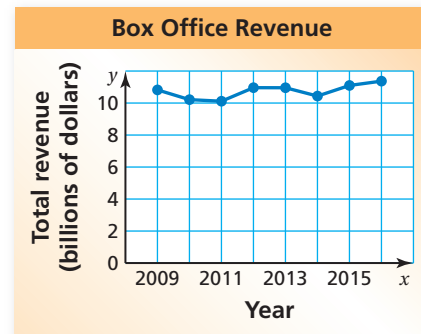
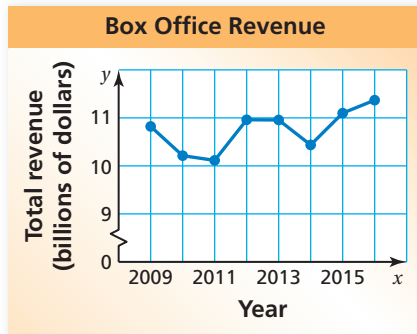
27. **MP LOGIC** The table shows the numbers of lift tickets and ski rentals sold to different groups. Is it possible to determine how much each lift ticket costs using the information for Groups 1 and 2? Groups 1 and 3? Justify your answers.

28. **DIG DEEPER!** Find the values of a and b so the system shown has the solution $(2, 3)$. Does the system have any other solutions for these values of a and b ? Explain.

$$\begin{aligned} 12x - 2by &= 12 \\ 3ax - by &= 6 \end{aligned}$$

EXAMPLE 3 Identifying a Misleading Data Display

Which line graph is misleading? Explain.

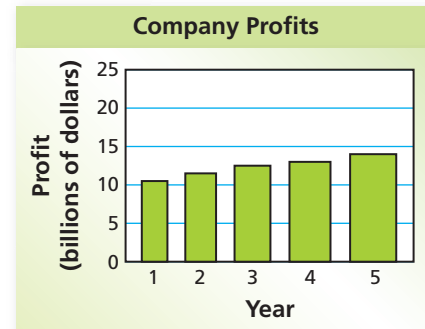


The vertical axis of the line graph on the left has a break (↗) and begins at 9. This graph makes it appear that the total revenue fluctuated drastically from 2009 to 2016. The graph on the right has an unbroken axis. It is more honest and shows that the total revenue changed much less from 2009 to 2016.

► So, the graph on the left is misleading.

Try It

6. Which bar graph is misleading? Explain.

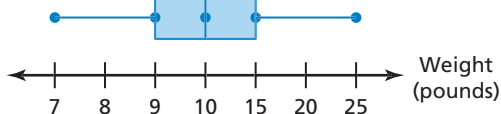


Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

CHOOSING A DATA DISPLAY Choose an appropriate data display for the situation. Explain your reasoning.

- the percent of band students playing each instrument
- a comparison of the amount of time spent using a tablet computer and the remaining battery life



- IDENTIFYING A MISLEADING DISPLAY** Is the box-and-whisker plot misleading? Explain.

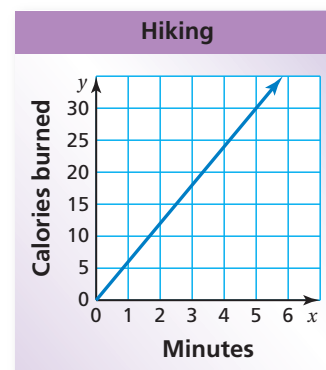
13. **INTERPRETING A LINEAR FUNCTION** The table shows the percent y (in decimal form) of battery power remaining x hours after you turn on a laptop computer.

Hours, x	0	2	4
Power Remaining, y	1.0	0.6	0.2

- Write and graph a linear function that relates y to x .
- Interpret the slope, the x -intercept, and the y -intercept.
- After how many hours is the battery power at 75%?

14. **MODELING REAL LIFE** The number y of calories burned after x minutes of kayaking is represented by the linear function $y = 4.5x$. The graph shows the number of calories burned by hiking.

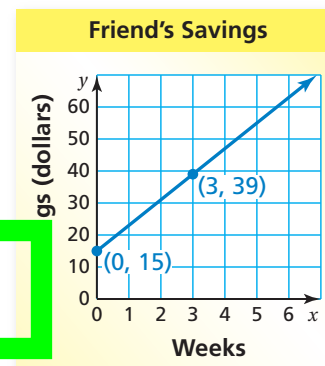
- Which activity burns more calories per minute?
- You perform each activity for 45 minutes. How many total calories do you burn? Justify your answer.



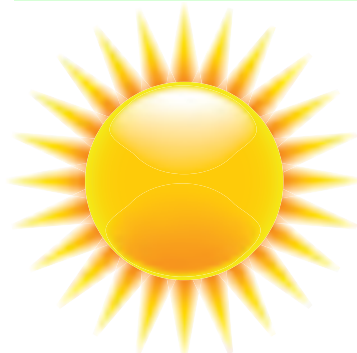
15. **DIG DEEPER!** You and a friend race each other. You give your friend a 50-foot head start. The distance y (in feet) your friend runs after x seconds is represented by the linear function $y = 14x + 50$. The table shows your distance at various times throughout the race. For what distances will you win the race? Explain.

Time (seconds), x	2	4	6	8
Distance (feet), y	38	76	114	152

16. **MP REASONING** You and your friend are saving money to buy bicycles that cost \$175 each. You have \$45 to start and save an additional \$5 each week. The graph shows the amount y (in dollars) that your friend has after x weeks. Who can buy a bicycle first? Justify your answer.



