BIL Counter Evidence to Ed Reports Alignment, Grade 7

GATEWAY TWO: Rigor and Mathematical Practices

Rigor and Balance

Each grade's instructional materials reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by helping students develop conceptual understanding, procedural skill and fluency, and application.

Indicator 2a -- Attention to conceptual understanding: Materials develop conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings.

Ed Reports Review	BIL Counter Evidence
Indicator 2a -- The instructional materials do not always provide students opportunities to independently demonstrate conceptual understanding throughout the grade-level. The shift from conceptual understanding, most prevalent in the Exploration Section, to procedural understanding occurs within the lesson. The Examples and “Concepts, Skills, and Problem Solving” sections have a focus that is primarily procedural with limited opportunities to demonstrate conceptual understanding.	Conceptual problems are intentionally included throughout the program. Each section begins with an Exploration where students develop conceptual understanding. In every lesson, each example example is directly followed by a set of Try It exercises that provide students immediate opportunity to independently practice the concept. There is always a Self-Assessment for Concepts & Skills set that practices the skills and always includes at least one conceptual problem. Also, every Concepts, Skills, & Problem Solving set always contains at least one conceptual problem. For example: Self-Assessment for Concepts & Skills • ***1.1 #7-12, page 5*** • 2.2 #10 and #14, page 57 • 3.2 #7-8, page 99 • 7.2 #6-7, page 294 Concepts, Skills, & Problem Solving • ***1.4 #12-13, page 27*** • 3.2 #8-9, page 101 • 6.5 #8-9, page 263

Indicator 2c -- Attention to Applications: Materials are designed so that teachers and students spend sufficient time working with engaging applications of the mathematics, without losing focus on the major work of each grade

Ed Reports Review	BIL Counter Evidence
Indicator 2c -- The instructional materials present opportunities for students to engage in application of grade-level mathematics; however, the problems are scaffolded through teacher-led questions and procedural explanation. The last example of each lesson is titled, “Modeling Real Life,” which provides a real-life problem involving the key standards addressed for each lesson. This section provides a step-by-step solution for the problem; therefore, students do not fully engage in application. Throughout the series, there are examples of routine application problems that require both single and multi-step processes; however, there are limited opportunities to engage in non-routine problems.	In every lesson, each Modeling Real Life example is directly followed by a set of Self-Assessment For Problem Solving exercises that provide students immediate opportunity to independently engage in routine and non-routine application problems. Students have similar opportunities in the Concepts, Skills, & Problem Solving and Connecting Concepts. Examples of non-routine problems: Self-Assessment For Problem Solving • 1.1 #14, page 6 • 2.1 #17, page 52 • ***3.2 #9, page 100*** • 4.1 #11, page 130 • 6.3 #13-14, page 250 • 7.3 #9, page 303 Concepts, Skills, & Problem Solving • ***2.1 #48, page 54*** • 3.4 #46-47, page 114 • 4.5 #29, page 156 • 5.2 #26-28, page 194 Connecting Concepts • ***Chapter 6 #3, page 271*** • Chapter 10 #3, page 445

Boldfaced italic type indicates pages where additional explanation is annotated on the supporting pages of this document.

Indicator 2d -- Balance: The three aspects of rigor are not always treated together and are not always treated separately. There is a balance of the 3 aspects of rigor within the grade.

Ed Reports Review	BIL Counter Evidence
Indicator 2d -- The instructional materials for Big Ideas Math: Modeling in Real Life, Grade 7 partially meet expectations that the three aspects of rigor are not always treated together and are not always treated separately.	The Big Ideas Math: Modeling Real Life program consistently across Grades K - 8 strives for a balanced approach to rigor. Each section develops a concept from conceptual understanding (explorations) to procedural fluency (skill examples) to rigorous application (Modeling Real Life examples), engaging students in the mathematics and promoting active learning. Every set of practice problems reflects this balance, giving students the rigorous practice they need to achieve mastery. The Teaching Edition front matter was updated in a recent printing to provide detail on the program philosophy concerning rigor: • ***Front matter, page xxiii***
Indicator 2d -- The instructional materials present opportunities in most lessons for students to engage in each aspect of rigor, however, these are often treated together. There is an over-emphasis on procedural skill and fluency.	The following are examples where conceptual understanding is treated by itself or is the focus. Chapter Explorations • ***Chapter 4 Exploration #1-2, page 126*** • Chapter 6 Exploration #1-8, page 234 • Chapter 9 Exploration #1-3, page 360 The following are examples where application is treated by itself or is the focus. Connecting Concepts and Performance Task • Chapter 1, page 37 • Chapter 2, page 79 • Chapter 3, page 115 • ***Chapter 4, page 171*** • Chapter 8, page 349

Mathematical Practice - Content Connections

Materials meaningfully connect the Standards for Mathematical Content and the Standards for Mathematical Practice.

Indicator 2e -- The Standards for Mathematical Practice are identified and used to enrich mathematics content within and throughout each applicable grade.

Ed Reports Review	BIL Counter Evidence
Indicator 2e -- The MPs are identified within some sections in both the Teaching Edition (Exploration and Example sections) and Student Edition (Exploration 1 [within blue boxes], Concept, Skills & Problem Solving section). In the Student Edition, MPs are labeled with "MP" and a shortened of version of the MP, such as "Structure, Reasoning, Construct Arguments, Precision, etc." There is no document that correlates the abbreviated title with the Standards for Mathematical Practice. For example, the label "MP Number Sense" could align to several MPs. Additionally, Big Ideas Math: Modeling in Real Life, Grade 7, added "MP Logic" as a Mathematical Practice. This added practice does not align with the CCSSM Standards for Mathematical Practice.	We have provided a correlation online at *bigideasmath.com* for students, aligning the MP labels and other headings in the Student Edition with the Standards for Mathematical Practice. Big Ideas Learning will also send the correlation to existing users of our program. The correlation will also be included in future textbook printings. The page is attached here for your reference. • ***Front matter, page vi***

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Indicator 2f -- Materials carefully attend to the full meaning of each practice standard.	
Ed Reports Review	**BIL Counter Evidence**
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP1: Make Sense of Problems and Persevere in Solving Them. The instructional materials present few opportunities for students to make sense of problems and persevere in solving them.	While the examples are stepped out for students, they illustrate opportunities for students to make sense of problems and persevere in solving them when they independently solve the related problems. Students are encouraged to use the methods shown and the Problem-Solving Plan to think through and solve problems. For example: • 2.5 Example 3 & Self-Assessment for Problem Solving #12-13, page 76 • 3.2 Example 3 & Self-Assessment for Problem Solving #9-10, page 100 • 4.3 Example 4 & Self-Assessment for Problem Solving #18-19, page 142 ***• 9.3 Example 3 & Self-Assessment for Problem Solving #5-6, page 378*** Connecting Concepts at the end of each chapter encourage students to make sense of problems and persevere in solving them. For example: • Chapter 1, page 37 ***• Chapter 3, page 115*** • Chapter 5, page 223 • Chapter 9, page 397 All non-routine problems listed under Indicator 2c also cover MP1. Teaching Edition notes labeled MP1 give opportunities for the teacher to emphasize these habits to students and for students to use them going forward. For example: • 2.3 page T-61 • 5.4 page T-205 • 9.3 page T-375 ***• 10.4 page T-427*** The Teaching Edition front matter was updated in a recent printing to emphasize opportunities for in-class problem solving throughout the program. ***• Front Matter, page xxiv***
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP2: Reason Abstractly and Quantitatively. The instructional materials present few opportunities for students to Reason Abstractly and Quantitatively.	Students have ample opportunities to reason abstractly and quantitatively throughout our program. In addition to the Explorations where students investigate math to understand the reasoning behind the rules, students must use reasoning to solve problems and explain their answers. For example: Explorations • 1.1 Exploration 1, page 3 ***• 2.1 Exploration 1, page 49*** • 4.6 Exploration 1, page 157 • 7.1 Exploration 1, page 283 Concepts, Skills, & Problem Solving • 1.1 #48-51, page 8 ***• 1.4 #46-49, page 28*** • 2.2 #45, page 60 • 2.4 #10-12, page 71 • 3.2 #32, page 102 • 4.6 #44, page 164
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP4: Model with Mathematics. The instructional materials present few opportunities for students to model with mathematics.	Modeling with mathematics is covered throughout our program. Every Modeling Real Life example is directly followed by corresponding problems for students to engage in MP4. In addition, every Concepts, Skills, & Problem Solving set contains multiple opportunities for students to model with mathematics in the Modeling Real Life exercises. For example: Self-Assessments for Problem Solving: Concepts, Skills, & Problem Solving: ***• 7.3 Self-Assessment #8-9, page 303*** • 7.3 #33-34, page 306 • 9.2 Self-Assessment #9-10, page 372 • 9.2 #14,15,20,21, page 374 • 9.4 Self-Assessment #15-17, page 385 ***• 9.4 #41-43, page 388***

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Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP5: Use Appropriate Tools Strategically. The instructional materials present few opportunities for students to choose their own tool, therefore, the full meaning of MP5 is not being attended to.	Students have opportunities to choose tools strategically. For example: • ***2.1 Concepts, Skills, & Problem Solving #9-11, page 53*** • 7.4 Exploration 1 Math Practice note, page 307 In the Dynamic Student Edition, students have access to the following mathematical tools at all times. • Algebra tiles • Balance scale • Desmos geometry tool • Desmos graphing calculator • Four function calculator • Fraction models • Number line • Place value • Probability tools • Scientific calculator • Simulation
Indicator 2g.i -- Materials prompt students to construct viable arguments and analyze the arguments of others concerning key grade-level mathematics detailed in the content standards.	
Ed Reports Review	**BIL Counter Evidence**
Indicator 2g.i -- The Student Edition labels MP3 as "MP Construct Arguments," however, these activities do not always require students to construct arguments. In the Student Edition, "Construct Arguments" was labeled only once for students and "Build Arguments" was labeled once for students.	We suggest that when explaining or comparing answers, students must use what they have learned in building a logical progression of statements that defends their answers. The ability to critique someone else's reasoning also helps students analyze their own work and formulate good explanations. For example: • 1.1 Exploration 1c, page 3 • 2.1 Self-Assessment for Concepts & Skills #14-15, page 51 • 3.1 Self-Assessment for Concepts & Skills #15, page 93 • 4.2 #39, page 138 • 4.4 Self-Assessment for Concepts & Skills #6, page 147 • ***4.4 #34, page 150*** • 5.1 Explorations 1 and 2, page 183 • 5.3 #47, page 202 • 5.4 #55, page 210
Indicator 2g.ii -- Materials assist teachers in engaging students in constructing viable arguments and analyzing the arguments of others concerning key grade-level mathematics detailed in the content standards.	
Ed Reports Review	**BIL Counter Evidence**
Indicator 2g.ii -- There are some missed opportunities where the materials could assist teachers in engaging students in both constructing viable arguments and analyzing the arguments of others.	The Teaching Edition contains many instances of guidance, along with probing questions the teacher can ask, to engage students in constructing arguments and analyzing the arguments of others. These are often indicated with either a MP3 inline head or a red "?" icon. For example: MP3 inline head • ***1.2 page T-11*** • 4.6 page T-159 • 5.6 page T-220 • 9.4 page T-381 • 9.4 page T-382 Red "?" icon • 1.1 page T-3 • 1.5 page T-32 • ***2.1 page T-51*** • 4.7 page T-168 • 5.3 page T-196 • 6.5 page T-262 • 7.1 page T-284

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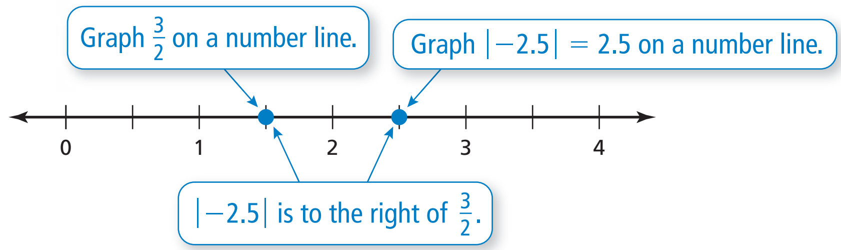
EXAMPLE 2 Comparing Rational Numbers

Compare $|-2.5|$ and $\frac{3}{2}$.



So, $|-2.5| > \frac{3}{2}$.

Remember

Two numbers that are the same distance from 0 on a number line, but on opposite sides of 0, are called *opposites*. The opposite of a number a is $-a$.

Try It **Copy and complete the statement using <, >, or =.**

4. $|9| \;\square\; |-9|$ **5.** $-\left|\frac{1}{2}\right| \;\square\; -\frac{1}{4}$ **6.** $7 \;\square\; -|-4.5|$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. VOCABULARY Which of the following numbers are integers?

$$9,\ 3.2,\ -1,\ \frac{1}{2},\ -0.25,\ 15$$

8. VOCABULARY What is the absolute value of a number?

COMPARING RATIONAL NUMBERS **Copy and complete the statement using <, >, or =. Use a number line to justify your answer.**

9. $3.5 \;\square\; \left|-\frac{7}{2}\right|$ **10.** $\left|\frac{11}{4}\right| \;\square\; |-2.8|$

11. WRITING You compare two numbers, a and b. Explain how $a > b$ and $|a| < |b|$ can both be true statements.

12. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$|6|$ 6 -6 $|-6|$

Indicator 2a - In #7-12, students demonstrate their conceptual understanding of rational numbers by using new vocabulary, using number lines, and explaining their reasoning.

EXAMPLE 3 Evaluating Expressions

Find the value of each expression when $x = 8$ and $y = -4$.

a. $\dfrac{x}{2y}$

$\dfrac{x}{2y} = \dfrac{8}{2(-4)}$ — Substitute 8 for x and -4 for y.

$= \dfrac{8}{-8}$ — Multiply 2 and -4.

$= -1$ — Divide 8 by -8.

The value of the expression is -1.

b. $-x^2 + 12 \div y$

$-x^2 + 12 \div y = -8^2 + 12 \div (-4)$ — Substitute 8 for x and -4 for y.

$= -(8 \cdot 8) + 12 \div (-4)$ — Write 8^2 as repeated multiplication.

$= -64 + 12 \div (-4)$ — Multiply 8 and 8.

$= -64 + (-3)$ — Divide 12 by -4.

$= -67$ — Add.

The value of the expression is -67.

Try It **Evaluate the expression when $a = -18$ and $b = -6$.**

7. $a \div b$ **8.** $\dfrac{a+6}{3}$ **9.** $\dfrac{b^2}{a} + 4$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. WRITING What can you conclude about two integers whose quotient is (a) positive, (b) negative, or (c) zero?

DIVIDING INTEGERS **Find the quotient.**

11. $-12 \div 4$ **12.** $\dfrac{-6}{-2}$ **13.** $15 \div (-3)$

14. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$\dfrac{10}{-5}$ $\dfrac{-10}{5}$ $\dfrac{-10}{-5}$ $-\dfrac{10}{5}$

To subtract one linear expression from another, add the opposite of each term in the expression. You can use a vertical or a horizontal method.

EXAMPLE 2 Subtracting Linear Expressions

Find each difference.

a. $(5x + 6) - (-x + 6)$

Vertical method: Align like terms vertically and subtract.

$$\begin{array}{r} (5x + 6) \\ \underline{-\ (-x + 6)} \end{array} \quad \xrightarrow{\text{Add the opposite.}} \quad \begin{array}{r} 5x + 6 \\ \underline{+\quad x - 6} \\ 6x \end{array}$$

The difference is $6x$.

b. $(7y + 5) - (8y - 6)$

Horizontal method: Use properties of operations to group like terms and simplify.

$$\begin{aligned} (7y + 5) - (8y - 6) &= (7y + 5) + (-8y + 6) && \text{Add the opposite.} \\ &= 7y + (-8y) + 5 + 6 && \text{Commutative Property of Addition} \\ &= [7y + (-8y)] + (5 + 6) && \text{Group like terms.} \\ &= -y + 11 && \text{Combine like terms.} \end{aligned}$$

The difference is $-y + 11$.

Common Error

When subtracting an expression, make sure you add the opposite of each term in the expression, not just the first term.

Try It **Find the difference.**

5. $(m - 3) - (-m + 12)$

6. $(-2c + 5) - (6.3c + 20)$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. **WRITING** Describe how to distinguish a linear expression from a nonlinear expression. Give an example of each.

8. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is x more than $3x - 1$?	Find $3x - 1$ decreased by x.
What is the difference of $3x - 1$ and x?	Subtract $(x + 1)$ from $3x$.

EXAMPLE 4 Using an Experimental Probability

Color	Frequency
Blue	3
Green	12
Red	9
Yellow	6

A bag contains 50 marbles. You randomly draw a marble from the bag, record its color, and then replace it. The table shows the results after 30 draws. Predict the number of red marbles in the bag.

Find the experimental probability of drawing a red marble.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{total number of trials}}$$

You draw red 9 times.

$$P(\text{red}) = \frac{9}{30} = \frac{3}{10}$$

You draw a total of 30 marbles.

To make a prediction, multiply the probability of drawing red by the total number of marbles in the bag.

$$\frac{3}{10} \cdot 50 = 15$$

So, you can predict that there are 15 red marbles in the bag.

Try It

5. An inspector randomly selects 200 pairs of jeans and finds 5 defective pairs. About how many pairs of jeans do you expect to be defective in a shipment of 5000?

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

6. **VOCABULARY** Explain what it means for an event to have a theoretical probability of 0.25 and an experimental probability of 0.3.

7. **DIFFERENT WORDS, SAME QUESTION** You flip a coin and record the results in the table. Which is different? Find "both" answers.

Heads	Tails
32	28

What is the experimental probability of flipping heads?

What fraction of the flips can you expect a result of heads?

What percent of the flips result in heads?

What is the relative frequency of flipping heads?

1.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Find the sum. Write fractions in simplest form.

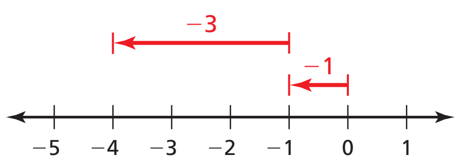
1. $\frac{5}{9} + \left(-\frac{2}{9}\right)$ **2.** $-8.75 + 2.43$ **3.** $-3\frac{1}{8} + \left(-2\frac{3}{8}\right)$

Add.

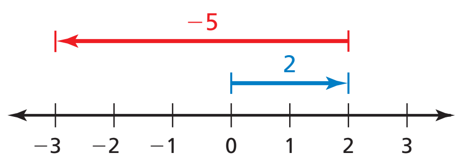
4. $2.48 + 6.711$ **5.** $12.807 + 7.116$ **6.** $18.7126 + 14.033$

Write an addition expression represented by the number line. Then find the sum.

7.



8.



Concepts, Skills, & Problem Solving

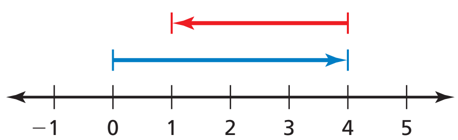
Indicator 2a - In #12-13, students use number lines to write and evaluate addition and subtraction expressions.

R COUNTERS **Use integer counters to find the difference.**
, p. 23.)

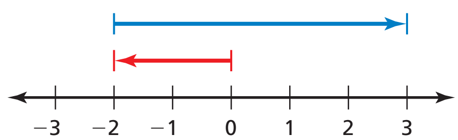
10. $1 - 4$ **11.** $-2 - (-6)$

USING NUMBER LINES **Write an addition expression and write a subtraction expression represented by the number line. Then evaluate the expressions.**

12.



13.



SUBTRACTING INTEGERS **Find the difference. Use a number line to verify your answer.**

14. $4 - 7$ **15.** $8 - (-5)$ **16.** $-6 - (-7)$ **17.** $-2 - 3$

18. $5 - 8$ **19.** $-4 - 6$ **20.** $-8 - (-3)$ **21.** $10 - 7$

22. $-8 - 13$ **23.** $15 - (-2)$ **24.** $-9 - (-13)$ **25.** $-7 - (-8)$

26. $-6 - (-6)$ **27.** $-10 - 12$ **28.** $32 - (-6)$ **29.** $0 - (-20)$

30. YOU BE THE TEACHER Your friend finds the difference. Is your friend correct? Explain your reasoning.

$7 - (-12) = 7 + 12 = 19$

3.2 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Simplify the expression.

1. $4f + 11f$
2. $b + 4b - 9b$
3. $-4z - 6 - 7z + 3$

Evaluate the expression when $x = -\frac{4}{5}$ and $y = \frac{1}{3}$.