17. **CRITICAL THINKING** Is every linear equation a linear function? Explain your reasoning.



18. **MP PROBLEM SOLVING** The heat index is calculated using the relative humidity and the temperature. For every 1 degree increase in the temperature from 94°F to 97°F at 75% relative humidity, the heat index rises 4°F. On a summer day, the relative humidity is 75%, the temperature is 94°F, and the heat index is 124°F. Estimate the heat index when the relative humidity is 75% and the temperature is 100°F. Use a function to justify your answer.

8.1 Exponents

Learning Target: Use exponents to write and evaluate expressions.

- Success Criteria:**
- I can write products using exponents.
 - I can evaluate expressions involving powers.
 - I can use exponents to solve real-life problems.

The expression 3^5 is called a *power*. The *base* is 3. The *exponent* is 5.



EXPLORATION 1

Using Exponent Notation

Work with a partner.

- a. Copy and complete the table.

Power	Repeated Multiplication Form	Value
$(-3)^1$	-3	-3
$(-3)^2$	$(-3) \cdot (-3)$	9
$(-3)^3$		
$(-3)^4$		
$(-3)^5$		
$(-3)^6$		
$(-3)^7$		

Math Practice

Build Arguments

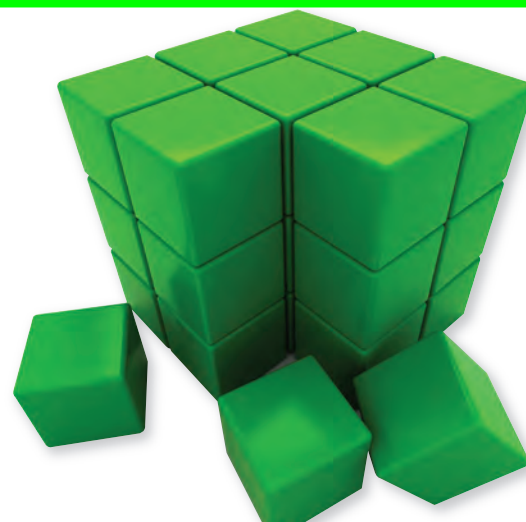
When is the value of $(-3)^n$ positive? negative?

- b. Describe what is meant by the expression $(-3)^n$. How can you find the value of $(-3)^n$?

EXPLORATION 2

Using Exponent Notation

Work with a partner. On a game show, each small cube is worth \$3. The small cubes are arranged to form a large cube. Show how you can use a power to find the total value of the large cube. Then write an explanation to convince a friend that your answer is correct.



Laurie's Notes

EXAMPLE 3

- This example involves two transformations. First the rectangle is rotated and then it is reflected. Work slowly and carefully through this example. Transparent paper and paperclips will be helpful for many students.
- ? "Can you visualize where the image will be after the two transformations?"
Answers will vary.
- Graph the original rectangle $ABCD$.
- ? "Does it matter whether you rotate the rectangle 90° clockwise or 90° counterclockwise?" yes
- To construct the image after the rotation, draw the segment OB , where O represents a point at the origin. Next locate vertex B' by drawing segment OB' so that the measure of $\angle BOB'$ is 90° and segments OB and OB' are the same length. After you locate vertex B' , the remaining vertices should be relatively easy to locate.
- Reflect rectangle $A'B'C'D'$ in the y -axis to obtain rectangle $A''B''C''D''$.
- Common Error:** Because the reflection resembles a translation, students may

? **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** "Are rectangles $ABCD$ and $A''B''C''D''$ the same size? Explain." Yes, because in both rotations and reflections the original figure and its image are the same size.

which you perform the transformations matters. (The answer is yes.) Have them thoroughly explain their reasoning.

Try It

- Think-Pair-Share:** Students should read the exercise independently and then work in pairs to solve the exercise. Have each pair compare their answer with another pair and discuss any discrepancies.

Self-Assessment for Concepts & Skills

- Students should work independently on these exercises.
- Popsicle Sticks:** Solicit explanations for Exercise 8.
- Thumbs Up:** Ask students to indicate their understanding of rotations.

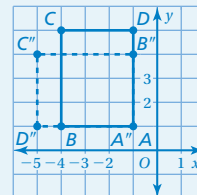
ELL Support

Allow students to work in pairs for extra support and to practice language. Check answers to Exercise 7 by having students indicate *yes* or *no* using a thumbs up or down signal. Then have each pair write the angle and direction of rotation on a whiteboard to hold up for your review. Have two pairs form a group to discuss their answers for Exercise 8. Monitor discussions and provide support. Have two groups present their explanations for one of the two answers and discuss as a class.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at BigIdeasMath.com.

Extra Example 3

The vertices of a rectangle are $A(-1, 1)$, $B(-4, 1)$, $C(-4, 5)$, and $D(-1, 5)$. Rotate the rectangle 90° clockwise about the origin, and then reflect it in the y -axis. What are the coordinates of the image?



$A''(-1, 1)$, $B''(-1, 4)$, $C''(-5, 4)$,
 $D''(-5, 1)$

Try It

- $P''(1, 2)$, $Q''(1, 0)$, $R''(-2, 0)$

Self-Assessment for Concepts & Skills

- yes; 90° counterclockwise
- What are the coordinates of the image after a 270° clockwise rotation about the origin?
 $A'(-4, 2)$, $B'(-4, 4)$,
 $C'(-1, 4)$; $A'(4, -2)$,
 $B'(4, -4)$, $C'(1, -4)$



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
8.EE.B.5, 8.EE.B.6

Learning Target

Graph proportional relationships.