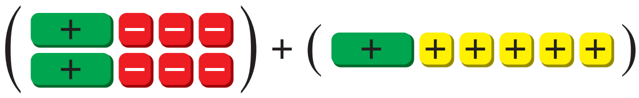
4. $x + y$
5. $2x + 6y$
6. $-x + 4y$

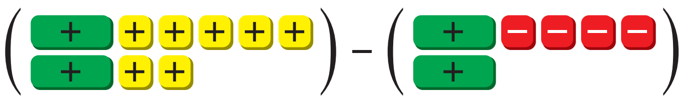
7. What is the surface area of a cube that has a side length of 5 feet?

A. 25 ft^2 **B.** 75 ft^2 **C.** 125 ft^2 **D.** 150 ft^2

Concepts, Skills, & Problem Solving

USING ALGEBRA TILES **Write the sum or difference modeled by the algebra tiles. Then use the algebra tiles to simplify the expression.** (See Exploration 1, p. 97.)

8. 

9. 

ADDING LINEAR EXPRESSIONS **Find the sum.**

10. $(n + 8) + (n - 12)$
11. $(7 - b) + (3b + 2)$
12. $(2w - 9) + (-4w - 5)$
13. $(2x - 6) + (4x - 12)$
14. $(-3.4k - 7) + (3k + 21)$
15. $\left(-\frac{7}{2}z + 4\right) + \left(\frac{1}{5}z - 15\right)$
16. $(6 - 2.7h) + (-1.3j - 4)$
17. $\left(\frac{7}{4}x - 5\right) + (2y - 3.5) + \left(-\frac{1}{4}x + 5\right)$

18. **MODELING REAL LIFE** While catching fireflies, you and a friend decide to have a competition. After m minutes, you have $(3m + 13)$ fireflies and your friend has $(4m + 6)$ fireflies.

a. How many total fireflies do you and your friend catch? Explain your reasoning.

b. The competition lasts 3 minutes. Who has more fireflies? Justify your answer.

6.5 Practice

Go to *BigIdeasMath.com* to get HELP with solving the exercises.

Review & Refresh

Identify the percent of change as an *increase* or a *decrease*. Then find the percent of change. Round to the nearest tenth of a percent if necessary.

1. 16 meters to 20 meters

2. 9 points to 4 points

3. 15 ounces to 5 ounces

4. 38 staples to 55 staples

Find the product. Write fractions in simplest form.

5. $\frac{4}{7}\left(-\frac{1}{6}\right)$

6. $-1.58(6.02)$

7. $-3\left(-2\frac{1}{8}\right)$

Concepts, Skills, & Problem Solving

COMPARING DISCOUNTS **The same item is on sale at two stores. Which one is the better price? Use percent models to justify your answer.** (See Exploration 1, p. 259.)

8. 60% off \$60 or 55% off \$50

9. 85% off \$90 or 70% off \$65

USING TOOLS **Copy and complete the table.**

	Original Price	Percent of Discount	Sale Price
10.	\$80	20%	
11.	\$42	15%	
12.	\$120	80%	
13.	\$112	32%	
14.	\$69.80	60%	
15.		25%	\$40
16.		5%	\$57
17.		80%	\$90
18.		64%	\$72
19.		15%	\$146.54
20.	\$60		\$45
21.	\$82		\$65.60
22.	\$95		\$61.75

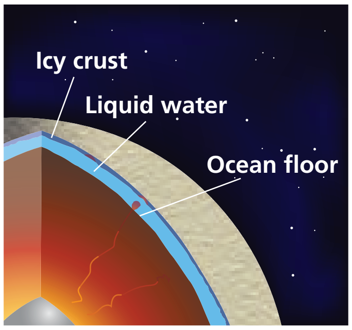
FINDING A SELLING PRICE **Find the selling price.**

23. Cost to store: \$50
Markup: 10%

24. Cost to store: \$80
Markup: 60%

25. Cost to store: \$140
Markup: 25%

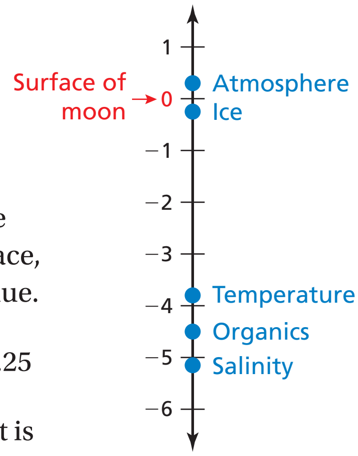
EXAMPLE 3 Modeling Real Life



A moon has an ocean underneath its icy surface. Scientists run tests above and below the surface. The table shows the elevations of each test. Which test is deepest? Which test is closest to the surface?

Test	Temperature	Salinity	Atmosphere	Organics	Ice
Elevation (miles)	−3.8	−5.15	0.3	−4.5	−0.25

To determine which test is deepest, find the least elevation. Graph the elevations on a vertical number line.



The number line shows that the salinity test is deepest. The number line also shows that the atmosphere test and the ice test are closest to the surface. To determine which is closer to the surface, identify which elevation has a lesser absolute value.

Atmosphere: $|0.3| = 0.3$ **Ice:** $|-0.25| = 0.25$

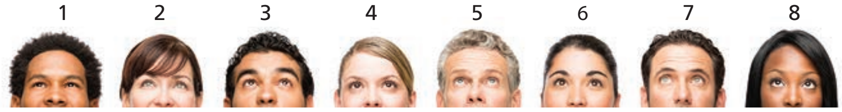
So, the salinity test is deepest and the ice test is closest to the surface.

Self-Assessment *for Problem Solving*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. An airplane is at an elevation of 5.5 miles. A submarine is at an elevation of −10.9 kilometers. Which is closer to sea level? Explain.

14. The image shows the corrective powers (in *diopters*) of contact lenses for eight people. The farther the number of diopters is from 0, the greater the power of the lens. Positive diopters correct *farsightedness* and negative diopters correct *nearsightedness*. Who is the most nearsighted? the most farsighted? Who has the best eyesight?



Patient	1	2	3	4	5	6	7	8
Power (diopters)	−1.25	0.75	2.5	−3.75	−2.5	−4.75	−7.5	1.5

EXAMPLE 3 Modeling Real Life

You solve a number puzzle on your phone. You start with 250 points. You finish the puzzle in 8 minutes 45 seconds and make 3 mistakes. What is your score?

Understand the problem.

You are given ways to gain points and lose points when completing a puzzle. You are asked to find your score after finishing the puzzle.

Make a plan.

Use a verbal model to solve the problem. Find the sum of the starting points, mistake penalties, and time bonus.

Solve and check.

Score = Starting points + Number of mistakes • Penalty per mistake + Time bonus

$= 250 + 3(-50) + 75$

10 min − 8 min 45 sec = 1 min 15 sec
= 75 sec

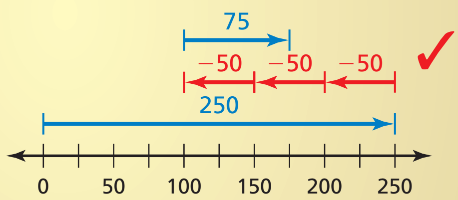
$= 250 + (-150) + 75$

$= 100 + 75$

$= 175$

So, your score is 175 points.

Another Method



Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

16. On a mountain, the temperature decreases by 18°F for each 5000-foot increase in elevation. At 7000 feet, the temperature is 41°F. What is the temperature at 22,000 feet? Justify your answer.

Player	Coins	Time
1	31	0:02:03
2	18	0:01:55
3	24	0:01:58
4	27	0:02:01

17. Players in a racing game earn 3 points for each coin they collect. Each player loses 5 points for each second that he or she finishes after the first-place finisher. The table shows the results of a race. List the players in order from greatest to least number of points.

EXAMPLE 3 Modeling Real Life

Skateboard kits cost d dollars and you have a coupon for \$2 off each one you buy. After assembly, you sell each skateboard for $(2d - 4)$ dollars. Find and interpret your profit on each skateboard sold.

Understand the problem.

You are given information about purchasing skateboard kits and selling the assembled skateboards. You are asked to find and interpret the profit made on each skateboard sold.

Make a plan.

Find the difference of the expressions representing the selling price and the purchase price. Then simplify and interpret the expression.

Solve and check.

You receive \$2 off of d dollars, so you pay $(d - 2)$ dollars for each kit.

Profit (dollars) = Selling price (dollars) − Purchase price (dollars)

$= (2d - 4) - (d - 2)$	Write the difference.
$= (2d - 4) + (-d + 2)$	Add the opposite.
$= 2d - d - 4 + 2$	Group like terms.
$= d - 2$	Combine like terms.

Your profit on each skateboard sold is $(d - 2)$ dollars. You pay $(d - 2)$ dollars for each kit, so you are doubling your money.

Look Back Assume each kit is \$40. Verify that you double your money.

When $d = 40$: You pay $d - 2 = 40 - 2 = \$38$.
You sell it for $2d - 4 = 2(40) - 4 = 80 - 4 = \$76$.
Because $\$38 \cdot 2 = \$76$, you double your money. ✓

Indicator 2c - #9 is non-routine because students use subtracting expressions to solve a bigger problem. Students have to interpret the expressions to determine which team wins. Then they have to subtract the losing team's points from the winning team's points to explain the scores after halftime.

ssessment for Problem Solving

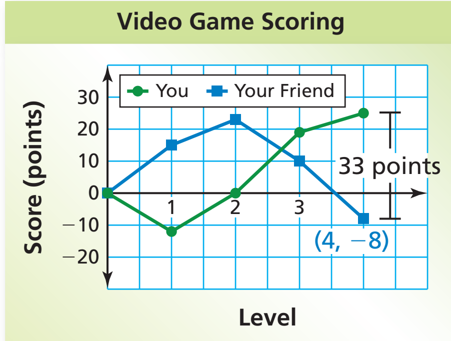
Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. DIG DEEPER! In a basketball game, the home team scores $(2m + 39)$ points and the away team scores $(3m + 40)$ points, where m is the number of minutes since halftime. Who wins the game? What is the difference in the scores m minutes after halftime? Explain.

10. Electric guitar kits originally cost d dollars online. You buy the kits on sale for 50% of the original price, plus a shipping fee of \$4.50 per kit. After painting and assembly, you sell each guitar online for $(1.5d + 4.5)$ dollars. Find and interpret your profit on each guitar sold.

EXAMPLE 3 Modeling Real Life

You and your friend play a video game. The line graph shows both of your scores after each level. What is your score after Level 4?



Understand the problem.

You are given a line graph that shows that after Level 4, your friend's score of −8 is 33 points less than your score. You are asked to find your score after Level 4.

Make a plan.

Use the information to write and solve an equation to find your score after Level 4.

Solve and check.

Words Your friend's score is 33 points less than your score.

Variable Let s be your score after Level 4.

Equation $-8 \quad = \quad s \quad - \quad 33$

$$-8 = s - 33 \qquad \text{Write equation.}$$

$$\underline{+33} \quad \underline{+33} \qquad \text{Addition Property of Equality}$$

$$25 = s \qquad \text{Simplify.}$$

Your score after Level 4 is 25 points.

Another Method After Level 4, your score is 33 points greater than your friend's score. So, your score is $-8 + 33 = 25$. ✓

Self-Assessment for Problem Solving

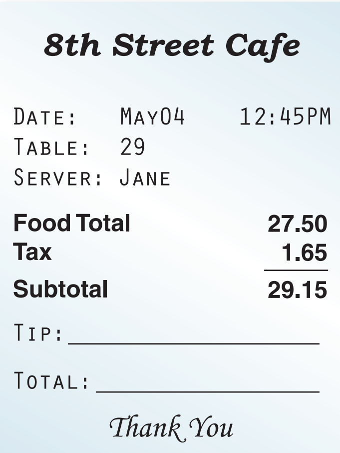
Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. You have \$512.50. You earn additional money by shoveling snow. Then you purchase a new cell phone for \$249.95 and have \$482.55 left. How much money do you earn shoveling snow?

11. **DIG DEEPER!** You swim 4 lengths of a pool and break a record by 0.72 second. The table shows your time for each length compared to the previous record holder. How much faster or slower is your third length than the previous record holder?

Length	Time (seconds)
1	−0.23
2	0.11
3	?
4	−0.42

EXAMPLE 4 Modeling Real Life



You are paying for lunch and receive the bill shown.

a. Find the percent of sales tax on the food total.

Answer the question: \$1.65 is what percent of \$27.50?

$a = p\% \cdot w$	Write the percent equation.
$1.65 = p\% \cdot 27.50$	Substitute 1.65 for *a* and 27.50 for *w*.
$0.06 = p\%$	Divide each side by 27.50.

Because 0.06 equals 6%, the percent of sales tax is 6%.

b. You leave a 16% tip on the food total. Find the total amount you pay for lunch.

Answer the question: What tip amount is 16% of \$27.50?

$a = p\% \cdot w$	Write the percent equation.
$= 0.16 \cdot 27.50$	Substitute 0.16 for *p%* and 27.50 for *w*.
$= 4.40$	Multiply.

The amount of the tip is \$4.40.

So, you pay a total of $\$29.15 + \$4.40 = \$33.55$.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. DIG DEEPER! A school offers band and chorus classes. The table shows the percents of the 1200 students in the school who are enrolled in band, chorus, or neither class. How many students are enrolled in both classes? Explain.

Class	Enrollment
Band	34%
Chorus	28%
Neither	42%

14. Water Tank A has a capacity of 550 gallons and is 66% full. Water Tank B is 53% full. The ratio of the capacity of Water Tank A to Water Tank B is 11 : 15.

a. How much water is in each tank?

b. What percent of the total volume of both tanks is filled with water?

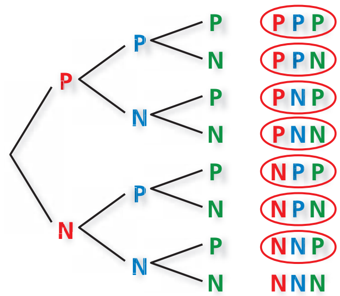
EXAMPLE 5 Modeling Real Life

On a game show, you choose one box from each pair of boxes shown. In each pair, one box contains a prize and the other does not. What is the probability of winning at least one prize?

Choice 1

Use a tree diagram to find the sample space. Let P = prize and N = no prize. Circle the outcomes in which you win 1, 2, or 3 prizes.

Choice 2



Choice 3

There are seven favorable outcomes in the sample space for winning at least one prize.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{at least one prize}) = \frac{7}{8} \qquad \text{Substitute.}$$

The probability of winning at least one prize is $\frac{7}{8}$, or 87.5%.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

8. A tour guide organizes vacation packages at a beachside town. There are 7 hotels, 5 cabins, 4 meal plans, 3 escape rooms, and 2 amusement parks. The tour guide chooses either a hotel or a cabin and then selects one of each of the remaining options. Find the total number of possible vacation packages.

9. **DIG DEEPER!** A fitness club with 100 members offers one free training session per member in either running, swimming, or weightlifting. Thirty of the fitness center members sign up for the free session. The running and swimming sessions are each twice as popular as the weightlifting session. What is the probability that a randomly chosen fitness club member signs up for a free running session?

YOU BE THE TEACHER **Your friend evaluates the expression. Is your friend correct? Explain your reasoning.**

42. $-2(-7) = -14$

43. $-10^2 = -100$

MP PATTERNS **Find the next two numbers in the pattern.**

44. $-12, 60, -300, 1500, \ldots$

45. $7, -28, 112, -448, \ldots$

46. **MP PROBLEM SOLVING** In a scavenger hunt, each team earns 25 points for each item that they find. Each team loses 15 points for every minute after 4:00 P.M. that they report to the city park. The table shows the number of items found by each team and the time that each team reported to the park. Which team wins the scavenger hunt? Justify your answer.

Team	Items	Time
A	13	4:03 P.M.
B	15	4:07 P.M.
C	11	3:56 P.M.
D	12	4:01 P.M.

Indicator 2c - #48 is non-routine because students have to decide when they can afford a speaker based on its price, which is decreasing each month, and how much they've saved.

47. **MP REASONI**
by 22,000 + (−
minutes it takes the plane to land. Explain your reasoning.

48. **MP PROBLEM SOLVING** The table shows the price of a bluetooth speaker each month for 4 months.

Month	Price (dollars)
June	165
July	$165 + (-12)$
August	$165 + 2(-12)$
September	$165 + 3(-12)$

a. Describe the change in the price of the speaker.

b. The table at the right shows the amount of money you save each month. When do you have enough money saved to buy the speaker? Explain your reasoning.

Amount Saved	
June	$35
July	$55
August	$45
September	$18

49. **DIG DEEPER!** Two integers, a and b, have a product of 24. What is the least possible sum of a and b?

50. **MP NUMBER SENSE** Consider two integers p and q. Explain why $p \times (-q) = (-p) \times q = -pq$.

YOU BE THE TEACHER **Your friend factors the expression. Is your friend correct? Explain your reasoning.**

36. $16p - 28 = 4(4p - 28)$

37. $\frac{2}{3}y - \frac{14}{3} = \frac{2}{3} \cdot y - \frac{2}{3} \cdot 7$

$= \frac{2}{3}(y - 7)$

FACTORING OUT A NEGATIVE NUMBER **Factor out the indicated number.**

38. Factor -4 out of $-8d + 20$.

39. Factor -6 out of $18z - 15$.

40. Factor -0.25 out of $7g + 3.5$.

41. Factor $-\frac{1}{2}$ out of $-\frac{1}{2}x + 6$.

42. Factor -1.75 out of $-14m - 5.25n$.

43. Factor $-\frac{1}{4}$ out of $-\frac{1}{2}x - \frac{5}{4}y$.

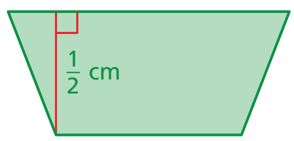
44. MP **STRUCTURE** A rectangle has an area of $(4x + 12)$ square units. Write three multiplication expressions that can represent the product of the length and the width of the rectangle.

45. **MODELING REAL LIFE** A square wrestling mat has a perimeter of $(12x - 32)$ feet. Explain how to use the expression to find the length (in feet) of the mat. Justify your answer.

46. **MODELING REAL LIFE** A table is 6 feet long and 3 feet wide. You extend the length of the table by inserting two identical table *leaves*. The extended table is rectangular with an area of $(18 + 6x)$ square feet. Write and interpret an expression that represents the length (in feet) of the extended table.

47. **DIG DEEPER!** A three-dimensional printing pen uses heated plastic to create three-dimensional objects. A kit comes with one 3D-printing pen and p packages of plastic. An art club purchases 6 identical kits for $(180 + 58.5p)$ dollars. Write and interpret an expression that represents the cost of one kit.

48. MP **STRUCTURE** The area of the trapezoid is $\left(\frac{3}{4}x - \frac{1}{4}\right)$ square centimeters. Write two different pairs of expressions that represent the possible base lengths (in centimeters). Justify your answers.



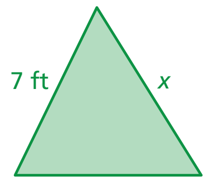
29. **MODELING REAL LIFE** A small airplane can hold 44 passengers. Fifteen passengers board the plane.

a. Write and solve an inequality that represents the additional numbers of passengers that can board the plane.

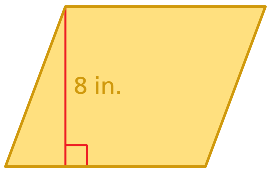
b. Can 30 more passengers board the plane? Explain.

GEOMETRY **Find the possible values of *x*.**

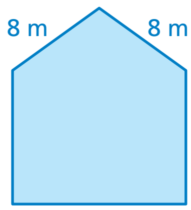
30. The perimeter is less than 28 feet.



31. The base is greater than the height.



32. The perimeter is less than or equal to 51 meters.



33. **MP REASONING** The inequality $d + s > -3$ is equivalent to $d > -7$. What is the value of s?

34. **MP LOGIC** You can spend up to $35 on a shopping trip.

a. You want to buy a shirt that costs $14. Write and solve an inequality that represents the remaining amounts of money you can spend if you buy the shirt.

b. You notice that the shirt is on sale for 30% off. How does this change your inequality in part (a)?

35. **DIG DEEPER!** If items plugged into a circuit use more than 2400 watts of electricity, the circuit overloads. A portable heater that uses 1050 watts of electricity is plugged into the circuit.

a. Find the additional numbers of watts you can plug in without overloading the circuit.

b. In addition to the portable heater, what two other items in the table can you plug in at the same time without overloading the circuit? Is there more than one possibility? Explain.