Success Criteria

- Graph an equation that represents a proportional relationship.
- Write an equation that represents a proportional relationship.
- Use graphs to compare proportional relationships.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at BigIdeasMath.com.

ELL Support

Point out the heading of Exploration 2, *Deriving an Equation*. Explain that the word *derive* means “figure out.” Ask students what other words they might use to explain the word *derive*. Examples may be *get* or *understand*. Suggest that they keep a notebook of synonyms to help expand their knowledge of vocabulary.

Exploration 1

a–b. See Additional Answers.

Exploration 2

- corresponding side lengths are proportional; *Sample answer:* The corresponding angles of the triangles are congruent, so the triangles are similar.
- $\frac{y-0}{x-0} = \frac{m}{1}$; $y = mx$
- $y = mx$ is the general equation for two quantities x and y that are in a proportional relationship; m represents the slope of the line, constant of proportionality, or unit rate.

Laurie's Notes

Preparing to Teach

- In previous courses, students determined whether two quantities are proportional. In this chapter, they have graphed linear equations and found slopes of lines. Now students will graph and write an equation that represents a proportional relationship.
- MP2 Reason Abstractly and Quantitatively:** Mathematically proficient students make sense of the quantities and their relationships in problem situations. To develop this proficiency, students must be asked to interpret the meaning of a slope as a unit rate.

Motivate

- As a warm-up and to connect to prior content, tell students that x and y are in a proportional relationship. Have students complete the ratio table.

Minutes, x	1	2	4	6
Gallons, y	8.5			

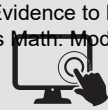
- MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask volunteers to justify their procedures and explain why their procedures show a proportional relationship.

Exploration 1

- In part (a), students should see how the additive structure in the table relates to the red and blue arrows in the graph, which gives them the rise and run.
- Common Error:** Students may confuse the slope ratio, saying, “slope is run over rise.” Tell students that they must “rise up” before they can “run,” so slope is rise over run.
- In part (b), students should use a similar additive structure to create their ratio tables. They should show that there is an increase of y fluid ounces of water for every increase of x fluid ounces of vinegar. So, the slope of the line is $\frac{y}{x}$, where y and x are real numbers based on the ratio chosen by students.

Exploration 2

- ? “What do you know about the two triangles?” *Sample answer: Their corresponding angles are congruent, so the triangles are similar.* If students do not mention similar, lead them to it. The triangles both have right angles and share a common angle at $(0, 0)$, so the third angles must be congruent.
- ? “When two triangles are similar, what do you know about their side lengths?” *Corresponding side lengths are proportional.*
- It may not be obvious to students that the vertical side of the smaller triangle has a length of m units and the horizontal side has a length of 1 unit. It may be even less obvious that the vertical side of the larger triangle has a length of y units and the horizontal side has a length of x units.
- Ask students to share their responses for part (c). They should realize that the equation $y = mx$ represents a proportional relationship, where m is the slope.



Check out the
digital flash cards.

BigIdeasMath.com

Laurie's Notes

Chapter Exploration

- **MP1 Make Sense of Problems and Persevere in Solving Them:** A scatter plot shows the relationship between two data sets. Students will find that there may be a positive linear relationship, a negative linear relationship, a nonlinear relationship, or no relationship.
- In Exercise 1, students may need help with scaling the axes. Students may want to use a broken vertical axis. You might let some students use a broken vertical axis and let others start at 0.
- If students use different scales for the vertical axis, have them display their scatter plots. Each plot will have a decreasing trend, but the steepness of the trend may look different.
- When students finish Exercise 1, ask volunteers to share their responses to parts (b) and (c).
- In Exercise 2, students should interpret the trend of each scatter plot in the context of the problem.
- Allow time for students to discuss the three descriptions and which scatter

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask volunteers to explain how they matched the descriptions and scatter plots. If students disagree, they should give supporting arguments for their own answers and explain what they dislike about the other answers.

Vocabulary

- These terms are from Chapter 6.1.
- Where have you seen these terms in the classroom?
- definition, b
- Allowing st
- understand
- When stud
- their Stude
- Chapter Re

ELL Supp

Explain that t
A date is a da
been collecte
a data display

Indicator 2g.ii - The Teaching Edition encourages teachers to ask probing questions to engage students in constructing arguments and analyzing the arguments of others. The note for the Chapter Exploration encourages teachers to have students explain how they matched the descriptions and scatter plots. If students disagree, they have to give arguments to support their answer and what about the other answers is wrong.

MP3 Construct viable arguments and critique the reasoning of others.

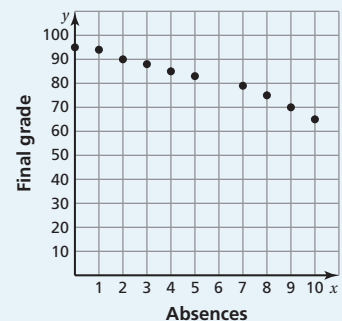
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Topics for Review

- Evaluating Expressions
- Making Data Displays
- Plotting Points
- Probability
- Writing a Linear Equation

Chapter Exploration

1. a. $(0, 95), (3, 88), (2, 90), (5, 83), (7, 79), (9, 70), (4, 85), (1, 94), (10, 65), (8, 75)$



- b. As the number of absences



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
Applying 8.SP.A.1

Learning Target

Use appropriate data displays to represent situations.

Success Criteria

- Choose appropriate data displays for situations.
- Identify misleading data displays.
- Analyze a variety of data displays.

Warm Up

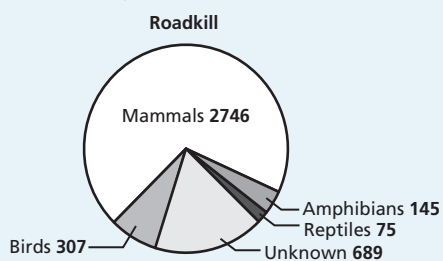
Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at BigIdeasMath.com.

ELL Support

Before beginning the lesson, review the types of data displays that students have studied in this chapter, scatter plots and two-way tables. Discuss the rationale for the use of each.

Exploration 1

a. *Sample answer:*



The circle graph shows the kinds of animals as parts of a whole.

- b. See Additional Answers.
- c. See Additional Answers.
- d. *Sample answer:* Reduce the speed limit on some roads.

Laurie's Notes

Preparing to Teach

- If your standards do not require choosing appropriate data displays for situations, you may skip this lesson and proceed to the next. However, knowing how to identify misleading data displays is useful in real life.

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** In this section, students will make decisions about how to display data. They will need to explain their reasoning for selecting a particular display. If two students select different data displays, it is important that they discuss the reasoning behind their choices.

Motivate

- The theme for the exploration is roadkill. While students may giggle at the thought, automobile accidents involving large animals can be serious.
- You may want to share your experiences with automobile accidents involving animals. Allow time for students to share personal stories too.
- Use the Internet to research and share vehicular data with students, such as the number of miles of roads in the U.S., the number of registered vehicles, the number of accidents, and the number of animal-related accidents.

Discuss

- Discuss the data displays with which students are familiar: pictograph, bar graph, line graph, circle graph, stem-and-leaf plot, histogram, dot plot, box-and-whisker plot, and scatter plot. Have students describe the features of each display.
- Discuss the different numerical tools students have for describing data: mean, median, mode, range, mean absolute deviation, quartile, and interquartile range.

Exploration 1

- Students need to decide which display makes sense for the type of data that they have. There may be more than one appropriate answer.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Discuss students' choices and their explanations.
- Possible data displays:
 - Part (a): a circle graph (what part of the whole set is each animal) or a bar graph (compare the different categories, although there is a large difference in bar heights: 75 to 2746)
 - Part (b): a scatter plot and line of best fit (pair data, show trend over time, and make predictions for the future) or a line graph
 - Part (c): a stem-and-leaf plot (spread of data), along with calculating the mean (about 16.7) and median (17.1)
 - Part (d): As a class, discuss students' ideas for minimizing the number of animals killed by vehicles.

Laurie's Notes

Scaffolding Instruction

- Students explored very large and very small numbers written in scientific notation using graphing calculators. Now they will convert between scientific notation and standard form.
- Emerging:** Students may struggle with converting numbers from standard form to scientific notation and vice versa. They will benefit from guided instruction for the Key Ideas and the examples.
- Proficient:** Students convert between scientific notation and standard form with ease. They should proceed to the Self-Assessment exercises.

Discuss

- Write the definition for **scientific notation**. There are two parts to the definition; the factor is a number n , with $1 \leq n < 10$, and it is multiplied by a power of 10 with an integer exponent.
- Emphasize that the factor is greater than or equal to 1 and less than 10, which means that the factor can be a decimal.

Key Idea

- Write the Key Idea.
 - If the number is greater than or equal to 10, the exponent is positive and the decimal point moves to the left.
 - If the number is between 0 and 1, the exponent is negative and the decimal point moves to the right.
- From their work in the previous section, students should find it reasonable that the exponent of the power of 10 is connected to place value.
- From previous courses, students should recall that when they “move the decimal point,” they are multiplying or dividing by a power of 10.

EXAMPLE 1

- Teaching Tip:** Have students underline the first nonzero digit and the digit to its right. In scientific notation, the decimal point is placed between these two digits.
- ?** “How do you read the number in part (a)?” **one hundred seventy-three million**
- FYI:** Drawing the movement of the decimal point under the numbers helps

? **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** “How do you know if the exponent of the power of 10 will be positive or negative?”
If the standard form of the number is greater than or equal to 10, the exponent is positive. If the standard form of the number is between 0 and 1, the exponent is negative.

Now the decimal point is under the 2. The number is thirty-two ten millionths.

Try It

- Students should work independently.
- Teaching Strategy:** Select a few students to share their work.

Teaching Strategy

When students are working alone or with partners on a problem, circulate to view different approaches. Students' work varies, revealing their thinking and conceptions about the mathematics. Make notes about the order in which you want to call on students, so you can control the sequence of responses. You do not want the first response to be the most polished or efficient. Look for work that clearly demonstrates the outcome(s).

Extra Example 1

- Write 2,450,000 in scientific notation.
 2.45×10^6
- Write 0.0000045 in scientific notation.
 4.5×10^{-6}

ELL Support

After demonstrating Example 1, have students work in pairs to discuss and complete Try It Exercises 1–6. Provide guiding questions such as, “Is the number greater than 10? Is the exponent positive or negative? How do you move the decimal point? How do you write the number in scientific notation?” Expect students to perform according to their language levels.