Item	Watts
Aquarium	200
Hair dryer	1200
Television	150
Vacuum cleaner	1100

36. **MP NUMBER SENSE** The possible values of x are given by $x + 8 \leq 6$. What is the greatest possible value of $7x$? Explain your reasoning.

22. **MODELING REAL LIFE** You can sand $\frac{4}{9}$ square yard of wood in $\frac{1}{2}$ hour. How many square yards can you sand in 3.2 hours? Justify your answer.

MP REASONING **Tell whether the rates are equivalent. Justify your answer.**

23. 75 pounds per 1.5 years
38.4 ounces per 0.75 year

24. $7\frac{1}{2}$ miles for every $\frac{3}{4}$ hour
$\frac{1}{2}$ mile for every 3 minutes

25. **MP PROBLEM SOLVING** The table shows nutritional information for three beverages.

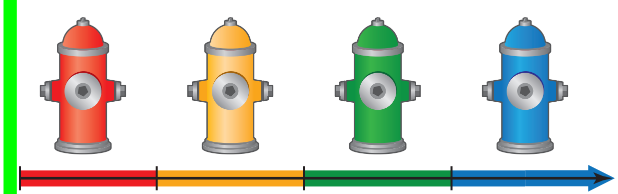
a. Which has the most calories per fluid ounce?

b. Which has the least sodium per fluid ounce?

Beverage	Serving Size	Calories	Sodium
Whole milk	1 c	146	98 mg
Orange juice	1 pt	210	10 mg
Apple juice	24 fl oz	351	21 mg

26. **MODELING REAL LIFE** A shuttle leaving Earth's atmosphere travels 15 miles every 2 seconds. When entering the Earth's atmosphere, the shuttle travels $2\frac{3}{8}$ miles per $\frac{1}{2}$ second. Find the difference in the distances traveled after 15 seconds when leaving and entering the atmosphere.

27. **RESEARCH** Fire hydrants are one of four different colors to indicate the rate at which water comes from the hydrant.



a. Use the Internet to find the ranges of rates indicated by each color.

b. Research why a firefighter needs to know the rate at which water comes out of a hydrant.

28. **DIG DEEPER!** You and a friend start riding bikes toward each other from opposite ends of a 24-mile biking route. You ride $2\frac{1}{6}$ miles every $\frac{1}{4}$ hour. Your friend rides $7\frac{1}{3}$ miles per hour.

a. After how many hours do you meet?

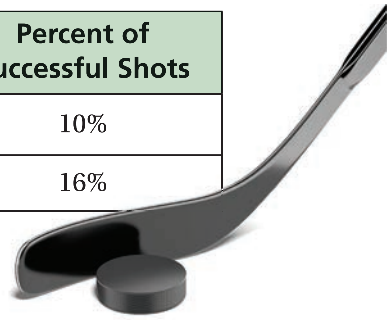
b. When you meet, who has traveled farther? How much farther?

6 Connecting Concepts

Using the Problem-Solving Plan

1. The table shows the percent of successful shots for each team in a hockey game. A total of 55 shots are taken in the game. The ratio of shots taken by the Blazers to shots taken by the Hawks is 6 : 5. How many goals does each team score?

Team	Percent of Successful Shots
Blazers	10%
Hawks	16%



Understand the problem. You know that 55 shots are taken in a hockey game and that the Blazers take 6 shots for every 5 shots taken by the Hawks. You also know the percent of successful shots for each team.

Make a plan. Use a ratio table to determine the number of shots taken by each team. Then use the percent equation to determine the number of successful shots for each team.

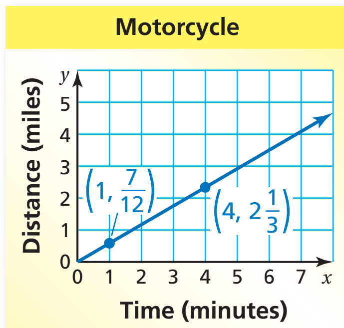
Solve and check. Use the plan to solve the probler

Indicator 2c - #3 is non-routine because students have to first read the graph to find the rate the motorcycle travels on a dirt road. Then, students must find the rate on a paved road. Lastly, students use their knowledge of percents to find the percent change in those rates.

2. Fill in the blanks with positive num
fractions is 37.5% of the first fractio

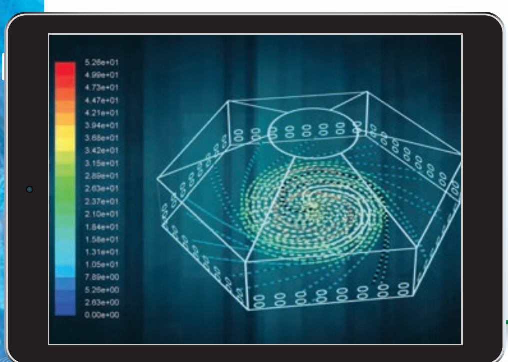
$$\frac{\square}{5} + \left(-\frac{\square}{4}\right)$$

3. The graph shows the distance traveled by a motorcycle on a dirt road. After turning onto a paved road, the motorcycle travels $\frac{1}{5}$ mile every $\frac{1}{4}$ minute. Find the percent of change in the speed of the motorcycle. Round to the nearest tenth of a percent if necessary.



Performance Task

Tornado Alley



At the beginning of this chapter, you watched a STEAM Video called "Tornado!" You are now ready to complete the performance task related to this video, available at *BigIdeasMath.com*. Be sure to use the problem-solving plan as you work through the performance task.

10 Connecting Concepts

Using the Problem-Solving Plan

1. A store pays $2 per pound for popcorn kernels. One cubic foot of kernels weighs about 45 pounds. What is the selling price of the container shown when the markup is 30%?

Understand the problem.

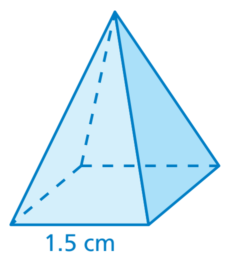
You are given the dimensions of a container of popcorn kernels and the price that a store pays for the kernels. You also know the weight of one cubic foot of popcorn kernels. You are asked to find the selling price of the container when the markup is 30%.

Make a plan.

Use the volume of the container to find the weight of the kernels. Then use the weight of the kernels to find the cost to the store. Finally, use the percent markup to find the selling price of the container.

Solve and check.

Use the plan to solve the problem. Then check your solution.



2. The pyramid shown has a square base. What is the height of the pyramid? Justify your answer.

3. A cylindrical can of soup has a height of 7 centimeters and a lateral surface area of 63π square centimeters. The can is redesigned to have a lateral surface area of 45π square centimeters without changing the radius of the can. What is the height of the new design? Justify your answer.

Performance Task

Volumes and Surface Areas of Small Objects

At the beginning of this chapter, you watched a STEAM Video called "Paper Measurements." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Professional Development

Rigorous by Design

1.5 Subtracting Rational Numbers

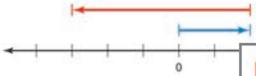
Learning Target: Find differences of rational numbers and find distances between numbers on a number line.

Success Criteria:
- I can explain how to model subtraction of rational numbers on a number line.
- I can find differences of rational numbers by reasoning about absolute values.
- I can find distances between numbers on a number line.

EXPLORATION 1 **Subtracting Rational Numbers**

Work with a partner.

a. Choose a unit fraction to represent the space between the tick marks on each number line. What expressions involving subtraction are being modeled? What are the differences?



Conceptual Understanding

Explorations help students reach a deeper level of conceptual understanding.

EXAMPLE 1 **Subtracting Rational Numbers**

Find $-4\frac{1}{7} - \frac{5}{7}$. **Estimate** $-4 - 1 = -5$

Rewrite the difference as a sum by adding the opposite.

$-4\frac{1}{7} - \frac{5}{7} = -4\frac{1}{7} + \left(-\frac{5}{7}\right)$

Because the signs are the same, add $\left|-4\frac{1}{7}\right|$ and $\left|-\frac{5}{7}\right|$.

$\left|-4\frac{1}{7}\right| + \left|-\frac{5}{7}\right| = 4\frac{1}{7} + \frac{5}{7}$ Find the absolute values.

$= 4 + \frac{1}{7} + \frac{5}{7}$ Write $4\frac{1}{7}$ as $4 + \frac{1}{7}$.

$= 4 + \frac{6}{7}$, or $4\frac{6}{7}$ Add fractions and simplify.

Because $-4\frac{1}{7}$ and $-\frac{5}{7}$ are both negative, use a negative sign in the difference.

So, $-4\frac{1}{7} - \frac{5}{7} = -4\frac{6}{7}$. **Reasonable?** $-4\frac{6}{7} \approx -5$ ✓

Try It **Find the difference. Write your answer in simplest form.**

1. $\frac{1}{3} - \left(-\frac{1}{3}\right)$ **2.** $-3\frac{1}{3} - \frac{2}{3}$ **3.** $4 - 5\frac{1}{2}$

Procedural Fluency

Lessons follow a gradual release model and give teachers opportunities for flexible instruction, providing opportunities for all levels of learners to attain procedural fluency. Self-Assessments provide students the opportunity to asses their understanding of the success criteria, taking ownership of their learning.

EXAMPLE 4 **Modeling Real Life**

You measure the height of the tide using the support beams of a pier. What is the mean hourly change in the height?

To find the mean hourly change in the height of the tide, divide the change in the height by the elapsed time.

$$\text{mean hourly change} = \frac{\text{final height} - \text{initial height}}{\text{elapsed time}}$$

The elapsed time from 2 P.M. to 8 P.M. is 6 hours.

$= \frac{8 - 59}{6}$ Substitute.

$= \frac{-51}{6}$ Subtract.

$= -8\frac{1}{2}$ Divide.

The mean change in the height of the tide is $-8\frac{1}{2}$ inches per hour.

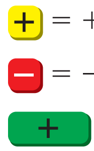
Application

Modeling Real Life examples bring problem solving into the classroom, promoting application of concepts and skills and reaching higher levels of DOK.

Indicator 2d - In this Chapter Exploration, one aspect of rigor (conceptual understanding) is emphasized. The focus is on using algebra tiles to introduce the conceptual understanding for solving equations.

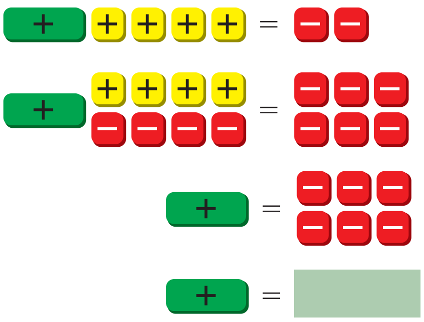
Getting Ready for Chapter 4

Chapter Exploration



1. Work with a partner. Use algebra tiles to model and solve each equation.

a. $x + 4 = -2$



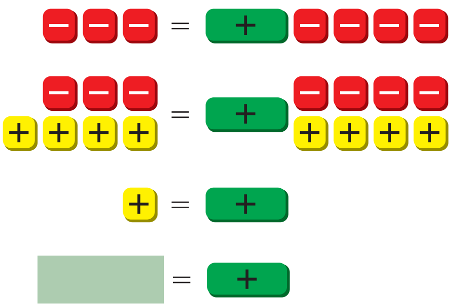
Model the equation $x + 4 = -2$.

Add four -1 tiles to each side.

Remove the zero pairs from the left side.

Write the solution of the equation.

b. $-3 = x - 4$



Model the equation $-3 = x - 4$.

Add four $+1$ tiles to each side.

Remove the zero pairs from each side.

Write the solution of the equation.

c. $x - 6 = 2$ **d.** $x - 7 = -3$ **e.** $-15 = x - 5$

f. $x + 3 = -5$ **g.** $7 + x = -1$ **h.** $-5 = x - 3$

2. **WRITE GUIDELINES** Work with a partner. Use your models in Exercise 1 to summarize the algebraic steps that you use to solve an equation.

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

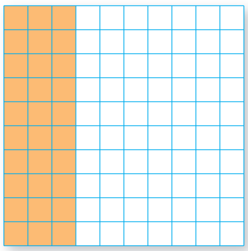
equivalent equations inequality solution set

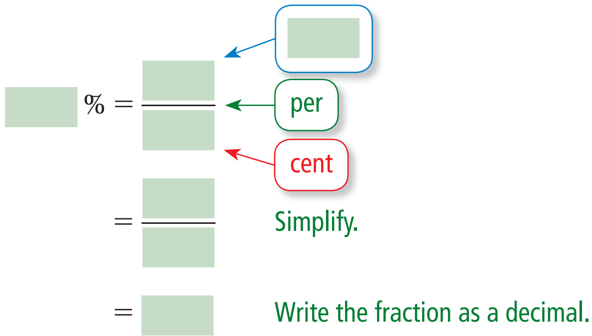
Getting Ready for Chapter 6

Chapter Exploration

Work with a partner. Write the percent of the model that is shaded. Then write the percent as a decimal.

1.





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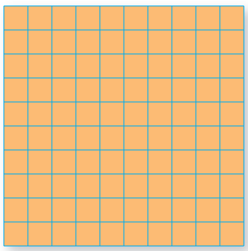

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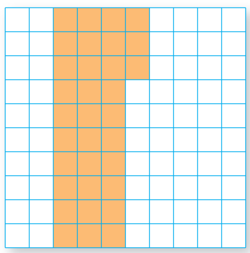

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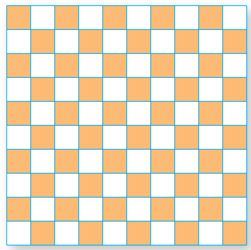

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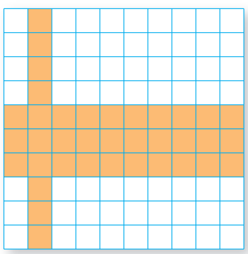

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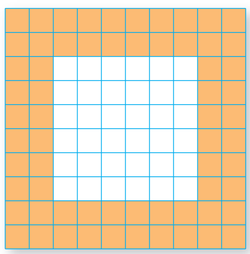

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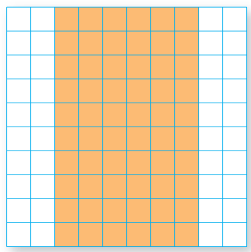


8. **WRITE A PROCEDURE** Work with a partner. Write a procedure for rewriting a percent as a decimal. Use examples to justify your procedure.

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

percent of change	percent of decrease	discount
percent of increase	percent error	markup

Getting Ready for Chapter 9

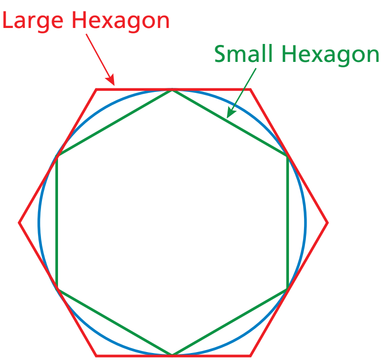
Chapter Exploration

Work with a partner.

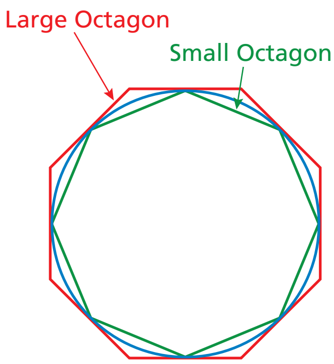
1. Perform the steps for each of the figures.

- Measure the perimeter of the larger polygon to the nearest millimeter.
- Measure the diameter of the circle to the nearest millimeter.
- Measure the perimeter of the smaller polygon to the nearest millimeter.
- Calculate the value of the ratio of the two perimeters to the diameter.
- Take the average of the ratios. This average is the approximation of π (the Greek letter *pi*).

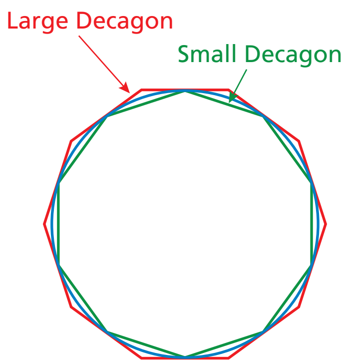
a.



b.



c.



Sides	Large Perimeter	Diameter of Circle	Small Perimeter	$\frac{\text{Large Perimeter}}{\text{Diameter}}$	$\frac{\text{Small Perimeter}}{\text{Diameter}}$	Average of Ratios
6						
8						
10						

2. Based on the table, what can you conclude about the value of π? Explain your reasoning.

3. The Greek mathematician Archimedes used the above procedure to approximate the value of π. He used polygons with 96 sides. Do you think his approximation was more or less accurate than yours? Explain your reasoning.

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

diameter of a circle	semicircle	adjacent angles
circumference	composite figure	vertical angles

1 Connecting Concepts

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Using the Problem-Solving Plan

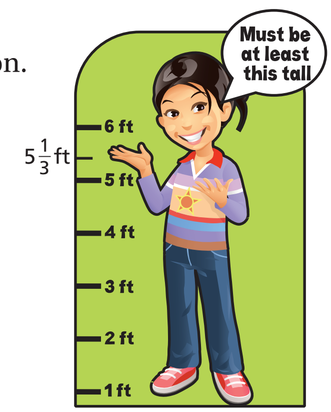
1. A land surveyor uses a coordinate plane to draw a map of a park, where each unit represents 1 mile. The park is in the shape of a parallelogram with vertices $(-2.5, 1.5)$, $(-1.5, -2.25)$, $(2.75, -2.25)$, and $(1.75, 1.5)$. Find the area of the park.

Understand the problem. You know the vertices of the parallelogram-shaped park and that each unit represents 1 mile. You are asked to find the area of the park.

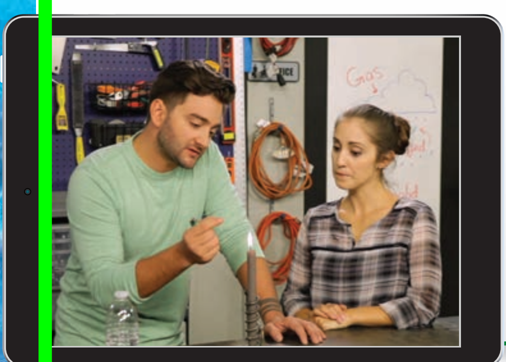
Make a plan. Use a coordinate plane to draw a map of the park. Then find the height and base length of the park. Find the area by using the formula for the area of a parallelogram.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. The diagram shows the height requirement for driving a go-cart. You are $5\frac{1}{4}$ feet tall. Write and solve an inequality to represent how much taller you must be to drive a go-cart.



Performance Task



Melting Matters

At the beginning of this chapter, you watched a STEAM Video called "Freezing Solid." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

2 Connecting Concepts

Using the Problem-Solving Plan

1. You feed several adult hamsters equal amounts of a new food recipe over a period of 1 month. You record the changes in the weights of the hamsters in the table. Use the data to answer the question "What is the typical weight change of a hamster that is fed the new recipe?"

Weight Change (ounces)				
−0.07	−0.03	−0.11	−0.04	−0.08
0.02	−0.08	−0.08	−0.06	−0.05
−0.11	−0.1	0	−0.07	−0.08

Understand the problem. You know the weight changes of 15 hamsters. You want to use this information to find the typical weight change.

Make a plan. Display the data in a dot plot to see the distribution of the data. Then use the distribution to determine the most appropriate measure of center.

Solve and check. Use the plan to solve the problem. Then check your solution.

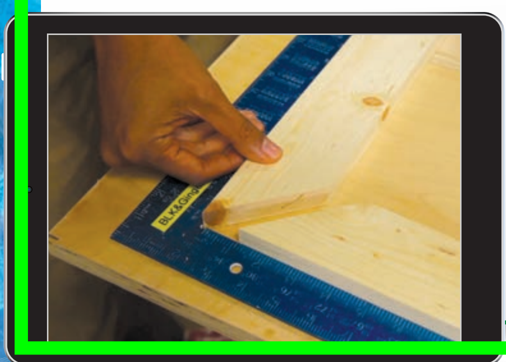
2. Evaluate the expression shown at the right. Write your answer in simplest form.

$$\frac{-\frac{1}{2}+\frac{2}{3}}{\frac{3}{5}\left(\frac{3}{4}-\frac{11}{8}\right)}$$

3. You drop a racquetball from a height of 60 inches. On each bounce, the racquetball bounces to a height that is 70% of its previous height. What is the change in the height of the racquetball after 3 bounces?

Performance Task

Precisely Perfect

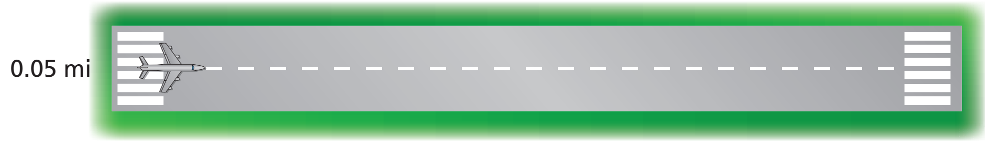


At the beginning of this chapter, you watched a STEAM Video called "Carpenter or Joiner." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

3 Connecting Concepts

Using the Problem-Solving Plan

1. The runway shown has an area of $(0.05x + 0.125)$ square miles. Write an expression that represents the perimeter (in feet) of the runway.



Understand the problem. You know the area of the rectangular runway in square miles and the width of the runway in miles. You want to know the perimeter of the runway in feet.

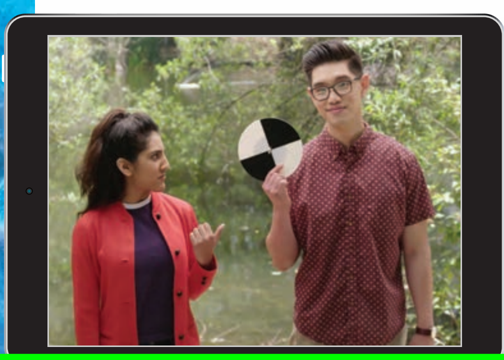
Make a plan. Factor the width of 0.05 mile out of the expression that represents the area to find an expression that represents the length of the runway. Then write an expression that represents the perimeter (in miles) of the runway. Finally, use a measurement conversion to write the expression in terms of feet.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. The populations of two towns after t years can be modeled by $-300t + 7000$ and $-200t + 5500$. What is the combined population of the two towns after t years? The combined population of the towns in Year 10 is what percent of the combined population in Year 0?

Performance Task

Chlorophyll in Plants



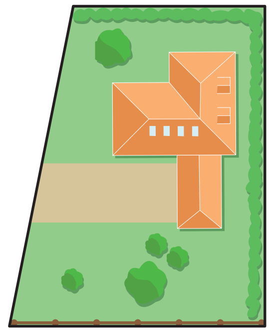
At the beginning of this chapter, you watched a STEAM Video called "Tropic Status." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Indicator 2d - In the exercises and the Performance Task, one aspect of rigor (application) is emphasized. Students use their learning from the chapter and previous chapters to complete the exercises.

4 Connecting Concepts

Using the Problem-Solving Plan

1. Fencing costs \$7 per foot. You install x feet of the fencing along one side of a property, as shown. The property has an area of 15,750 square feet. What is the total cost of the fence?



Understand the problem. You know the area, height, and one base length of the trapezoid-shaped property. You are asked to find the cost of x feet of fencing, given that the fencing costs \$7 per foot.

Make a plan. Use the formula for the area of a trapezoid to find the length of fencing that you buy. Then multiply the length of fencing by \$7 to find the total cost.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. A pool is in the shape of a rectangular prism with a length of 15 feet, a width of 10 feet, and a depth of 4 feet. The pool is filled with water at a rate no faster than 3 cubic feet per minute. How long does it take to fill the pool?

3. The table shows your scores on 9 out of 10 quizzes that are each worth 20 points. What score do you need on the final quiz to have a mean score of at least 17 points?

Quiz Scores				
15	14	16	19	18
19	20	15	16	?

Performance Task

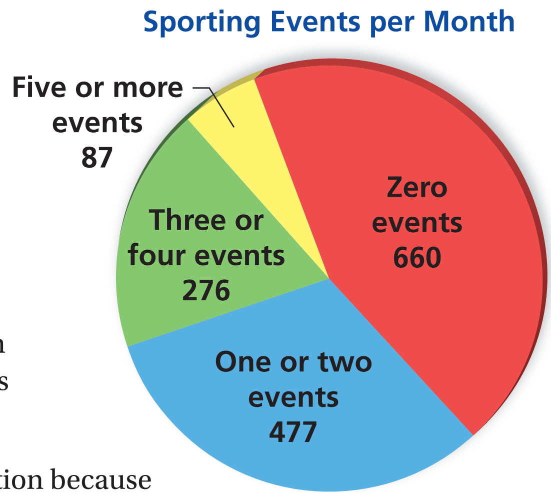
Distance and Brightness of the Stars

At the beginning of this chapter, you watched a STEAM Video called "Space Cadets." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

8 Connecting Concepts

Using the Problem-Solving Plan

1. In a city, 1500 randomly chosen residents are asked how many sporting events they attend each month. The city has 80,000 residents. Estimate the number of residents in the city who attend at least one sporting event each month.

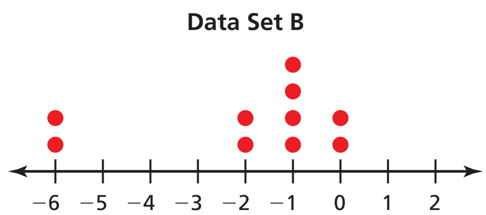


Understand the problem. You are given the numbers of sporting events attended each month by a sample of 1500 residents. You are asked to make an estimate about the population, all residents of the city.

Make a plan. The sample is representative of the population because it is selected at random and is large enough to provide accurate data. So, find the percent of people in the survey that attend at least one sporting event each month, and use the percent equation to make an estimate.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. The dot plots show the values in two data sets. Is the difference in the measures of center for the data sets significant?

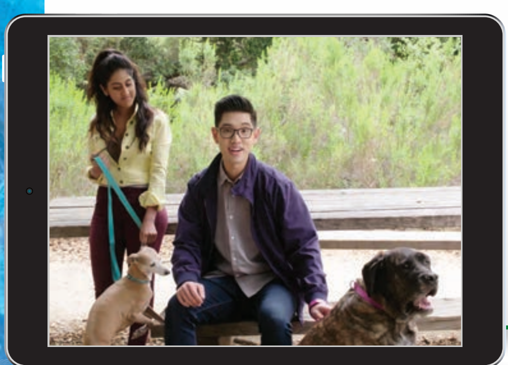


Yes	No
42	18

3. You ask 60 randomly chosen students whether they support a later starting time for school. The table shows the results. Estimate the probability that at least two out of four randomly chosen students do not support a later starting time.

Performance Task

Estimating Animal Populations



At the beginning of the this chapter, you watched a STEAM Video called "Comparing Dogs." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Indicator 2e - The front matter provides a correlation aligning the MP labels and other headings in the Student Edition with the Standards for Mathematical Practice.

Standards for Mathematical Practice

MP MP

1 Make sense of problems and persevere in solving them.

- Multiple representations are presented to help students move from concrete to representative and into abstract thinking.
- *Modeling Real Life Examples* and PROBLEM-SOLVING exercises encourage students to use problem-solving strategies, such as drawing a diagram, making a table, and solving a simpler problem. They also use a formal problem-solving plan: understand the problem, make a plan, and solve and check.

2 Reason abstractly and quantitatively.

- Visual problem-solving models help students create a coherent representation of the problem.
- *Explorations* allow students to investigate concepts to understand the REASONING behind the rules.
- Questions ask students to explain and justify their REASONING.
- Questions encourage students to apply NUMBER SENSE and formulate consistent and appropriate REASONING.

3 Construct viable arguments and critique the reasoning of others.

- *Explorations* help students make conjectures, use LOGIC, and CONSTRUCT ARGUMENTS to support their conjectures.
- Exercises, such as YOU BE THE TEACHER; DIFFERENT WORDS, SAME QUESTION; and WHICH ONE DOESN'T BELONG?, provide students the opportunity to critique the reasoning of others.

4 Model with mathematics.

- Real-life situations are translated into diagrams, tables, equations, and graphs to help students analyze relations and to draw conclusions.
- Real-life problems are provided to help students apply the mathematics they are learning to everyday life.
- MODELING REAL LIFE examples and exercises help students see that math is used across content areas, other disciplines, and in their own experiences.

5 Use appropriate tools strategically.

- *Graphic Organizers* support the thought process of what, when, and how to solve problems.
- A variety of tools, such as number lines and graph paper, manipulatives, and digital tools, are available as students CHOOSE TOOLS and begin USING TOOLS to solve problems.

6 Attend to precision.

- PRECISION exercises encourage students to formulate consistent and appropriate reasoning.
- Cooperative learning opportunities support precise communication.

7 Look for and make use of structure.

- *Learning Targets* and *Success Criteria* at the start of each chapter and section help students understand what they are going to learn.
- *Explorations* provide students the opportunity to see PATTERNS and STRUCTURE in mathematics.
- Real-life problems help students use the STRUCTURE of mathematics to break down and solve more difficult problems.

8 Look for and express regularity in repeated reasoning.

- Opportunities are provided to help students make generalizations through REPEATED REASONING.
- Students are continually encouraged to check for reasonableness in their solutions.

The colored words above are used throughout the program to indicate exercises that correlate to the Standards for Mathematical Practice.

EXAMPLE 3 Modeling Real Life

A restaurant launches a mobile app that allows customers to rate their food on a scale from −5 to 5. So far, customers have given the lasagna scores of 2.25, −3.5, 0, −4.5, 1.75, −1, 3.5, and −2.5. Should the restaurant consider changing the recipe? Explain.

Understand the problem.

You are given eight scores for lasagna. You are asked to determine whether the restaurant should make changes to the lasagna recipe.

Make a plan.

Use the mean score to determine whether people generally like the lasagna. Then decide whether the recipe should change.

Solve and check.

Divide the sum of the scores by the number of scores. Group together scores that are convenient to add.

$$\text{mean} = \frac{0 + (-3.5 + 3.5) + (2.25 + 1.75) + [(-4.5) + (-2.5) + (-1)]}{8}$$

$$= \frac{0 + 0 + 4 + (-8)}{8}$$

$$= \frac{-4}{8}, \text{ or } -0.5$$

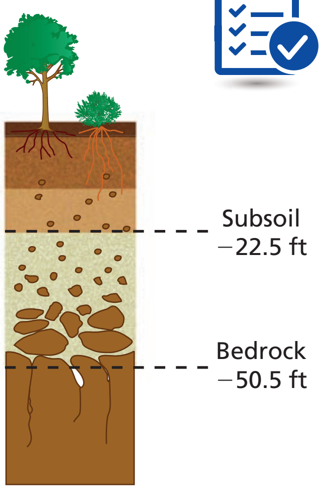
The mean score is below the "mediocre" score of 0.

So, the restaurant should consider changing the recipe.

Look Back
Only 3 of the 8 scores were better than "mediocre." So, it makes sense to conclude that the restaurant should change the recipe. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.



12. **DIG DEEPER!** Soil is composed of several layers. A geologist measures the depths of the *subsoil* and the *bedrock*, as shown. Find and interpret two quotients involving the depths of the subsoil and the bedrock.

13. The restaurant in Example 3 receives additional scores of −0.75, −1.5, −1.25, 4.75, −0.25, −0.5, 5, and −0.5 for the lasagna. Given the additional data, should the restaurant consider changing the recipe? Explain.

EXAMPLE 3 Modeling Real Life

Skateboard kits cost *d* dollars and you have a coupon for $2 off each one you buy. After assembly, you sell each skateboard for (2*d* − 4) dollars. Find and interpret your profit on each skateboard sold.

Understand the problem.

You are given information about purchasing skateboard kits and selling the assembled skateboards. You are asked to find and interpret the profit made on each skateboard sold.

Make a plan.

Find the difference of the expressions representing the selling price and the purchase price. Then simplify and interpret the expression.

Solve and check.

You receive $2 off of d dollars, so you pay $(d - 2)$ dollars for each kit.

Profit (dollars) = Selling price (dollars) − Purchase price (dollars)

$= (2d - 4) - (d - 2)$ Write the difference.

$= (2d - 4) + (-d + 2)$ Add the opposite.

$= 2d - d - 4 + 2$ Group like terms.

$= d - 2$ Combine like terms.

Your profit on each skateboard sold is $(d - 2)$ dollars. You pay $(d - 2)$ dollars for each kit, so you are doubling your money.

Look Back Assume each kit is $40. Verify that you double your money.

When $d = 40$: You pay $d - 2 = 40 - 2 = \$38$.
You sell it for $2d - 4 = 2(40) - 4 = 80 - 4 = \$76$.
Because $\$38 \cdot 2 = \$76$, you double your money. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. **DIG DEEPER!** In a basketball game, the home team scores $(2m + 39)$ points and the away team scores $(3m + 40)$ points, where m is the number of minutes since halftime. Who wins the game? What is the difference in the scores m minutes after halftime? Explain.

10. Electric guitar kits originally cost d dollars online. You buy the kits on sale for 50% of the original price, plus a shipping fee of $4.50 per kit. After painting and assembly, you sell each guitar online for $(1.5d + 4.5)$ dollars. Find and interpret your profit on each guitar sold.

EXAMPLE 4 Modeling Real Life

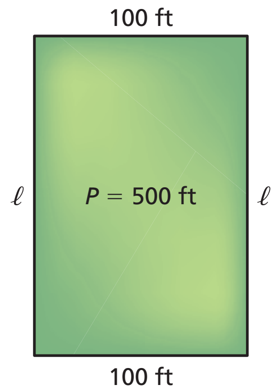
You install 500 feet of invisible fencing along the perimeter of a rectangular yard. The width of the yard is 100 feet. What is the length of the yard?

Understand the problem.

You are given that the perimeter of a rectangular yard is 500 feet and the width is 100 feet. You are asked to find the length of the yard.

Make a plan.

Draw a diagram of the yard. Then use the formula for the perimeter of a rectangle to write and solve an equation to find the length of the yard.



Solve and check.

$P = 2\ell + 2w$	Perimeter of a rectangle
$500 = 2\ell + 2(100)$	Substitute for P and w.
$500 = 2\ell + 200$	Multiply.
$300 = 2\ell$	Subtract 200 from each side.
$150 = \ell$	Divide each side by 2.

So, the length of the yard is 150 feet.

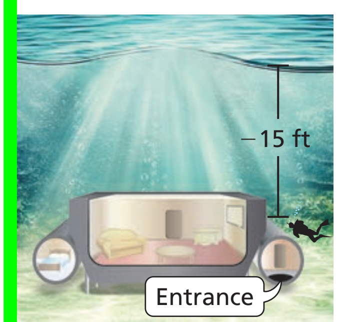
Another Method Use a different form of the formula for the perimeter of a rectangle, $P = 2(\ell + w)$.

$500 = 2(\ell + 100)$	Substitute for P and w.
$250 = \ell + 100$	Divide each side by 2.
$150 = \ell$	Subtract 100 from each side.

So, the length of the yard is 150 feet. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

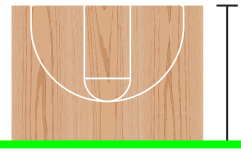


18. You must scuba dive to the entrance of your room at Jules' Undersea Lodge in Key Largo, Florida. The diver is 1 foot deeper than $\frac{2}{3}$ of the elevation of the entrance. What is the elevation of the entrance?

19. **DIG DEEPER!** A car drives east along a road at a constant speed of 46 miles per hour. At 4:00 P.M., a truck is 264 miles away, driving west along the same road at a constant speed. The vehicles pass each other at 7:00 P.M. What is the speed of the truck?

EXAMPLE 3 Modeling Real Life

The center circle of the basketball court has a radius of 3 feet and is painted blue. The rest of the court is stained brown. One gallon of wood stain covers 150 square feet. How many gallons of wood stain do you portions of the cour



Indicator 2f - In Example 4, the Problem-Solving Plan is shown to help students make sense of problems and persevere in solving them. Students then use the Problem-Solving Plan to help them solve #5-6.

MP1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.... Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Understand the problem.

You are given dimens are asked to determin wood stain needed to the court when one g 150 square feet.

Make a plan.

Find the entire area o of the center circle ar

Solve and check.

Area of Rectangl

$A = \ell w$

$= 84(50)$

$= 4200$

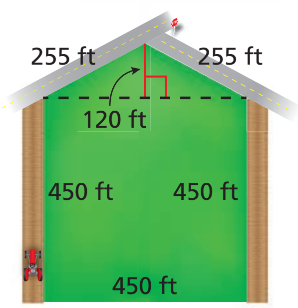
The area that is s

Because one you need 41

Check Reasonableness
The circle covers a small area of the court. So, it makes sense that you need just less than $\frac{84(50)}{150} = 28$ gallons of wood stain. ✓

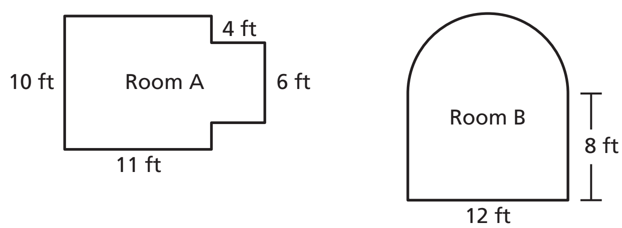
Self-Asses

Solve each exercise. Then rate your understanding of the success criteria in your journal.



5. A farmer wants to seed and fence a section of land. Fencing costs \$27 per yard. Grass seed costs \$2 per square foot. How much does it cost to fence and seed the pasture?

6. **DIG DEEPER!** In each room shown, you plan to put down carpet and add a wallpaper border around the ceiling. Which room needs more carpeting? more wallpaper?



1 Connecting Concepts

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

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- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Using the Problem-Solving Plan

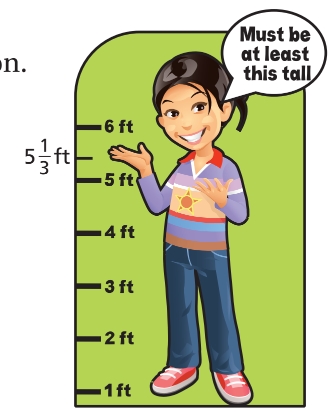
1. A land surveyor uses a coordinate plane to draw a map of a park, where each unit represents 1 mile. The park is in the shape of a parallelogram with vertices $(-2.5, 1.5)$, $(-1.5, -2.25)$, $(2.75, -2.25)$, and $(1.75, 1.5)$. Find the area of the park.

Understand the problem. You know the vertices of the parallelogram-shaped park and that each unit represents 1 mile. You are asked to find the area of the park.

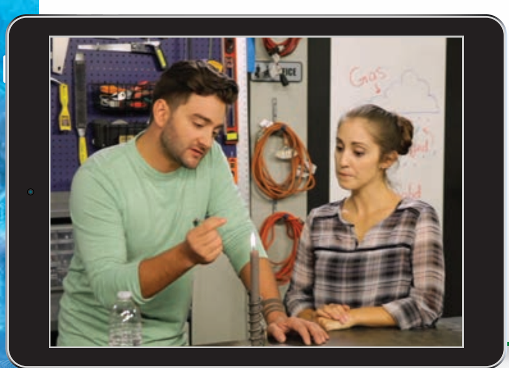
Make a plan. Use a coordinate plane to draw a map of the park. Then find the height and base length of the park. Find the area by using the formula for the area of a parallelogram.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. The diagram shows the height requirement for driving a go-cart. You are $5\frac{1}{4}$ feet tall. Write and solve an inequality to represent how much taller you must be to drive a go-cart.



Performance Task



Melting Matters

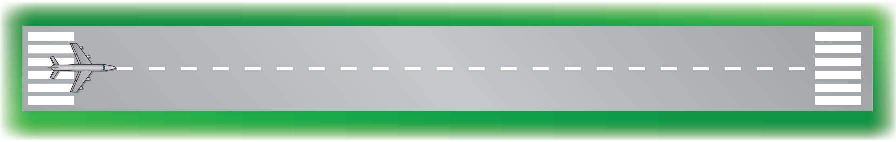
At the beginning of this chapter, you watched a STEAM Video called "Freezing Solid." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

3 Connecting Concepts

Using the Problem-Solving Plan

1. The runway shown has an area of $(0.05x + 0.125)$ square miles. Write an expression that represents the perimeter (in feet) of the runway.

0.05 mi



Understand the problem. You know the area of the rectangular runway in square miles and the width of the runway in miles. You want to know the perimeter of the runway in feet.