Beginner: Write the number.

Intermediate: State the number.

Advanced: Explain why the number is written as it is.

Try It

- 5×10^4
- 2.5×10^7
- 6.83×10^2
- 5×10^{-3}
- 3.3×10^{-7}
- 5.06×10^{-4}

Laurie's Notes

Extra Example 2

Approximate $\sqrt{23}$ to the nearest
(a) integer and (b) tenth.

- a. 5
b. 4.8

Try It

4. a. 3
b. 2.8
5. a. -4
b. -3.6
6. a. -5
b. -4.9
7. a. 4
b. 4.5

Extra Example 3

Which is greater, $\sqrt{0.49}$ or 0.71? 0.71

Try It

8. π ; π is to the right of $\sqrt{8}$ on a number line.
9. $\sqrt{26}$; $\sqrt{26}$ is to the right of $\sqrt[3]{65}$ on a number line.
10. $-\sqrt{2}$; $-\sqrt{2}$ is to the right of $-\sqrt[3]{10}$ on a number line.

EXAMPLE 2

- It is important for students to make an estimate before using a calculator. Use reasoning first!

? "What are the first 10 perfect squares?" 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

? "What type of number do you get when you take the square root of any of

? **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** "Between what two perfect squares is $\sqrt{71}$? How do you know?" 8 and 9 because $\sqrt{64} = 8$ and $\sqrt{81} = 9$, so $\sqrt{71}$ has to be a number between 8 and 9.

? "Is $\sqrt{71}$ closer to 8 or 9? Why? It is closer to 8 because 71 is closer to 64 than to 81.

- You may wish to allow students to calculate squares of decimals using a calculator.
- You could explore more about square roots using a calculator approximation. For example, $\sqrt{71} \approx 8.4261498$. So, you can rationalize that $\sqrt{71}$ is between 8 and 9, between 8.4 and 8.5, between 8.42 and 8.43, etc., by truncating the decimal.

Try It

- Ask volunteers to share their thinking about each problem.

EXAMPLE 3

- MP2 Reason Abstractly and Quantitatively:** A number line is used as a visual model. Students will ask where to place $\sqrt{35}$. Knowing that $\sqrt{35}$ is between $\sqrt{25}$ and $\sqrt{36}$ does not tell you where to graph it on the number line.

? "Is $\sqrt{35}$ closer to 5 or 6? Why?" It is closer to 6 because 35 is closer to 36 than to 25.

- Similarly, $\sqrt[3]{80}$ is between $\sqrt[3]{64} = 4$ and $\sqrt[3]{125} = 5$, but closer to 4 than to 5.
- Students should use calculators to check their approximations. Make sure they are comfortable using a calculator to find square roots and cube roots.
- Plot the approximations on a number line. Because $6 > 4$, $\sqrt{35} > \sqrt[3]{80}$.

Try It

- Ask volunteers to share their thinking about each problem.

ELL Support

After demonstrating Examples 2 and 3, have students work in pairs to discuss and complete Try It Exercises 4–10. Provide guiding questions such as, "What is the closest perfect square (or perfect cube)? What is its square root (or cube root)? What is the approximation to the nearest tenth?" Expect students to perform according to their language levels.

Beginner: Write tables of squares (or cubes) and identify the approximations.

Intermediate: Use phrases and simple sentences to answer the guiding questions and contribute to discussion.

Advanced: Use detailed sentences to answer and contribute to discussion.

Laurie's Notes

EXAMPLE 4

- Ask a volunteer to read the problem. Then ask another volunteer to interpret the problem. If the student cannot interpret the problem, ask them to read the problem again and ask a third student to explain it.
- You may need to review *mean* with students.

? "Is it equivalent to write $\frac{x + 3.5}{5} = 1.5$ instead of $\frac{3.5 + x}{5} = 1.5$? Explain." Yes, addition is commutative.

- FR:** It may be helpful to write the third step with parentheses: $5\left(\frac{\quad}{5}\right)$.
- MP2 Reason Abstractly and Quantitatively:** Ask students to explain the impact of trying to achieve a mean of 1.5 miles run per day when you ran 0 miles on two of the days.
- Note:** This is a classic question. When all of the data are known except for one, what is needed in order to achieve a particular average? Students often ask this in the context of wanting to know what they have to score on a test to achieve a certain average.

✓ Self-Assessment for Problem Solving

- Allow time in class for students to practice using the problem-solving plan. Remember, some students may only be able to complete the first step.
- Divide students into three groups. Assign each group one of Exercises 17–19. The groups may choose to subdivide into smaller groups, but they should work with the assigned problem. Students need to read and interpret the problem. Each group should write a verbal model or make a plan to solve the problem. Select two members of each group to present their verbal model or plan to the class.
- Students should solve the problems independently and then have their neighbors check their work.
- 👍 **Thumbs Up:** Have students indicate their understanding of the success criteria.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at BigIdeasMath.com.

Closure

- Solve $8x + 9 - 4x = 25$. Check your solution. $x = 4$

Extra Example 4

You have scored 7, 10, 8, and 9 on four quizzes. Find the score you need on the fifth quiz so that your mean

score is 8. $\frac{x + 7 + 10 + 8 + 9}{5} = 8$; 6

Self-Assessment for Problem Solving

- 53 action figures
- 50 min
- \$21;
 $6(38) + 6c = 354$
 $228 + 6c = 354$
 $6c = 126$
 $c = 21$

Learning Target

Write and solve multi-step equations.