Make a plan. Factor the width of 0.05 mile out of the expression that represents the area to find an expression that represents the length of the runway. Then write an expression that represents the perimeter (in miles) of the runway. Finally, use a measurement conversion to write the expression in terms of feet.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. The populations of two towns after t years can be modeled by $-300t + 7000$ and $-200t + 5500$. What is the combined population of the two towns after t years? The combined population of the towns in Year 10 is what percent of the combined population in Year 0?

Indicator 2f - In each exercise, students have to use their knowledge of the current chapter and previous concepts to solve them. This helps students make sense of problems and persevere in solving them. For instance, #1 requires the skills of factoring linear expressions, area, perimeter, and unit conversions.

Performance Task

Chlorophyll in Plants

At the beginning of this chapter, you watched a STEAM Video called "Tropic Status." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

5 Connecting Concepts

Using the Problem-Solving Plan

1. The table shows the toll y (in dollars) for traveling x miles on a turnpike. You have \$8.25 to pay your toll. How far can you travel on the turnpike?

Distance, x (miles)	25	30	35	40
Toll, y (dollars)	3.75	4.50	5.25	6.00

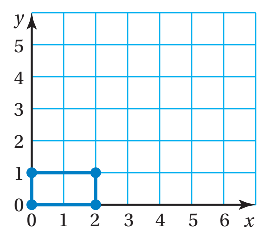
Understand the problem. The table shows the tolls for traveling several different distances on a turnpike. You have \$8.25 to pay the toll. You are asked to find how far you can travel on the turnpike with \$8.25 for tolls.

Make a plan. First, determine the relationship between x and y and write an equation to represent the relationship. Then use the equation to determine the distance you can travel.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. A company uses a silo in the shape of a rectangular prism to store bird seed. The base of the silo is a square with side lengths of 20 feet. Are the height and the volume of the silo proportional? Justify your answer.

3. A rectangle is drawn in a coordinate plane as shown. In the same coordinate plane, create a scale drawing of the rectangle that has a vertex at (0, 0) and a scale factor of 3.



Performance Task

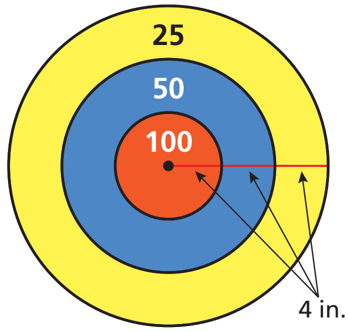
Mixing Paint

At the beginning of this chapter, you watched a STEAM Video called "Painting a Large Room." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

9 Connecting Concepts

Using the Problem-Solving Plan

1. A dart is equally likely to hit any point on the board shown. Find the theoretical probability that a dart hitting the board scores 100 points.

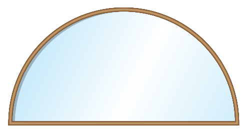


Understand the problem. You are given the dimensions of a circular dart board. You are asked to find the theoretical probability of hitting the center circle.

Make a plan. Find the area of the center circle and the area of the entire dart board. To find the theoretical probability of scoring 100 points, divide the area of the center circle by the area of the entire dart board.

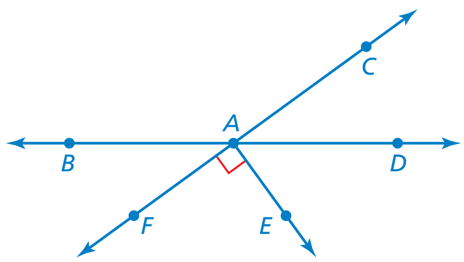
Solve and check. Use the plan to solve the problem. Then check your solution.

2. A scale drawing of a window is shown. Find the perimeter and the area of the actual window. Justify your answer.



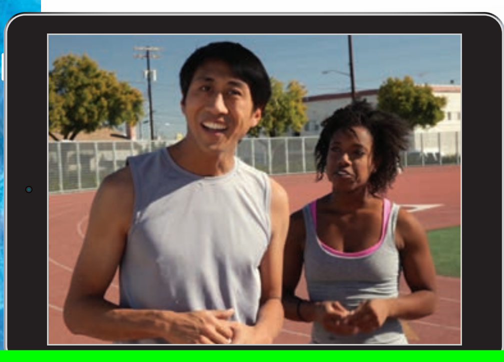
1 cm : 2 ft

3. $\angle CAD$ makes up 20% of a pair of supplementary angles. Find the measure of $\angle DAE$. Justify your answer.



Performance Task

Finding the Area and Perimeter of a Track



At the beginning of the this chapter, you watched a STEAM video called "Track and Field". You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Learning Target

Convert between different forms of rational numbers.

Success Criteria

- Explain the difference between terminating and repeating decimals.
- Write fractions and mixed numbers as decimals.
- Write decimals as fractions and mixed numbers.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Explain that the word *convert* means "to change or transform." Its Latin parts mean "together" (*con*) and "to turn" (*vertere*), so *convert* roughly means "turning together." In everyday life, convert is often associated with a person changing his or her view about something or changing a substance into a different form. In math, however, it means changing a quantity expressed in one form into another, such as changing a fraction into a decimal.

Exploration 1

a. $\frac{7}{10}$; $1\frac{29}{100}$; $12\frac{831}{1000}$; $\frac{41}{10{,}000}$;

b. *Sample answer:* The factors (other than 1) are multiples of 2 and 5; yes

Exploration 2

a–c. See Additional Answers.

Laurie's Notes

STATE STANDARDS
7.NS.A.2b, 7.NS.A.2d

Preparing to Teach

- In prior courses, students divided decimals that terminated or they rounded the results. Now students will write fractions and mixed numbers as decimals, including repeating decimals. In the next course, students will expand this understanding to write repeating decimals as fractions.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** In this lesson, students will write fractions as decimals and vice versa. Students should always check the reasonableness of their answers. For instance, $\frac{7}{11}$ is greater than $\frac{1}{2}$. So, when you write $\frac{7}{11}$ as a decimal, the result should be greater than 0.5.

Motivate

- Ask students to form a "name fraction," where the numerator is the number of letters in the student's first name and the denominator is the number of letters in the student's last name.
- Before class, go through your class roster and select two students whose name fractions are nearly equivalent, but one is a **terminating decimal** and the other is a **repeating decimal**. Discuss writing the fractions as decimals.
- When you discuss the repeating decimal, share that today's lesson is about writing rational numbers, which may be repeating decimals.

Exploration 1

- ? Tell students that *decimal fractions* are those with powers of 10 in the denominator, such as $\frac{3}{10}$ or $\frac{21}{100}$.
- Review the proper way to read decimals using place value. For example, 0.9 is *not* read as "point nine." It is read as "nine-tenths" and written as $\frac{9}{10}$.
- Discuss part (b). Students should notice that the denominators of decimal fractions are always powers of 10.
- ? "What are *prime numbers*?" Numbers greater than 1 whose only factors are 1 and itself. Examples: 2, 3, 5, 23, 41.
- ? "What are the *prime factors* of the denominators of decimal fractions?" 2 and 5

Exploration 2

- Students should recognize some of the decimals *terminate* and others *repeat.*
- Discuss the question in the Math Practice note. Have students use equivalent fractions to convert some of the fractions to decimals.
- ? "Will equivalent fractions work for all of the fractions? Explain." No, some of the fractions have denominators that are not powers or factors of 10.

Laurie's Notes

Extra Example 3

Solve each proportion.

a. $\frac{u}{6} = \frac{3}{4}$ $u = 4.5$

b. $\frac{4}{13} = \frac{12}{h}$ $h = 39$

Try It

7. $x = 8$ **8.** $y = 2.5$

9. $z = 15$

Extra Example 4

Find the value of *x* so that the ratios 5 : 6 and 4 : *x* are equivalent. $x = 4.8$

Try It

10. $x = 3$ **11.** $x = 20$

12. $x = 7.5$

EXAMPLE 3

- "How are parts (a) and (b) different?" *Sample answer:* In part (a), the variable is in the numerator and in part (b), the variable is in the denominator. Part (b) involves one numerator that is a factor of the other numerator.
- "Can you easily use the Multiplication Property of Equality to solve both proportions? Explain." No, using the Multiplication Property of Equality would be difficult in part (b) because the variable is in the denominator
- **MP1 Make Sense of Problems and Persevere in Solving Them:** As you work through the problems with students, share with them the wisdom of analyzing the problem first to decide what method makes the most sense.
- **Common Error:** Students sometimes confuse the multiplication of fractions and the Cross Products Property.

Try It

- **Think-Pair-Share:** Students should read each exercise independently and then work in pairs to complete the exercises. Then have each pair compare their answers with another pair and discuss any discrepancies.
- Ask students to share their strategies. Although Example 3 was solved using the Cross Products Property, some students may solve Exercises 7 and 8 by using mental math and recognizing equivalent fractions.

EXAMPLE 4

- Ask students to read and analyze the problem to decide which method makes the most sense. They should realize that mental math is not convenient because 8 is not a factor of 20.
- Work through the problem as shown.

Try It

- **Neighbor Check:** Have students work independently and then have their neighbors check their work. Have students discuss any discrepancies.

ELL Support

After demonstrating Example 4, have students practice language by working in pairs to complete Try It Exercises 10–12. Have one student ask another, "What proportion do you write? By what number do you multiply each side? What is your answer?" Have students alternate roles.

Beginner: Write the steps and/or use one-word answers.

Intermediate: Use simple sentences such as, "The proportion is two-fourths equals *x* divided by six."

Advanced: Use detailed sentences such as, "To find the value of *x*, write the proportion two-fourths equals *x* divided by six."

Learning Target

Find perimeters and areas of composite figures.

Success Criteria

- Use a grid to estimate perimeters and areas.
- Identify the shapes that make up a composite figure.
- Find the perimeters and areas of shapes that make up composite figures.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Explain that a composite figure is made up of more than one shape. It may contain rectangles, triangles, circles, or other geometric shapes. *Composite* is related to the word *compose*, which comes from the Latin language and means "put together." The perimeter is the distance around a figure. The prefix *peri-* means "around" and as students learned in the chapter opener, *meter* comes from the Latin word for "measure."

Exploration 1

Check students' work.

Laurie's Notes

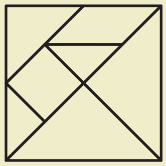
Check out the Dynamic Classroom.
BigIdeasMath.com

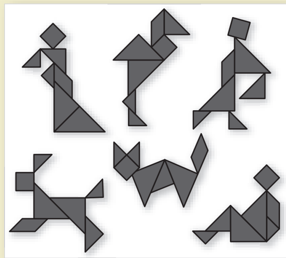
STATE STANDARDS
7.G.B.4, 7.G.B.6

Preparing to Teach

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students have worked with perimeter and area formulas. In making sense of **composite figures**, they need to view the figures as composed of smaller, familiar figures.

Motivate





- Arrange a set of tangram pieces in a square. Tell students that the area of the square is 16 square units.
- ? "What are the dimensions of the square?" 4 units by 4 units
- Now rearrange all of the tangram pieces to make a new shape.
- ? "Can you find the area of each of these? Explain." Yes, the area is 16 square units because each new figure is composed of the same 7 pieces that made the square.

Exploration 1

- Share with students that builders and contractors submit bids for work that they want to do. If more than one bid is received, the consumer selects the builder or contractor based upon a number of factors, one of which is the cost that is quoted.
- In this exploration, assume that each pair of students is bidding on the job. They could even have a name for their two-person company. Their bid sheet (work done) should be neat and organized and easily understood by the pool's owner—you!
- Explain the phrase *$5 per linear foot* so that all students understand. In construction, when the width of material (in this case, the tile) is predetermined, the cost is given in terms of the length, not in terms of area (square feet). From the drawing, students should realize that side length of each tile is 1 linear foot.
- Students should count the number of tiles surrounding the pool to determine how much tile is needed and then multiply by $5 per linear foot.
- Students need to design the custom-made tarp for the pool. Encourage students to lower the cost by ordering an irregular-shaped tarp that covers only the surface of the pool with minimal overlap of the deck.
- Make sure each pair includes a labor charge based on the information given. They will set their own hourly wage. Is it realistic?
- Remind students to estimate their total profits.
- Discuss general results with the whole class. Compare the quotes, separating out the material and labor.
- **Extension:** List the hourly wage and the number of hours of labor that each company charges. Use this data to find the mean, median, and range of the hourly wages and the total labor charge.

Learning Target

Find the volume of a prism.

Success Criteria

- Use a formula to find the volume of a prism.
- Use the formula for the volume of a prism to find a missing dimension.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Students may be familiar with the word *volume* as it is used in everyday language. Ask them to describe what they know, which may be a measure of sound. Explain that in math, volume is a measure of the amount of space that a three-dimensional figure occupies. It is measured using cubic units of measurement. Relate the word *cube* to the word *cubic*. For example, a prism with dimensions 2 feet, 3 feet, and 2 feet has a volume of 12 cubic feet. So, 12 cubes measuring 1 foot by 1 foot by 1 foot would fit inside it.

Exploration 1

a. See Additional Answers.

b. See Additional Answers.

c. $V = Bh$, where B is the area of the base and h is the height of the prism.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
7.G.B.6

Preparing to Teach

- Students should know how to find areas of two-dimensional figures, surface areas of three-dimensional figures, and volumes of rectangular prisms using $V = \ell wh$ or by counting unit cubes. Now they will use a formula to find volumes of prisms.
- **MP8 Look for and Express Regularity in Repeated Reasoning:** To develop a formula for the volume of a prism, students will consider repeated layers with the same base. Mathematically proficient students notice that each layer increases the volume by the number of units of the area of the base.

Motivate

- **True Story:** Baseball legend Ken Griffey Jr. owed teammate Josh Fogg some money and paid him back in pennies. Griffey stacked 60 cartons, each holding \$25 worth of pennies, in Fogg's locker.

? Ask the following questions.
  - "How does this story relate to the volume of a prism?" The volume of the carton is being measured in pennies.
  - "How big is a carton that holds \$25 worth of pennies?" Answers will vary.
  - "How many pennies were in each carton?" 2500
  - "How much did Griffey owe Fogg?" \$1500
  - "How much do you think each carton weighed?" A \$25 carton of pennies weighs about 16 pounds.

Exploratio

Indicator 2f - This note appears in the Teaching Edition to point out that students have to make sense of the general volume formula to be able to answer the questions in Exploration 1. This way, students don't have to memorize each volume formula, they can remember the general one and apply it to each solid.

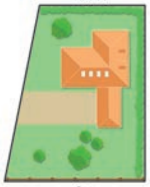
- In part (a), st
  layer has 6 c
- If students ar
  volume of ea
- **Big Idea:** The
  many layers?
- In part (b), tell students that the sliced cubes are exactly half a cubic unit
- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students can memorize formulas and have little understanding of why the formula makes sense. It is important throughout this chapter that students see that the formulas are all similar. The volume is found by finding the area of the base (B) and then multiplying by the number of layers (h).
- **Common Misconception:** The height of a prism does not need to be the vertical measure. Demonstrate this by holding a rectangular prism (a tissue box is fine). Ask students to identify the base (a face of the prism) and the height (an edge). Chances are students will identify the (standard) bottom of the box as the base. Now, rotate the tissue box so that the base is vertical. Again ask students to identify the base and height. Students may stick with their first answers or switch to the "bottom face" as the base.
- Give students time to discuss the solids. If you have physical models, ask six volunteers to describe how to find the volume of the solid. Expect volunteers to point to the bases and the heights as they explain how to find the volumes.

In-Class Problem Solving

4 Connecting Concepts

Using the Problem-Solving Plan

1. Fencing costs \$7 per foot. You install x feet of the fencing along one side of a property, as shown. The property has an area of 15,750 square feet. What is the total cost of the fence?



Understand the problem. You know the area, height, and one base length of the trapezoid-shaped property. You are asked to find the cost of x feet of fencing, given that the fencing costs \$7 per foot.

Make a plan. Use the formula for the area of a trapezoid to find the length of fencing that you buy. Then multiply the length of fencing by \$7 to find the total cost.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. A pool is in the shape of a rectangular prism with a length of 15 feet, a width of 10 feet, and a depth of 4 feet. The pool is filled with water at a rate no faster than 3 cubic feet per minute. How long does it take to fill the pool?

3. The table shows your scores on 9 out of 10 quizzes that are each worth 20 points. What score do you need on the final quiz to have a mean score of at least 17 points?

Quiz Scores				
15	14	16	19	18
19	20	15	16	?

Connecting Concepts pages combine previously learned skills with concepts from the current chapter, so students practice problem solving for high-stakes assessments. Students use the Problem-Solving Plan along with a variety of problem-solving strategies.

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Performance Task

Distance and Brightness of the Stars

At the beginning of this chapter, you watched a STEAM Video called "Space Cadets." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Self-Assessment for Problem Solving gives teachers the opportunity for continual formative assessment and allows students to communicate mathematically in every lesson.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

18. A polar vortex causes the temperature to decrease from 3°C at 3:00 P.M. to −2°C at 4:00 P.M. The temperature continues to change by the same amount each hour until 8:00 P.M. Find the total change in temperature from 3:00 P.M. to 8:00 P.M.

19. DIG DEEPER! While on vacation, you map several locations using a coordinate plane in which each unit represents 1 mile. A cove is at (3, −7), an island is at (−5, 4), and you are currently at (3, 4). Are you closer to the cove or the island? Justify your answer.

1.1 Rational Numbers

Learning Target: Understand absolute values and ordering of rational numbers.

Success Criteria:
- I can graph rational numbers on a number line.
- I can find the absolute value of a rational number.
- I can use a number line to compare rational numbers.

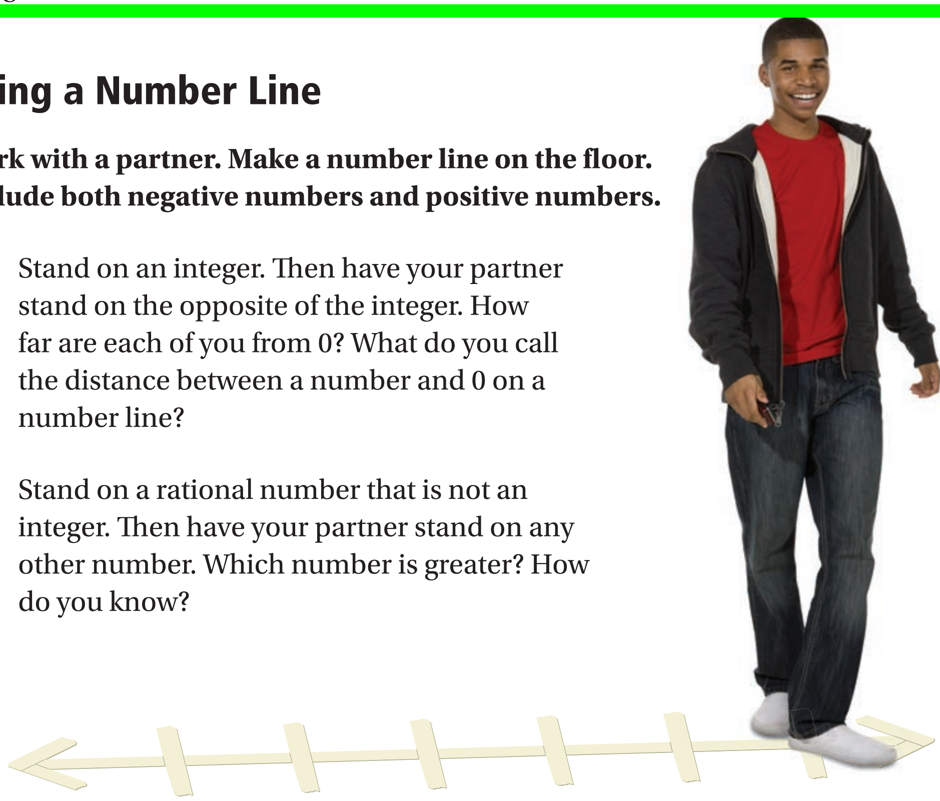
Recall that **integers** are the set of whole numbers and their opposites. A **rational number** is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

EXPLORATION 1 Using a Number Line

Work with a partner. Make a number line on the floor. Include both negative numbers and positive numbers.

a. Stand on an integer. Then have your partner stand on the opposite of the integer. How far are each of you from 0? What do you call the distance between a number and 0 on a number line?

b. Stand on a rational number that is not an integer. Then have your partner stand on any other number. Which number is greater? How do you know?



c. Stand on any number other than 0 on the number line. Can your partner stand on a number that is:

- greater than your number and farther from 0?
- greater than your number and closer to 0?
- less than your number and the same distance from 0?
- less than your number and farther from 0?

For each case in which it was not possible to stand on a number as directed, explain why it is not possible. In each of the other cases, how can you decide where your partner can stand?

Math Practice

Find Entry Points
What are some ways to determine which of two numbers is greater?

2.1 Multiplying Integers

Learning Target: Find products

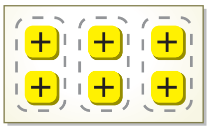
Success Criteria:
- I can expla
- I can find
- I can find

Indicator 2f - In Exploration 1, students use their knowledge of multiplying whole numbers to reason about multiplying integers. In part a, students reason about how to use a number line to show the product of a positive and a negative integer. Then in part b, students use a pattern to find products of integers. Finally in part c, students use what they learned in the prior parts to complete a table and write general rules for multiplying integers.

MP2 Reason abstractly and quantitatively - Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

EXPLORATION 1 Understan

Work with a pa



a. The numb
How can y

b. Use the ta

2	•			
1	•			
0	•			
−1	•	2	=	
−2	•	2	=	
−3	•	2	=	

−3	•	0	=	
−3	•	−1	=	
−3	•	−2	=	

c. **INDUCTIVE REASONING** Complete the table. Then write general rules for multiplying (i) two integers with the same sign and (ii) two integers with different signs.

Expression	Type of Product	Product	Product: Positive or Negative
$3 \cdot 2$	Integers with the same sign		
$3 \cdot (-2)$			
$-3 \cdot 2$			
$-3 \cdot (-2)$			
$6 \cdot 3$			
$2 \cdot (-5)$			
$-6 \cdot 5$			
$-5 \cdot (-3)$			

Math Practice

Construct Arguments

Construct an argument that you can use to convince a friend of the rules you wrote in Exploration 1(c).

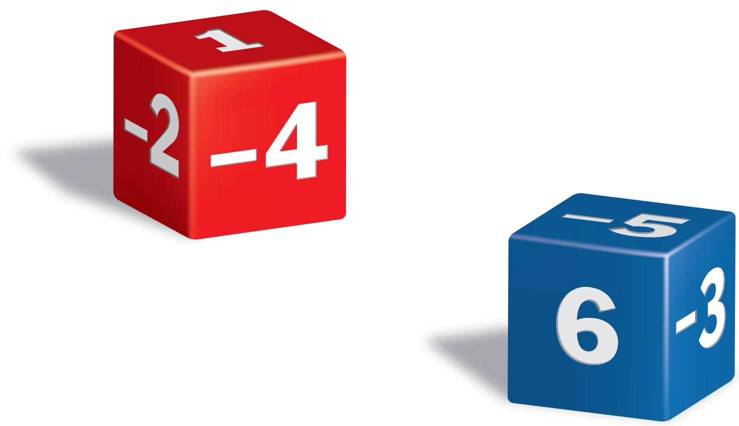
4.6 Solving Inequalities Using Multiplication or Division

Learning Target: Write and solve inequalities using multiplication or division.

Success Criteria:
- I can apply the Multiplication and Division Properties of Inequality to produce equivalent inequalities.
- I can solve inequalities using multiplication or division.
- I can apply inequalities involving multiplication or division to solve real-life problems.

EXPLORATION 1 Writing Inequalities

Work with a partner. Use two number cubes on which the odd numbers are negative on one of the number cubes and the even numbers are negative on the other number cube.



- Roll the number cubes. Write an inequality that compares the numbers.
- Roll one of the number cubes. Multiply each side of the inequality by the number and record your result.
- Repeat the previous two steps nine more times.

a. When you multiply each side of an inequality by the same number, does the inequality remain true? Explain your reasoning.

b. When you divide each side of an inequality by the same number, does the inequality remain true? Use inequalities generated by number cubes to justify your answer.

c. Use your results in parts (a) and (b) to make a conjecture about how to solve an inequality of the form $ax < b$ for x when $a > 0$ and when $a < 0$.

Math Practice

Use Counterexamples
Use a counterexample to show that $2a \geq a$ is not true for every value of a.

7.1 Probability

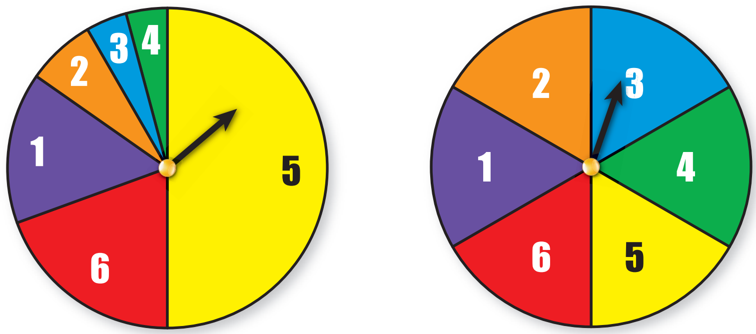
Learning Target: Understand how the probability of an event indicates its likelihood.

Success Criteria:
- I can identify possible outcomes of an experiment.
- I can use probability and relative frequency to describe the likelihood of an event.
- I can use relative frequency to make predictions.

EXPLORATION 1 Determining Likelihood

Work with a partner. Use the spinners shown.

Spinner 1 **Spinner 2**



a. For each spinner, determine which numbers you are more likely to spin and which numbers you are less likely to spin. Explain your reasoning.

b. Spin each spinner 20 times and record your results in two tables. Do the data support your answers in part (a)? Explain why or why not.

Spinner 1	
Number	Frequency
1	
2	
3	
4	
5	
6	

Spinner 2	
Number	Frequency
1	
2	
3	
4	
5	
6	

Math Practice

Recognize Usefulness of Tools

How does organizing the data in tables help you to interpret the results?

c. How can you use percents to describe the likelihood of spinning each number? Explain.

39. **OPEN-ENDED** Write a negative number whose absolute value is greater than 3.

40. **MODELING REAL LIFE** The *summit elevation* of a volcano is the elevation of the top of the volcano relative to sea level. The summit elevation of Kilauea, a volcano in Hawaii, is 1277 meters. The summit elevation of Loihi, an underwater volcano in Hawaii, is −969 meters. Which summit is higher? Which summit is closer to sea level?

41. **MODELING REAL LIFE** The *freezing point* of a liquid is the temperature at which the liquid becomes a solid.

a. Which liquid in the table has the lowest freezing point?

b. Is the freezing point of mercury or butter closer to the freezing point of water, 0°C?

Liquid	Freezing Point (°C)
Butter	35
Airplane fuel	−53
Honey	−3
Mercury	−39
Candle wax	53

ORDERING RATIONAL NUMBERS **Order the values from least to greatest.**

42. $8, |3|, -5, |-2|, -2$

43. $|-6.3|, -7.2, 8, |5|, -6.3$

44. $|3.5|, |-1.8|, 4.6, 3\frac{2}{5}, |2.7|$

45. $\left|-\frac{3}{4}\right|, \frac{5}{8}, \left|\frac{1}{4}\right|, -\frac{1}{2}, \left|-\frac{7}{8}\right|$

46. **MP PROBLEM SOLVING** The table shows golf scores, relative to *par*.

a. The player with the lowest score wins. Which player wins?

b. Which player is closest to par?

c. Which player is farthest from par?

Player	Score
1	+5
2	0
3	−4
4	−1
5	+2

47. **DIG DEEPER!** You use the table below to record the temperature at the same location each hour for several hours. At what time is the temperature coldest? At what time is the temperature closest to the freezing point of water, 0°C?

Time	10:00 A.M.	11:00 A.M.	12:00 P.M.	1:00 P.M.	2:00 P.M.	3:00 P.M.
Temperature (°C)	−2.6	−2.7	−0.15	1.6	−1.25	−3.4

MP REASONING **Determine whether $n \geq 0$ or $n \leq 0$.**

48. $n + |-n| = 2n$

49. $n + |-n| = 0$

TRUE OR FALSE? **Determine whether the statement is *true* or *false*. Explain your reasoning.**

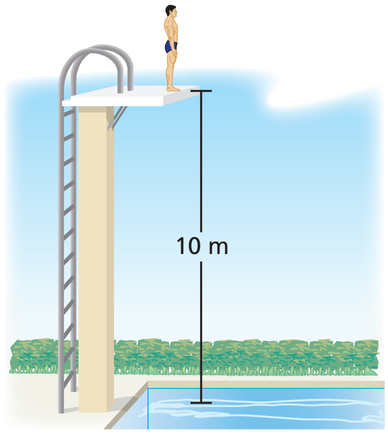
50. If $x < 0$, then $|x| = -x$.

51. The absolute value of every rational number is positive.

31. MP **STRUCTURE** A scientist records the water temperature and the air temperature in Antarctica. The water temperature is -2°C. The air is 9°C colder than the water. Which expression can be used to find the air temperature? Explain your reasoning.

$-2 + 9$ $\quad$ $-2 - 9$ $\quad$ $9 - 2$

32. **MODELING REAL LIFE** A shark is 80 feet below the surface of the water. It swims up and jumps out of the water to a height of 15 feet above the surface. Find the vertical distance the shark travels. Justify your answer.



33. **MODELING REAL LIFE** The figure shows a diver diving from a platform. The diver reaches a depth of 4 meters. What is the change in elevation of the diver?

34. **OPEN-ENDED** Write two different pairs of negative integers, x and y, that make the statement $x - y = -1$ true.

USING PROPERTIES **Tell how the Commutative and Associative Properties of Addition can help you evaluate the expression using mental math. Then evaluate the expression.**

35. $2 - 7 + (-2)$ $\quad$ **36.** $-6 - 8 + 6$ $\quad$ **37.** $8 + (-8 - 5)$

38. $-39 + 46 - (-39)$ $\quad$ **39.** $[13 + (-28)] - 13$ $\quad$ **40.** $-2 + (-47 - 8)$

ALGEBRA **Evaluate the expression when $k = -3$, $m = -6$, and $n = 9$.**

41. $4 - n$ $\quad$ **42.** $m - (-8)$ $\quad$ **43.** $-5 + k - n$ $\quad$ **44.** $|m - k|$

45. **MODELING REAL LIFE** The table shows the record monthly high and low temperatures for a city in Alaska.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High (°F)	56	57	56	72	82	92	84	85	73	64	62	53
Low (°F)	−35	−38	−24	−15	1	29	34	31	19	−6	−21	−36

a. Which month has the grea

Indicator 2f - In #46-49, students use reasoning and their knowledge of multiplying integers to make statements about the products of integers.

b. What is the range of temperatures for the year?

MP **REASONING** **Tell whether the difference of the two integers is *always*, *sometimes*, or *never* positive. Explain your reasoning.**

46. two positive integers $\quad$ **47.** a positive integer and a negative integer

48. two negative integers $\quad$ **49.** a negative integer and a positive integer

MP **NUMBER SENSE** **For what values of a and b is the statement true?**

50. $|a - b| = |b - a|$ $\quad$ **51.** $|a - b| = |a| - |b|$

31. MODELING REAL LIFE You read 105 pages of a novel over 7 days. What is the mean number of pages you read each day?

USING ORDER OF OPERATIONS **Evaluate the expression.**

32. $-8 - 14 \div 2 + 5$

33. $24 \div (-4) + (-2) \cdot (-5)$

EVALUATING EXPRESSIONS **Evaluate the expression when $x = 10$, $y = -2$, and $z = -5$.**

34. $x \div y$

35. $12 \div 3y$

36. $\frac{2z}{y}$

37. $\frac{-x + y}{6}$

38. $100 \div (-z^2)$

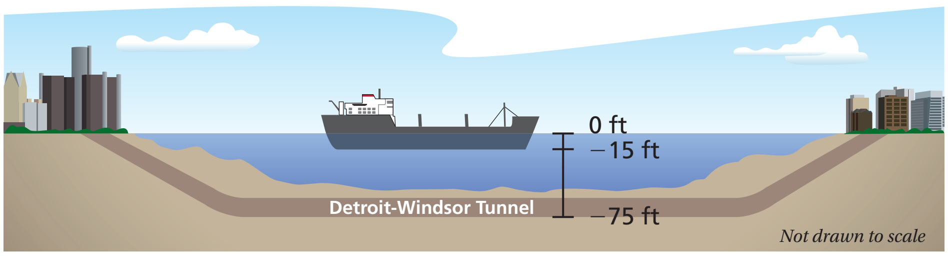
39. $\frac{10y^2}{z}$

40. $\left|\frac{xz}{-y}\right|$

41. $\frac{-x^2 + 6z}{y}$

42. MP **PATTERNS** Find the next two numbers in the pattern $-128, 64, -32, 16, \ldots$. Explain your reasoning.

43. MODELING REAL LIFE The Detroit-Windsor Tunnel is an underwater highway that connects the cities of Detroit, Michigan, and Windsor, Ontario. How many times deeper is the roadway than the bottom of the ship?



44. MODELING REAL LIFE A snowboarder descends from an elevation of 2253 feet to an elevation of 1011 feet in 3 minutes. What is the mean change in elevation per minute?

45. MP **REASONING** The table shows a golfer's scores relative to *par* for three out of four rounds of a tournament.

a. What was the golfer's mean score per round for the first 3 rounds?

b. The golfer's goal for the tournament is to have a mean score no greater than −3. Describe how the golfer can achieve this goal.

Scorecard	
Round 1	+1
Round 2	−4
Round 3	−3
Round 4	?

46. MP **PROBLEM SOLVING** The regular admission price for an amusement park is \$72. For a group of 15 or more, the admission price is reduced by \$25 per person. How many people need to be in a group to save \$500?

47. DIG DEEPER! Write a set of five different integers that has a mean of −10. Explain how you found your answer.

2.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

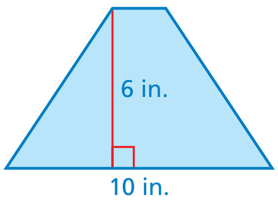
Review & Refresh

Write the fraction or mixed number as a decimal.

1. $\frac{5}{16}$ **2.** $-\frac{9}{22}$ **3.** $6\frac{8}{11}$ **4.** $-\frac{26}{24}$

Find the area of the figure.

5.



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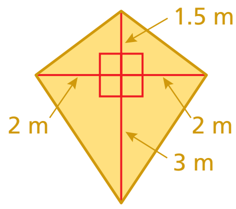

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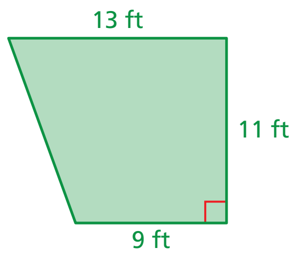
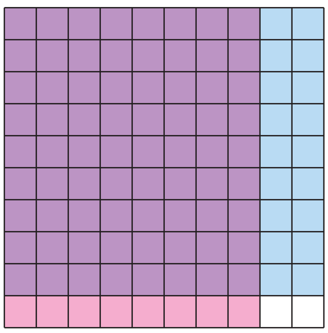


Concepts, Skills, & Problem Solving

FINDING PRODUCTS OF RATIONAL NUMBERS **Write a multiplication expression represented by the area model. Then find the product.** (See Exploration 1, p. 67.)

8.



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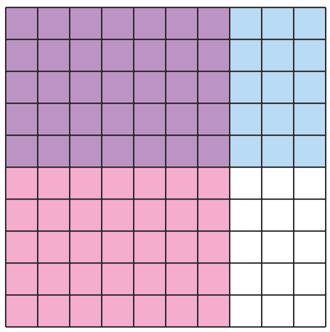


MP REASONING **Without multiplying, tell whether the value of the expression is positive or negative. Explain your reasoning.**

10. $-1\left(\frac{4}{5}\right)$ **11.** $\frac{4}{7} \cdot \left(-3\frac{1}{2}\right)$ **12.** $-0.25(-3.659)$

MULTIPLYING RATIONAL NUMBERS **Find the product. Write fractions in simplest form.**

13. $-\frac{1}{4} \times \left(-\frac{4}{3}\right)$ **14.** $\frac{5}{6}\left(-\frac{8}{15}\right)$ **15.** $-2\left(-1\frac{1}{4}\right)$

16. $-3\frac{1}{3} \cdot \left(-2\frac{7}{10}\right)$ **17.** $0.4 \times (-0.03)$ **18.** $-0.05 \times (-0.5)$

19. $-8(0.09)(-0.5)$ **20.** $\frac{5}{6} \cdot \left(-4\frac{1}{2}\right) \cdot \left(-2\frac{1}{5}\right)$ **21.** $\left(-1\frac{2}{3}\right)^3$

SUBTRACTING LINEAR EXPRESSIONS **Find the difference.**

19. $(-2g + 7) - (g + 11)$

20. $(6d + 5) - (2 - 3d)$

21. $(4 - 5y) - (2y - 16)$

22. $(2n - 9) - (-2.4n + 4)$

23. $\left(-\frac{1}{8}c + 16\right) - \left(\frac{3}{8} + 3c\right)$

24. $\left(\frac{9}{4}x + 6\right) - \left(-\frac{5}{4}x - 24\right)$

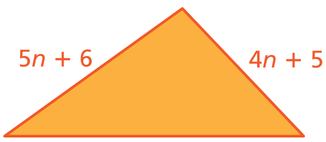
25. $\left(\frac{1}{3} - 6m\right) - \left(\frac{1}{4}n - 8\right)$

26. $(1 - 5q) - (2.5s + 8) - (0.5q + 6)$

27. **YOU BE THE TEACHER** Your friend finds the difference $(4m + 9) - (2m - 5)$. Is your friend correct? Explain your reasoning.

$$\begin{aligned}(4m + 9) - (2m - 5) &= 4m + 9 - 2m - 5\\ &= 4m - 2m + 9 - 5\\ &= 2m + 4\end{aligned}$$

28. **GEOMETRY** The expression $17n + 11$ represents the perimeter of the triangle. What is the length of the third side? Explain your reasoning.



29. **MP LOGIC** Your friend says the sum of two linear expressions is always a linear expression. Is your friend correct? Explain.

30. **MODELING REAL LIFE** You burn 265 calories running and then 7 calories per minute swimming. Your friend burns 273 calories running and then 11 calories per minute swimming. You each swim for the same number of minutes. Find and interpret the difference in the amounts of calories burned by you and your friend.

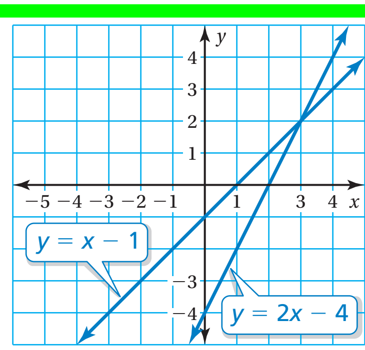
31. **DIG DEEPER!** You start a new job. After w weeks, you have $(10w + 120)$ dollars in your savings account and $(45w + 25)$ dollars in your checking account.

a. What is the total amount of money in the accounts? Explain.

b. How much money did you have before you started your new job? How much money do you save each week?

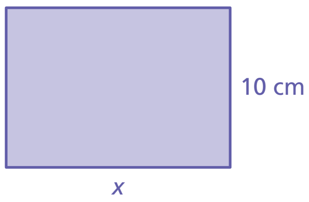
c. You want to buy a new phone for \$150, and still have \$500 left in your accounts afterwards. Explain how to determine when you can buy the phone.

32. **MP REASONING** Write an expression in simplest form that represents the vertical distance between the two lines shown. What is the distance when $x = 3$? when $x = -3$?

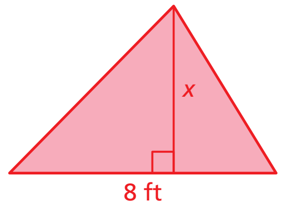


GEOMETRY **Write and solve an inequality that represents *x*.**

41. Area ≥ 120 cm^2



42. Area < 20 ft^2



43. **MODELING REAL LIFE** A device extracts no more than 37 liters of water per day from the air. How long does it take to collect at least 185 liters of water? Explain your reasoning.

44. **MP REASONING** Students in a science class are divided into 6 equal groups with at least 4 students in each group for a project. Describe the possible numbers of students in the class.

45. **PROJECT** Choose two novels to research.

a. Use the Internet to complete the table below.

b. Use the table to find and compare the average number of copies sold per month for each novel. Which novel do you consider to be the most successful? Explain.

c. Assume each novel continues to sell at the average rate. For what numbers of months will the total number of copies sold exceed twice the current number sold for each novel?

Author	Name of Novel	Release Date	Current Number of Copies Sold
1.			
2.			