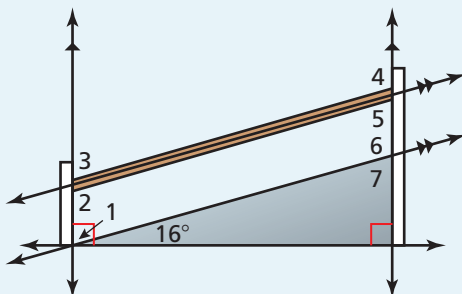
Success Criteria

- Apply properties to produce equivalent equations.
- Solve multi-step equations.
- Use multi-step equations to model and solve real-life problems.

Laurie's Notes

Extra Example 4

The ramp has a 16° incline. At what angles do you need to attach a rail to two parallel posts so that the rail is parallel to the incline of the ramp?



$\angle 2 = 106^\circ$, $\angle 3 = 74^\circ$, $\angle 4 = 106^\circ$,
 $\angle 5 = 74^\circ$

Self-Assessment for Problem Solving

10. 20; *Sample answer:* Because the lines are parallel, the a° angle and the angle adjacent to the a° angle also form a right angle. Using alternate interior angles, the measure of the angle adjacent to the a° angle is 70° . So, $a + 70 = 90$, and $a = 20$.

11. 73°

Learning Target

Find missing angle measures created by the intersections of lines.

Success Criteria

- Identify congruent angles when a transversal intersects parallel lines.
- Find angle measures when a transversal intersects parallel lines.

EXAMPLE 4

- Ask a volunteer to read the problem and another to explain what the problem is asking.
- Students could complete a Four Square as you work through the problem-solving plan.

Work through the solution as shown.

? "Is there another way to solve this problem? Explain." *yes; Sample answer:* You could use $\angle 2$ and $\angle 3$ instead of $\angle 4$ and $\angle 5$. There are many other ways to solve this problem. Some students may suggest using the sum of the angles in a triangle to find the measure of $\angle 2$. This idea will be formally developed in the next section.

answers for reasonableness.

- Point out that you would only need to find one of the angle measures if you were actually hanging the rail. Once you place the rail using one of the angles, the other angles will fall into place because the posts are parallel.

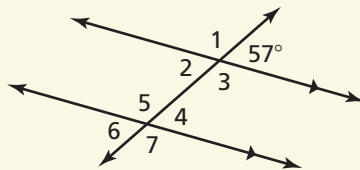
✓ Self-Assessment for Problem Solving

- Encourage students to use a Four Square to complete these exercises. Until students become comfortable with the problem-solving plan, they may only be ready to complete the first square.
- Neighbor Check:** Have students work independently to complete their Four Squares and then have their neighbors check their work. Have students discuss any discrepancies.
- Thumbs Up:** Have students indicate their understanding of each success criterion.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at BigIdeasMath.com.

Closure

- Find the measure of each angle. Explain your reasoning.



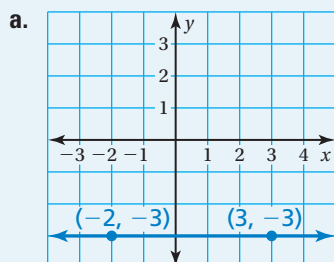
$\angle 1 = 123^\circ$, $\angle 2 = 57^\circ$, $\angle 3 = 123^\circ$, $\angle 4 = 57^\circ$, $\angle 5 = 123^\circ$, $\angle 6 = 57^\circ$, $\angle 7 = 123^\circ$;
Explanations will vary.

Try It

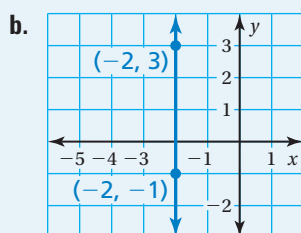
1. $-\frac{1}{5}$
2. $\frac{1}{3}$

Extra Example 2

Find the slope of each line.



0



undefined

ELL Support

After demonstrating Example 1, have students work in groups to discuss and complete Try It Exercises 1 and 2. Provide the guiding questions: Is the slope positive or negative? What formula do you use? What is the rise? What is the run? What is the slope? Expect students to perform according to their language levels.

Beginner: Write out the formula with the appropriate values.

Intermediate: Use phrases or simple sentences to answer the guiding questions.

Advanced: Use detailed sentences and help guide discussion.

Try It

3. 0
4. undefined

Laurie's Notes

Try It

- Have students use whiteboards to find each slope. Ask them to show their work so you can quickly evaluate their understanding.
- Common Error:** Students may confuse the slope ratio, saying, "slope is run over rise." Tell students that they must "rise up" before they can "run," so slope is rise over run.

EXAMPLE 2

? "How does a slope of $\frac{1}{2}$ compare to a slope of $\frac{1}{5}$? Describe the lines." A slope of $\frac{1}{2}$ runs 2 units for every 1 unit it rises. A slope of $\frac{1}{5}$ runs 5 units for each 1 unit it rises. A slope of $\frac{1}{5}$ is not as steep.

? "What would a slope of $\frac{1}{10}$ look like?" A slope of $\frac{1}{10}$ is less steep than a slope of $\frac{1}{5}$, so it is almost flat.

? "How steep do you think a horizontal line is?" Listen for students to describe a horizontal line as having no rise. In part (a), they will see it has a slope of 0.