46. **MP LOGIC** When you multiply or divide each side of an inequality by the same negative number, you must reverse the direction of the inequality symbol. Explain why.

MP NUMBER SENSE **Describe all numbers that satisfy *both* inequalities. Include a graph with your description.**

47. $4m > -4$ and $3m < 15$

48. $\frac{n}{3} \geq -4$ and $\frac{n}{-5} \geq 1$

49. $2x \geq -6$ and $2x \geq 6$

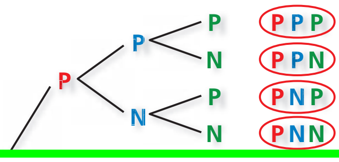
50. $-\frac{1}{2}s > -7$ and $\frac{1}{3}s < 12$

EXAMPLE 5 Modeling Real Life

On a game show, you choose one box from each pair of boxes shown. In each pair, one box contains a prize and the other does not. What is the probability of winning at least one prize?

Choice 1

Use a tree diagram to find the sample space. Let P = prize and N = no prize. Circle the outcomes in which you win 1, 2, or 3 prizes.



Choice 2

Choice 3

There are seven f
at least one prize

P

P(at least on

The probabi

Indicator 2f - In #8-9, students model with mathematics by being presented real-life situations. Students can draw a tree diagram in #8 if they want.

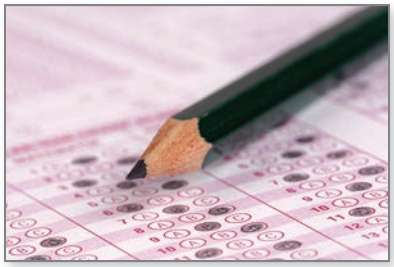
MP4 Model with mathematics - Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.... Mathematically proficient students who can apply what they know... are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Self-Ass

Solve each exercise. Then rate your understanding of the success criteria in your journal.

8. A tour guide organizes vacation packages at a beachside town. There are 7 hotels, 5 cabins, 4 meal plans, 3 escape rooms, and 2 amusement parks. The tour guide chooses either a hotel or a cabin and then selects one of each of the remaining options. Find the total number of possible vacation packages.

9. **DIG DEEPER!** A fitness club with 100 members offers one free training session per member in either running, swimming, or weightlifting. Thirty of the fitness center members sign up for the free session. The running and swimming sessions are each twice as popular as the weightlifting session. What is the probability that a randomly chosen fitness club member signs up for a free running session?



31. MP **REASONING** You randomly guess the answers to two questions on a multiple-choice test. Each question has three choices: A, B, and C.

a. What is the probability that you guess the correct answers to both questions?

b. Suppose you can eliminate one of the choices for each question. How does this change the probability that both of your guesses are correct?

32. MP **REASONING** You forget the last two digits of your cell phone password.

a. What is the probability that you randomly choose the correct digits?

b. Suppose you remember that both digits are even. How does this change the probability that you choose the correct digits?

33. **MODELING REAL LIFE** A combination lock has 3 wheels, each numbered from 0 to 9. You try to guess the combination by writing five different numbers from 0 to 999 on a piece of paper. Find the probability that the correct combination is written on the paper.

34. **MODELING REAL LIFE** A train has one engine and six train cars. Find the total number of ways an engineer can arrange the train. (The engine must be first.)

35. MP **REPEATED REASONING** You have been assigned a nine-digit identification number.

a. Should you use the Fundamental Counting Principle or a tree diagram to find the total number of possible identification numbers? Explain.

b. How many identification numbers are possible?

c. **RESEARCH** Use the Internet to find out why the possible number of Social Security numbers is not the same as your answer to part (b).

36. **DIG DEEPER!** A social media account password includes a number from 0 to 9, an uppercase letter, a lowercase letter, and a special character, in that order.

a. There are 223,080 password combinations. How many special characters are there?

b. What is the probability of guessing the account password if you know the number and uppercase letter, but forget the rest?

37. MP **PROBLEM SOLVING** From a group of 5 scientists, an environmental committee of 3 people is selected. How many different committees are possible?

EXAMPLE 3 Modeling Real Life

A tsunami warning siren can be heard up to 2.5 miles away in all directions. From how many square miles can the siren be heard?

Understand the problem.

You are given the description of a region in which a siren can be heard. You are asked to find the number of square miles within the range of the siren.

Make a plan.

Two and a half miles from the siren in all directions is a circular region with a radius of 2.5 miles. So, find the area of a circle with a radius of 2.5 miles.

Solve and check.

$A = \pi r^2$	Write formula for area.
$\approx 3.14 \cdot 2.5^2$	Substitute 3.14 for π and 2.5 for r.
$= 3.14 \cdot 6.25$	Evaluate 2.5^2.
$= 19.625$	Multiply.

So, the siren can be heard from about 20 square miles.

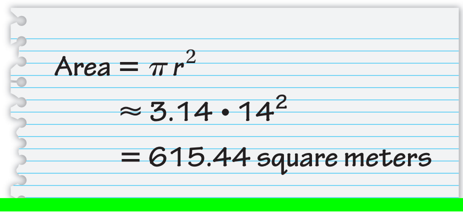
Check Reasonableness The number of square miles should be greater than $3 \cdot 2^2 = 12$, but less than $4 \cdot 3^2 = 36$. Because $12 < 20 < 36$, the answer is reasonable. ✓

Self-Assessment for Problem Solving

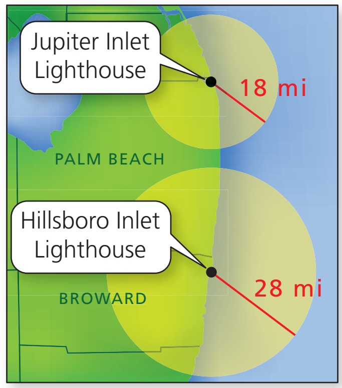
Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. A local event planner wants to cover a circular region with mud for an obstacle course. The region has a circumference of about 157 feet. The cost to cover 1 square foot with mud is \$1.50. Approximate the cost to cover the region with mud.

10. **DIG DEEPER!** A manufacturer recommends that you use a frying pan with a radius that is within 1 inch of the radius of your stovetop burner. The area of the bottom of your frying pan is 25π square inches. The circumference of your cooktop burner is 9π inches. Does your frying pan meet the manufacturer's recommendation?



13. **YOU BE THE TEACHER** Your friend finds the area of a circle with a diameter of 7 meters. Is your friend correct? Explain.



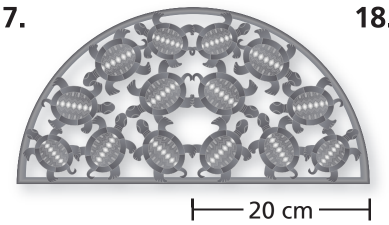
14. **MODELING REAL LIFE** The diameter of a flour tortilla is 12 inches. What is the total area of two tortillas?

15. **MODELING REAL LIFE** The diameter of a coaster is 7 centimeters. What is the total area of five coasters?

16. **MP PROBLEM SOLVING** The Hillsboro Inlet Lighthouse lights up how much more area than the Jupiter Inlet Lighthouse?

FINDING THE AREA OF A SEMICIRCLE **Find the area of the semicircle.**

17.



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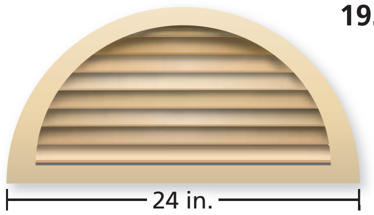

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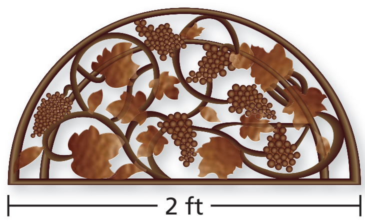
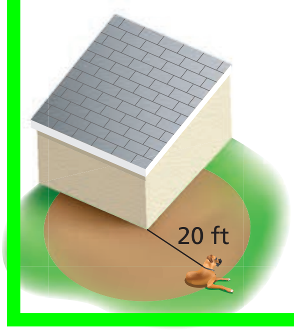


20. **MODELING REAL LIFE** The plate for a microscope has a circumference of 100π millimeters. What is the area of the plate?

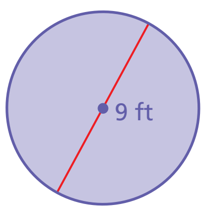


21. **MODELING REAL LIFE** A dog is leashed to the corner of a house. How much running area does the dog have? Explain how you found your answer.

22. **MP REASONING** Target A has a circumference of 20 feet. Target B has a diameter of 3 feet. Both targets are the same distance away. Which target is easier to hit? Explain your reasoning.

23. **DIG DEEPER!** A circular oil spill has a radius of 2 miles. After a day, the radius of the oil spill increases by 3 miles. By how many square miles does the area of the oil spill increase?

24. **FINDING AN AREA** Find the area of the circle in square yards.



25. **MP REPEATED REASONING** What happens to the circumference and the area of a circle when you double the radius? triple the radius? Justify your answer.

26. **CRITICAL THINKING** Is the area of a semicircle with a diameter of x *greater than*, *less than*, or *equal to* the area of a circle with a diameter of $\frac{1}{2}x$? Explain.

EXAMPLE 5 Modeling Real Life

You enclose a flower bed using landscaping boards with lengths of 3 yards, 4 yards, and 5 yards. Estimate the area of the flower bed.

Understand the problem.

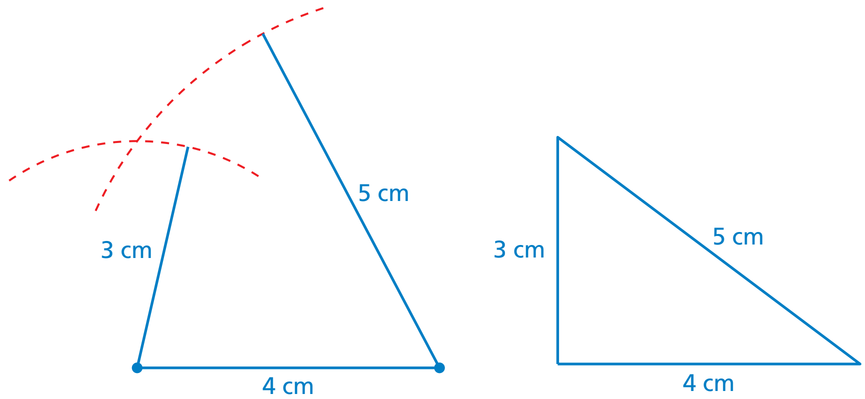
You know the lengths of boards used to enclose a triangular region. You are asked to estimate the area of the triangular region.

Make a plan.

Draw a triangle with side lengths of 3 yards, 4 yards, and 5 yards using a scale of 1 cm : 1 yd. Use the drawing to estimate the base and height of the flower bed. Then use the formula for the area of a triangle to estimate the area.

Solve and check.

Draw the triangle.



Another Method
Using a ruler, the height from the largest angle to the 5-centimeter side is about 2.4 centimeters. So, the area is about $\frac{1}{2}(2.4)(5) = 6 \text{ yd}^2$. ✓

The shape of the flower bed appears to be a right triangle with a base length of 4 yards and a height of 3 yards.

So, the area of the flower bed is about $A = \frac{1}{2}(4)(3) = 6$ square yards.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

15. A triangular pen has fence lengths of 6 feet, 8 feet, and 10 feet. Create a scale drawing of the pen.

16. The front of a cabin is the shape of a triangle. The angles of the triangle are 40°, 70°, and 70°. Can you determine the height of the cabin? If not, what information do you need?

17. **DIG DEEPER!** Two rooftops have triangular patios. One patio has side lengths of 9 meters, 10 meters, and 11 meters. The other has side lengths of 6 meters, 10 meters, and 15 meters. Which patio has a greater area? Explain.

CONSTRUCTING QUADRILATERALS **Draw a quadrilateral with the given angle measures, if possible.**

31. 60°, 60°, 120°, 120°

32. 50°, 60°, 110°, 150°

33. 20°, 30°, 150°, 160°

34. 10°, 10°, 10°, 150°

CONSTRUCTING SPECIAL QUADRILATERALS **Construct a quadrilateral with the given description.**

35. a rectangle with side lengths of 1 inch and 2 inches

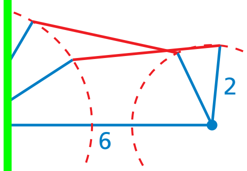
36. a kite with side lengths of 4 centimeters and 7 centimeters

37. a trapezoid with base angles of 40°

38. a rhombus with side length

Indicator 2f - In #41-43, students model with mathematics by being presented real-life situations. Students write and evaluate expressions for area for these situations.

39. MP **REASONING** A quadri
2 units, and 3 units as show
be formed given a fourth si



40. MP **REASONING** What typ four side lengths of 7 units? Use drawings to support your conclusion.

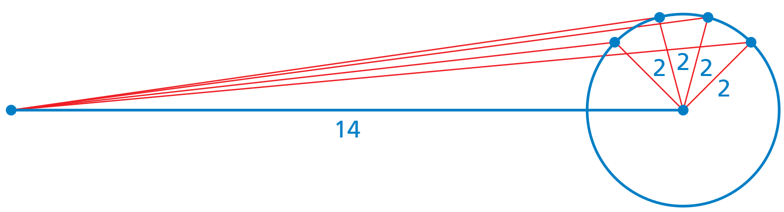
41. **MODELING REAL LIFE** A triangular section of a farm is enclosed by fences that are 2 meters, 6 meters, and 7 meters long. Estimate the area of the section.

42. **MODELING REAL LIFE** A chemical spill expert sets up a triangular caution zone using cones. Cones A and B are 14 meters apart. Cones B and C are 22 meters apart. Cones A and C are 34 meters apart. Estimate the area of the caution zone.

43. **MODELING REAL LIFE** A search region is in the shape of an equilateral triangle. The measure of one side of the region is 20 miles. Make a scale drawing of the search region. Estimate the area of the search region.

44. MP **REASONING** A triangle has fixed side lengths of 2 and 14.

a. How many triangles can you construct? Use the figure below to explain your reasoning.



b. Is the unknown side length of the triangle also fixed? Explain.

2.1 Practice

Go to *BigIdeasMath.com* to get HELP with solving the exercises.

Review & Refresh

Find the distance between the two numbers on a number line.

1. -4.3 and 0.8 **2.** -7.7 and -6.4 **3.** $-2\frac{3}{5}$ and -1

Divide.

4. $27 \div 9$ **5.** $48 \div 6$ **6.** $56 \div 4$ **7.** $153 \div 8$

8. What is the prime factorization of 84?

A. $2^2 \times 3^2$ **B.** $2^3 \times 7$ **C.** $3^3 \times 7$ **D.** $2^2 \times 3 \times 7$

Concepts, Skills, & Problem Solving

MP **CHOOSE TOOLS** **Use a number line or integer counters to find the product.** (See Exploration 1, p. 49.)

9. $2(-4)$ **10.** $-6(3)$ **11.** $4(-5)$

MULTIPLYING INTEGERS **Find the product.**

12. $6 \cdot 4$ **13.** $7(-3)$ **14.** $-2(8)$ **15.** $-3(-4)$

16. $-6 \cdot 7$

20. $-5(10)$

24. $-10 \cdot 11$

Indicator 2f - In #9-11 students can choose whether to use a number line or integer counters to find the product of two integers.

MP5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.... Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

EVALUATING EXPRESSIONS **Evaluate the expression.**

30. $(-4)^2$ **31.** -6^2 **32.** $-5 \cdot 3 \cdot (-2)$

33. $3 \cdot (-12) \cdot 0$ **34.** $-5(-7)(-20)$ **35.** $5 - 8^2$

36. $-5^2 \cdot 4$ **37.** $-2 \cdot (-3)^3$ **38.** $2 + 1 \cdot (-7 + 5)$

39. $4 - (-2)^3$ **40.** $4 \cdot (25 \cdot 3^2)$ **41.** $-4(3^2 - 8) + 1$

7.4 Simulations

Learning Target: Design and use simulations to find probabilities of compound events.

Success Criteria:
- I can design a simulation to model a real-life situation.
- I can recognize favorable outcomes in a simulation.
- I can use simulations to find experimental probabilities.

EXPLORATION 1 Using a Simulation

Work with a partner. A basketball player makes 80% of her free throw attempts.

a. Is she likely to make at least two of her next three free throws? Explain your reasoning.

b. The table shows 30 randomly generated numbers from 0 to 999. Let each number represent three shots. How can you use the digits of these numbers to represent made shots and missed shots?

838	617	282	341	785
747	332	279	082	716
937	308	800	994	689
198	025	853	591	813
672	289	518	649	540
865	631	227	004	840

c. Use the table to estimate the probability that of her next three free throws, she makes

- exactly two free throws.
- at most one free throw.
- at least two free throws.
- at least two free throws in a row.

d. The experiment used in parts (b) and (c) is called a *simulation*. Another player makes $\frac{3}{5}$ of her free throws. Describe a simulation that can be used to estimate the probability that she makes three of her next four free throws.

Math Practice

Choose Tools
What tools can you use to randomly generate data?

1.1 Rational Numbers

Learning Target: Understand absolute values and ordering of rational numbers.

Success Criteria:
- I can graph rational numbers on a number line.
- I can find the absolute value of a rational number.
- I can use a number line to compare rational numbers.

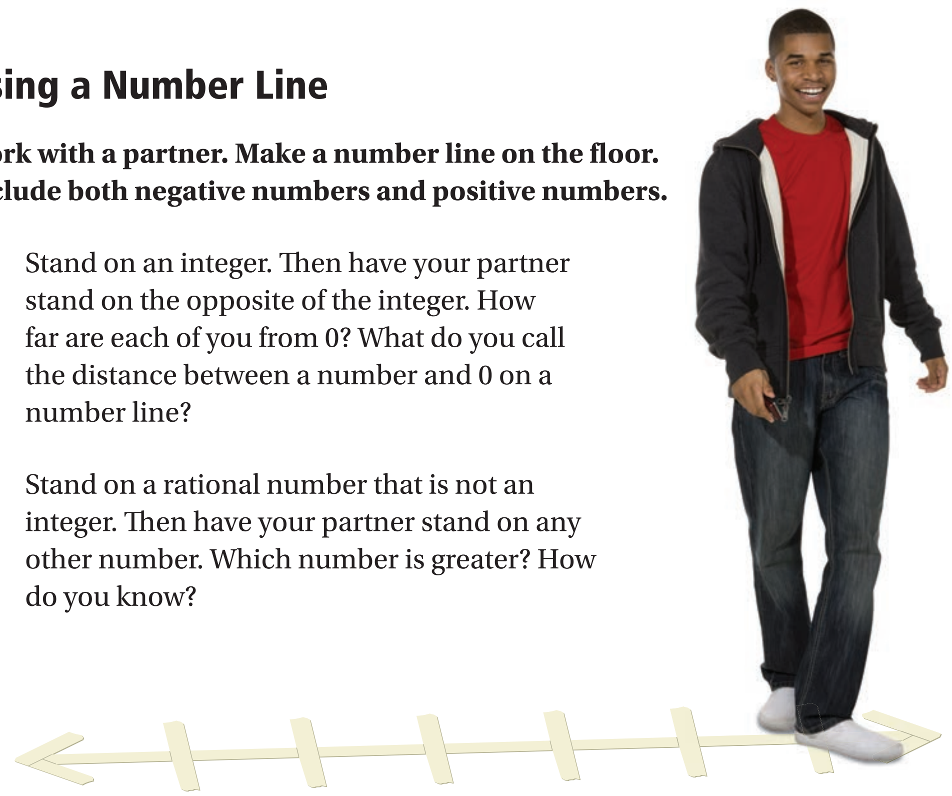
Recall that **integers** are the set of whole numbers and their opposites. A **rational number** is a number that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

EXPLORATION 1 Using a Number Line

Work with a partner. Make a number line on the floor. Include both negative numbers and positive numbers.

a. Stand on an integer. Then have your partner stand on the opposite of the integer. How far are each of you from 0? What do you call the distance between a number and 0 on a number line?

b. Stand on a rational number that is not an integer. Then have your partner stand on any other number. Which number is greater? How do you know?



c. Stand on any number other than 0 on the number line. Can your partner stand on a number that is:
- greater than your number and farther from 0?
- greater than your number and closer to 0?
- less than your number and the same distance from 0?
- less than your number and farther from 0?

For each case in which it was not possible to stand on a number as directed, explain why it is not possible. In each of the other cases, how can you decide where your partner can stand?