As the lines get less steep, the lines approach a horizontal line.

- Work through part (a). There is no change in y . So, the change in y is 0.
- In part (b), ask a series of questions similar to part (a).

? "How does a slope of $\frac{9}{2}$ compare to a slope of $\frac{3}{2}$? Describe the lines." A slope of $\frac{9}{2}$ runs 2 units for every 9 units it rises. A slope of $\frac{3}{2}$ runs 2 units for every 3 units it rises. A slope of $\frac{9}{2}$ is steeper.

? "What would a slope of 10 look like?" A slope of 10 is steeper than a slope of $\frac{9}{2}$, so it is almost vertical.

? "How steep do you think a vertical line is?" Listen for students to describe a vertical line as having a slope of infinity.

Because division by zero is undefined, the slope of the line is undefined.

Try It

- Students should solve the problems on whiteboards. Discuss any discrepancies as a class.

Summary

- Students have computed the slopes of several lines. Have students discuss the Summary by working with a partner to provide an example of each type of slope.



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
8.EE.C.8b, 8.EE.C.8c

Learning Target

Understand how to solve systems of linear equations by elimination.

Success Criteria

- Add or subtract equations in a system.
- Use the Multiplication Property of Equality to produce equivalent equations.
- Solve a system of linear equations by elimination.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at BigIdeasMath.com.

ELL Support

Provide students with examples of elimination from everyday life, such as sports teams that lose playoff games or individuals voted off a reality show. If possible, have them provide their own examples. Then ask students how they think elimination is used to solve a

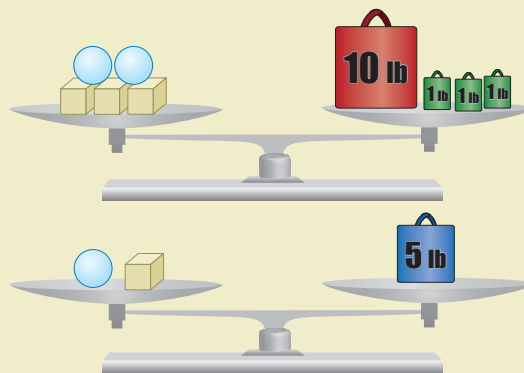
Laurie's Notes

Preparing to Teach

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students have the prerequisite knowledge for investigating a third technique for solving a system of linear equations. They will use more than one approach. Encourage students to persevere and try to understand why these approaches work.
- There are multiple avenues for solving a system using the elimination technique. Students may solve the system using several different approaches, all of which can lead to the same answer.

Motivate

- Draw a sketch of the two balance scales on the board. Make it clear that the right sides represent 13 pounds of weight and 5 pounds of weight.
- Have students draw a sketch of a balance scale with 4 cubes and 3 balls on the left and 18 pounds on the right. Relate this to adding equations.



- **?** Ask, "Is this scale balanced? Explain." **Yes, if you add equal weights to each side of a balanced scale, then the scale remains balanced.**
- Ask students to draw a sketch of a balance scale with 2 cubes and 1 ball on the left and 8 pounds on the right. Relate this to subtracting equations.
- **?** Ask, "Is this scale balanced? Explain." **Yes, if you subtract equal weights from both sides of a balanced scale, then the scale remains balanced.**

Exploration 1

Indicator 2g.ii - The Teaching Edition encourages teachers to ask probing questions to engage students in constructing arguments and analyzing the arguments of others. The notes for Exploration 1 encourages teachers to ask probing questions while students are working with a partner to complete the exploration. By doing this, students have the opportunity to critique the reasoning of others.

Exploration 1 is for students to discover that they can add equations in a system to eliminate a variable and solve

to examine the original system. Students may from Exploration 1 in Section 5.2, but it is good for them for solving the same system.

that students should solve the system using a method or graphing.

- the equations will only produce an equation in one variable when the equations in the system have one pair of like terms with opposite coefficients.
- System 1: yes, $(1, -2)$;
System 2: no; *Sample answer:* Replace Equation 2 with $6x - 4y = 26$; $(3, -2)$
 - Answers will vary.

- If students cannot find an entry point, have them write the system and then write $4x = -4$ below it.

- **?** Ask, "Is there are way to obtain $4x = -4$ from the original system? Explain." **Yes, add the two equations together.**
- **?** "How do you know that $4x = -4$ is an equivalent equation?" **Addition Property of Equality**
- In part (b), many students will add the two equations in System 1. Ask some students to subtract and compare the results.
- **?** "How can you obtain an equation in one variable in System 2?" **Sample answer: Multiply Equation 2 by 2 and then add the two equations together.**

Laurie's Notes

Scaffolding Instruction

- In the exploration, students read two-way tables. Now they will construct two-way tables and identify relationships between the categories of a two-way table.
- Emerging:** Students may be able to read a two-way table, but they need practice describing relationships between the data. They will benefit from guided instruction for the examples.
- Proficient:** Students read and interpret data in a two-way table with ease. They should work through Examples 2 and 3 before completing the Self-Assessment exercises.

Discuss

- Define **two-way table**. Emphasize that information is known about two categories from the same source. The focus in this lesson is drawing conclusions from the data in a two-way table.
- Refer to the exploration in explaining "information about two categories from the same source," such as soccer shirts-size and color.
- Define **joint frequency**. Each entry in the two-way table is a frequency for two categories, hence the name joint frequency.

EXAMPLE 1

- ? "What category do the rows represent?" **test grade: passed or failed**
 ? "What category do the columns represent?" **preparation of the student: studied or did not study**

Try It

- Have students display their answers on whiteboards. Discuss any discrepancies.

EXAMPLE 2

- Define **marginal frequencies**. Tell students that the sums of the rows and columns appear in the *margins* of the two-way table.
- Expand the two-way table. Label the new row and new column "Total."
- Add the rows and columns. Identify the sums using the labels shown.
- Ask students general questions about the row and column totals.
- Make sure students understand that 30 students were surveyed, not 60. Because each student is tallied twice, once for each category, you do not add $22 + 8 + 23 + 7$ to find the number surveyed. The sum of the rows and the sum of the columns should be equal.
- ? "What can you conclude about the data?" **Sample answer: Of the 30 students, all but one of those who studied for the test passed.**

Try It

- ? **Extension:** Ask students percent questions such as, "What percent of the students in the survey are *not* planning to attend either event?" **about 26%**

Extra Example 1

You randomly survey students about whether they like orange juice. The two-way table shows your results. How many female students in the survey like orange juice?

		Gender	
		Male	Female
Orange Juice	No	12	22
	Yes	37	29

29

Try It

- 1 student

Extra Example 2

Find and interpret the marginal frequencies for the survey in Extra Example 1. **A total of 34 students do not like orange juice. A total of 66 students like orange juice. A total of 49 male students participated in the survey. A total of 51 female students participated in the survey. A total of 100 students participated in the survey.**

ELL Support

Have students work in pairs to discuss and complete Try It Exercises 1 and 2. Have students ask each other questions that can be answered from the tables. For Exercise 2, have students find and interpret the marginal frequencies. Expect students to perform according to their language levels.

Beginner: Write the answer(s).

Intermediate: Use simple sentences to state the answer(s).

Advanced: Use detailed sentences to state the answer(s).

Try It

- 51 students will attend the game; 25 students will not attend the game; 40 students will attend the dance; 36 students will not attend the dance; 76 students were surveyed.



Check out the
Dynamic Classroom.

BigIdeasMath.com



STATE STANDARDS
8.F.A.3

Learning Target

Understand differences between linear and nonlinear functions.

Success Criteria

- Recognize linear functions represented as tables, equations, and graphs.
- Compare linear and nonlinear functions.

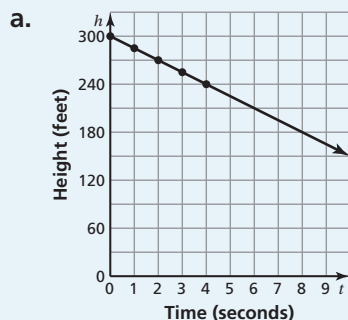
Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at BigIdeasMath.com.

ELL Support

Discuss the words *linear* and *nonlinear*. Ask students what the word *linear* means. Clarify any misunderstandings. Write the word *non/linear* on the board with a slash as shown. Ask students what the prefix *non-* means and what it does to the word *linear*. Guide them to understand that *non-* means “no” or “not,” and when used as a prefix the word has the opposite meaning.

Exploration 1



Sample answer: Plot points and connect them with a line; linear

b. See Additional Answers.

Laurie's Notes

Preparing to Teach

- **MP4 Model with Mathematics & MP8 Look for and Express Regularity in Repeated Reasoning:** The goal is for students to recognize when a pattern in real life is linear or nonlinear. Using falling objects, students will look for numeric patterns. The presence or absence of a *constant rate of change* will help students determine whether the data represent a linear or nonlinear function.

Motivate

- “How many of you would like to try skydiving? Why?”
- Share with students that the first successful parachute jump made from a moving airplane was made by Captain Albert Berry in St. Louis, in 1912.
- The first parachute jump from a balloon was completed by André-Jacques Garnerin in 1797 over Monceau Park in Paris.
- Tell students that today they will explore whether the function that describes the height of a skydiver is linear or nonlinear.
- Students will study many types of **nonlinear functions**, such as quadratic functions, radical functions, and rational functions, in future mathematics courses.

Exploration 1

- Discuss the two falling objects—one with a parachute and one that is free-falling.

“Is there a difference in the rate at which two objects fall when one is attached to a parachute and the other is left to free-fall? Explain.” Listen for discussion of rate. It is unlikely students will bring up acceleration.

small figurine to model a parachute-controlled fall. Then model a free-fall.

- Students may graph the skydiver equation using the slope and y-intercept, but

“Do the scales for the x- and y-axis need to be the same? Explain.” **No, one scale represents the height, which includes larger numbers and the other scale represents time, which includes smaller numbers.**

- After students have plotted the points, ask about the two graphs. First note that the two graphs begin at the same height (y-intercept), 300 feet.
- “How far has the skydiver fallen after 4 seconds?” **60 feet** “How far has the bowling ball fallen after 4 seconds?” **256 feet**
- “What is the difference in the way the graphs look?” **The graph of the skydiver appears to be a line and the graph of the bowling ball is not.**
- “Describe the fall of the skydiver.” **falling at a constant rate of 15 feet per second**
- “Describe the fall of the bowling ball. Why is it not a straight line?” Listen for students to describe that the bowling ball is picking up speed as it falls.
- “How are the two equations different?” Students should recognize that there is an exponent in the bowling ball equation but not the skydiver equation.