Math Practice

Find Entry Points

What are some ways to determine which of two numbers is greater?

EXAMPLE 2 Evaluating Expressions

The expression $(-2)^2$ indicates to multiply the number in parentheses, -2, by itself.
The expression -2^2, however, indicates to find the opposite of 2^2.

a. **Find $(-2)^2$.**

$(-2)^2 = (-2) \cdot (-2)$ — Write $(-2)^2$ as repeated multiplication.

$= 4$ — Multiply.

b. **Find -2^2.**

$-2^2 = -(2 \cdot 2)$ — Write 2^2 as repeated multiplication.

$= -4$ — Multiply 2 and 2.

c. **Find $-2 \cdot 17 \cdot (-5)$.**

$-2 \cdot 17 \cdot (-5) = -2 \cdot (-5) \cdot 17$ — Commutative Property of Multiplication

$= 10 \cdot 17$ — Multiply -2 and -5.

$= 170$ — Multiply 10 and 17.

Remember

Use order of operations when evaluating an expression.

d. **Find $-6(-3 + 4) + 6$.**

$-6(-3 + 4) + 6 = -6(1) + 6$ — Perform operation in parentheses.

$= -6 + 6$ — Multiplication Property of 1

$= 0$ — Additive Inverse Property

Try It **Evaluate the expression.**

7. $8 \cdot (-15) \cdot 0$ **8.** $24 - 3^3$ **9.** $10 - 7(3 - 5)$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. **WRITING** What can you conclude about two integers whose product is (a) positive and (b) negative?

EVALUATING AN EXPRESSION **Evaluate the expression.**

11. $4(-8)$ **12.** $-5(-7)$ **13.** $12 - 3^2 \cdot (-2)$

MP **REASONING** **Tell whether the statement is *true* or *false*. Explain your reasoning.**

14. The product of three positive integers is positive.

15. The product of three negative integers is positive.

EXAMPLE 3 Simplifying Algebraic Expressions

a. Simplify $\frac{3}{4}y + 12 - \frac{1}{2}y - 6$.

$$\frac{3}{4}y + 12 - \frac{1}{2}y - 6 = \frac{3}{4}y + 12 + \left(-\frac{1}{2}y\right) + (-6)$$ Rewrite as a sum.

$$= \frac{3}{4}y + \left(-\frac{1}{2}y\right) + 12 + (-6)$$ Commutative Property of Addition

$$= \left[\frac{3}{4} + \left(-\frac{1}{2}\right)\right]y + 12 + (-6)$$ Distributive Property

$$= \frac{1}{4}y + 6$$ Combine like terms.

b. Simplify $-3y - 5y + 4z + 9z$.

$$-3y - 5y + 4z + 9z = (-3 - 5)y + (4 + 9)z$$ Distributive Property

$$= -8y + 13z$$ Simplify.

Try It **Simplify the expression.**

7. $14 - 3z + 8 + z$ **8.** $2.5x + 4.3x - 5$ **9.** $2s - 9s + 8t - t$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. WRITING Explain how to identify the terms and like terms of $3y - 4 - 5y$.

SIMPLIFYING ALGEBRAIC EXPRESSIONS **Simplify the expression.**

11. $7p + 6p$ **12.** $\frac{4}{5}n - 3 + \frac{7}{10}n$ **13.** $2w - g - 7w + 3g$

14. VOCABULARY Is the expression $3x + 2x - 4$ in simplest form? Explain.

15. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$-4 + 6 + 3x$	$3x + 9 - 7$
$5x - 10 - 2x$	$5x - 4 + 6 - 2x$

33. **MODELING REAL LIFE** You make a profit of \$0.75 for every bracelet you sell. Write and solve an equation to determine how many bracelets you must sell to earn enough money to buy the soccer cleats shown.

34. **MODELING REAL LIFE** A rock climber averages $12\frac{3}{5}$ feet climbed per minute. How many feet does the rock climber climb in 30 minutes? Justify your answer.

OPEN-ENDED **Write (a) a multiplication equation and (b) a division equation that has the given solution.**

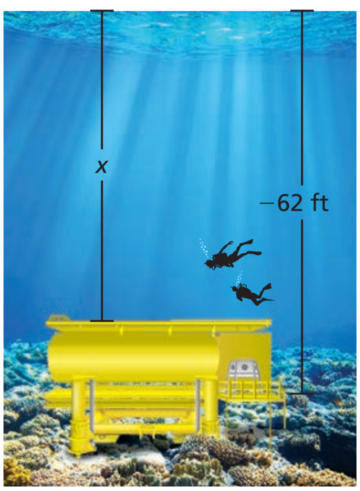
35. -3 **36.** -2.2 **37.** $-\frac{1}{2}$ **38.** $-1\frac{1}{4}$

39. MP **REASONING** Which method(s) can you use to solve $-\frac{2}{3}c = 16$?

Multiply each side by $-\frac{2}{3}$.	Multiply each side by $-\frac{3}{2}$.
Divide each side by $-\frac{2}{3}$.	Multiply each side by 3, then divide each side by -2.

40. **MODELING REAL LIFE** A stock has a return of $-\$1.26$ per day. Find the number of days until the total return is $-\$10.08$. Justify your answer.

41. MP **PROBLEM SOLVING** In a school election, $\frac{3}{4}$ of the students vote. There are 1464 votes. Find the number of students. Justify your answer.



42. **DIG DEEPER!** The diagram shows Aquarius, an underwater ocean laboratory located in the Florida Keys National Marine Sanctuary. The equation $\frac{31}{25}x = -62$ can be used to calculate the depth of Aquarius. Interpret the equation. Then find the depth of Aquarius. Justify your answer.

43. MP **PROBLEM SOLVING** The price of a bike at Store A is $\frac{5}{6}$ the price at Store B. The price at Store A is \$150.60. Find how much you save by buying the bike at Store A. Justify your answer.

44. **CRITICAL THINKING** Solve $-2|m| = -10$.

45. MP **NUMBER SENSE** In 4 days, your family drives $\frac{5}{7}$ of the total distance of a trip. The total distance is 1250 miles. At this rate, how many more days will it take to reach your destination? Justify your answer.

EXAMPLE 2 Checking Solutions

a. Tell whether -2 is a solution of $y - 5 \geq -6$.

$y - 5 \geq -6$ Write the inequality.

$-2 - 5 \overset{?}{\geq} -6$ Substitute -2 for y.

$-7 \ngeq -6$ ✗ Simplify.

So, -2 is *not* a solution of the inequality.

b. Tell whether -2 is a solution of $-5.5y < 14$.

$-5.5y < 14$ Write the inequality.

$-5.5(-2) \overset{?}{<} 14$ Substitute -2 for y.

$11 < 14$ ✓ Simplify.

So, -2 is a solution of the inequality.

Try It **Tell whether -5 is a solution of the inequality.**

3. $x + 12 > 7$ **4.** $1 - 2p \leq -9$ **5.** $n \div 2.5 \geq -3$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

6. MP **REASONING** Do $x < 5$ and $5 < x$ represent the same inequality? Explain.

7. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

A number k is less than or equal to -3.

A number k is at least -3.

A number k is at most -3.

A number k is no more than -3.

CHECKING SOLUTIONS **Tell whether -4 is a solution of the inequality.**

8. $c + 6 \leq 3$ **9.** $6 > p \div (-0.5)$ **10.** $-7 < 2g + 1$

GRAPHING AN INEQUALITY **Graph the inequality on a number line.**

24. $r \leq -9$ **25.** $g > 2.75$ **26.** $x \geq -3\frac{1}{2}$ **27.** $1\frac{1}{4} > z$

28. **MODELING REAL LIFE** Each day at lunchtime, at least 53 people buy food from a food truck. Write and graph an inequality that represents this situation.

CHECKING SOLUTIONS **Tell whether the given value is a solution of the inequality.**

29. $4k < k + 8; k = 3$ **30.** $\frac{w}{3} \geq w - 12; w = 15$

31. $7 - 2y > 3y + 13; y = -1$ **32.** $\frac{3}{4}b - 2 \leq 2b + 8; b = -4$

33. MP **PROBLEM SOLVING** A single subway ride for a student costs $1.25. A monthly pass costs $35.

a. Write an inequality that represents the numbers of times you can ride the subway each month for the monthly pass to be a better deal.

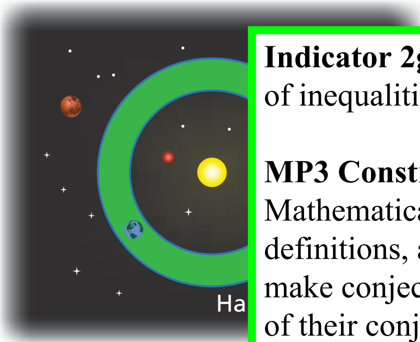
b. You ride the subway about 45 times per month. Should you buy the monthly pass? Explain.

34. MP **LOGIC** Consider the inequality $b > -2$.

a. Describe the values of b that are solutions of the inequality.

b. Describe the values of b that are *not* solutions of the inequality. Write an inequality that represents these values.

c. What do all the values in parts (a) and (b) represent? Is this true for any similar pair of inequalities? Explain your reasoning.



Indicator 2g.i - In #34, students have to construct arguments about the solutions of inequalities.

MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

36. **DIG DEEPER** around the p length as a si package can of 108 inches

a. Write an dimensi

b. Find thre reasonal

5.1 Ratios and Ratio Tables

Learning Target: Understand ratios of rational numbers and use ratio tables to represent equivalent ratios.

Success Criteria:
- I can write and interpret ratios involving rational numbers.
- I can use various operations to create tables of equivalent ratios.
- I can use ratio tables to solve ratio problems.

EXPLORATION 1 Describing Ratio Relationships

Work with a partner. Use the recipe shown.

Chicken Soup

stewed tomatoes	9 ounces	chopped spinach	9 ounces
chicken broth	15 ounces	grated parmesan	5 tablespoons
chopped chicken	1 cup		

a. Identify several ratios in the recipe.

b. You halve the recipe. Describe your ratio relationships in part (a) using the new quantities. Is the relationship between the ingredients the same as in part (a)? Explain.

EXPLORATION 2 Completing Ratio Tables

Work with a partner. Use the ratio tables shown.

x	5			
y	1			

x	$\frac{1}{4}$			
y	$\frac{1}{2}$			

Math Practice

Communicate Precisely

How can you determine whether the ratios in each table are equivalent?

a. Complete the first ratio table using multiple operations. Use the same operations to complete the second ratio table.

b. Are the ratios in the first table equivalent? the second table? Explain.

c. Do the strategies for completing ratio tables of whole numbers work for completing ratio tables of fractions? Explain your reasoning.

41. **MP STRUCTURE** You add the same numbers of pennies and dimes to the coins shown. Is the new ratio of pennies to dimes proportional to the original ratio of pennies to dimes? If so, illustrate your answer with an example. If not, show why with a counterexample.

a. 

b. 

42. **MP REASONING** You are 13 years old, and your cousin is 19 years old. As you grow older, is your age proportional to your cousin's age? Explain your reasoning.

43. **MODELING REAL LIFE** The shadow of the moon during a solar eclipse travels 2300 miles in 1 hour. In the first 20 minutes, the shadow traveled $766\frac{2}{3}$ miles. How long does it take for the shadow to travel 1150 miles? Justify your answer.

44. **MODELING REAL LIFE** In 60 seconds, a car in a parade travels 0.2 mile. The car traveled the last 0.05 mile in 12 seconds. How long did it take for the car to travel 0.1 mile? Justify your answer.

45. **OPEN-ENDED** Describe (a) a real-life situation where you expect two quantities to be proportional and (b) a real-life situation where you do *not* expect two quantities to be proportional. Explain your reasoning.

46. **MP PROBLEM SOLVING** A specific shade of red nail polish requires 7 parts red to 2 parts yellow. A mixture contains 35 quarts of red and 8 quarts of yellow. Is the mixture the correct shade? If so, justify your answer. If not, explain how you can fix the mixture to make the correct shade of red.

47. **MP LOGIC** The quantities x and y are proportional. Use each of the integers 1–5 to complete the table. Justify your answer.

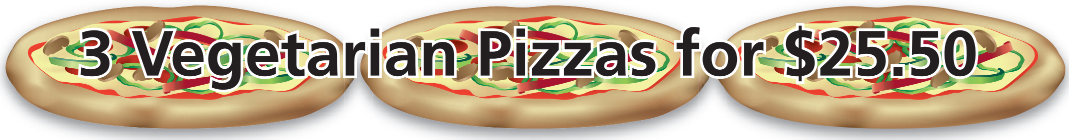
x	10		6	
y				0.5

48. **CRITICAL THINKING** Ratio A and Ratio B form a proportion. Ratio B and Ratio C also form a proportion. Do Ratio A and Ratio C form a proportion? Justify your answer.

43. **MODELING REAL LIFE** There are 144 people in an audience. The ratio of adults to children is 5 to 3. How many are adults?

44. MP **PROBLEM SOLVING** You have \$50 to buy T-shirts. You can buy 3 T-shirts for \$24. Do you have enough money to buy 7 T-shirts? Justify your answer.

45. MP **PROBLEM SOLVING** You buy 10 vegetarian pizzas and pay with \$100. How much change do you receive?



46. **MODELING REAL LIFE** A person who weighs 120 pounds on Earth weighs 20 pounds on the Moon. How much does a 93-pound person weigh on the Moon?

47. MP **PROBLEM SOLVING** Three pounds of lawn seed covers 1800 square feet. How many bags are needed to cover 8400 square feet?

48. **MODELING REAL LIFE** There are 180 white lockers in a school. There are 3 white lockers for every 5 blue lockers. How many lockers are in the school?

CONVERTING MEASURES **Use a proportion to complete the statement. Round to the nearest hundredth if necessary.**

49. 6 km ≈ ☐ mi **50.** 2.5 L ≈ ☐ gal **51.** 90 lb ≈ ☐ kg

SOLVING A PROPORTION **Solve the proportion.**

52. $\frac{2x}{5} = \frac{9}{15}$ **53.** $\frac{5}{2} = \frac{d-2}{4}$ **54.** $\frac{4}{k+3} = \frac{8}{14}$

55. MP **LOGIC** It takes 6 hours for 2 people to build a swing set. Can you use the proportion $\frac{2}{6} = \frac{5}{h}$ to determine the number of hours h it will take 5 people to build the swing set? Explain.

56. MP **STRUCTURE** The ratios $a : b$ and $c : d$ are equivalent. Which of the following equations are proportions? Explain your reasoning.

$\frac{b}{a} = \frac{d}{c}$ $\frac{a}{c} = \frac{b}{d}$ $\frac{a}{d} = \frac{c}{b}$ $\frac{c}{a} = \frac{d}{b}$

57. **CRITICAL THINKING** Consider the proportions $\frac{m}{n} = \frac{1}{2}$ and $\frac{n}{k} = \frac{2}{5}$. What is $\frac{m}{k}$? Explain your reasoning.

Laurie's Notes

Key Idea

- Write the first Key Idea on the board.
- **Model:** As you discuss the first Key Idea, ask half the students to model 2 + 5 using integer counters and the other half to model using number lines. Repeat for −2 + (−5).
  - When the signs are the *same*, the counters will be the *same color*.
  - When the signs are the *same*, both arrows will be going in the same *direction*.
- Write the second Key Idea on the board.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** "When you add two integers with different signs, how do you know if the sum is positive or negative?" Students answered a similar question in Example 1, but now they should be using the concept of absolute value, even if they don't use the precise language. You want to hear something about the size of the number, meaning its absolute value.
- **MP8 Look for and Express Regularity in Repeated Reasoning:** Write these problems on the board: 14 + (−8) = ? and (−14) + 8 = ?. Ask, "How are these problems alike? How are they different?" *Sample answers:* They each

Indicator 2g.ii - The Teaching Edition encourages teachers to ask probing questions to engage students in constructing arguments and analyzing the arguments of others. The note for the Key Idea encourages teachers to have students explain how they know if a sum of integers with different signs is positive or negative.

MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Extra Example 2

Find −5 + (−3). −8

ELL Support

Have students practice language by working in pairs to complete Try It Exercises 4–6. Expect students to perform as follows.

Beginner: Write the answer.

Intermediate: State the answer. For example, "twenty."

Advanced: Explain using a sentence. For example, "The sum of seven and thirteen is twenty."

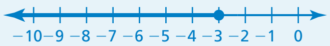
Try It

4. 20
5. −13
6. −17

Extra Example 2

Solve $7y \leq -21$. Graph the solution.

$y \leq -3$;



ELL Support

After demonstrating Example 2, have students practice language by working in pairs to complete Try It Exercises 4–6. Have one student ask another, "What is the first step? Which operation do you need to perform? What is the solution? How do you graph the solution?" Have students alternate roles.

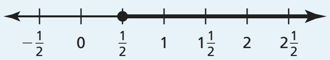
Beginner: Write out the problem and provide one-word answers.

Intermediate: Use phrases or simple sentences such as, "First, I divide by four."

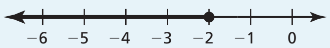
Advanced: Answer with detailed sentences such as, "I divide both sides of the inequality by four."

Try It

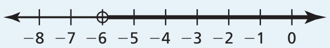
4. $b \geq \frac{1}{2}$



5. $k \leq -2$



6. $q > -6$



Laurie's Notes

EXAMPLE 2

- "What operation is being performed on x?" multiplication
- "How do you undo multiplication?" division
- Solve, graph, and check.

Try It

- **Think-Pair-Share:** Students should read each exercise independently and then work in pairs to solve and graph the inequalities. Have each pair compare their answers with another pair and discuss any discrepancies.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Notice that although all of the coefficients are positive, sometimes the constant is negative. This is important in helping students understand when the direction of the inequality symbol is going to be reversed. The focus is on the sign of the coefficient, not the sign of the constant.
- For Exercise 6, remind students that after solving this inequality, the result will be $-6 < q$. Students can rewrite this as $q > -6$. The direction of the inequality symbol is reversed *only* because the two sides of the inequality are being reversed. Reversing the sign has nothing to do with the negative constant (-6).
- These problems integrate decimal operations.

Key Idea

- These properties look identical to what students have been using in the lesson, *except* now the direction of the inequality symbol must be reversed for the inequality to remain true because they are multiplying or dividing by a *negative* quantity!
- The short version of the property: When you multiply or divide by a negative quantity, reverse the direction of the inequality symbol.
- **Common Error:** When students solve $2x < -4$, they sometimes reverse the inequality symbol because there is a negative number in the problem. You reverse the inequality symbol when you multiply or divide each side by a negative number to eliminate a negative coefficient. You do not reverse the inequality symbol just because there is a negative constant.
- "What happens to the inequality symbol when you multiply both sides of an inequality by 0?" The inequality symbol changes to an equal sign $(0 = 0)$.

Laurie's Notes

Formative Assessment Tip

3-Read Modeling

This technique helps students make sense of word problems by focusing their attention on understanding the situation rather than finding the answer. The *problem stem* (the word problem without the question) is read three times with a different goal each time. First, read the problem stem to the class and ask, "What is this situation about?" Second, lead students in a choral read or have students read the problem stem to a partner. Then ask, "What quantities and units are involved?" Third, ask students to think about what is missing while they choral or partner read. Then ask, "What mathematical questions could you ask about this situation?" After each question is shared, ask, "Can this question be answered with the given information?" Discuss why or why not. Then have students work in groups to solve a question based on the problem stem. You can assign a specific question or allow groups to choose.

EXAMPLE 3

- **3-Read Modeling:** Read the problem stem and scale drawing.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** "Is the actual chip larger or smaller than the chip in the scale drawing? Explain." smaller; 1 centimeter represents only 2 millimeters.
- Measure to find the side length of the chip in the scale drawing.
- To find the actual perimeter and area of the chip, you can begin by setting up and solving a proportion to find the side length of the actual chip. Be careful to label units and use precise language. As shown in the solution, the numerator of each ratio is a drawing distance and the denominator of each ratio is an actual distance.
- Some students will want to bypass the step of writing the proportion and use mental math. Remind them that they are practicing a *process* that will enable them to solve more difficult problems that they may not be able to solve using mental math.
- **MP6 Attend to Precision:** Work slowly through part (c). Remind students that when finding the scale factor, it is necessary to have the same units in the numerator and in the denominator.

Self-Assessment for Problem Solving

- Allow time in class for students to practice using the problem-solving plan. Remember, some students may only be able to complete the first step.
- Have students use *Think-Pair-Share* to solve these problems. It is important for students to spend time working independently first.

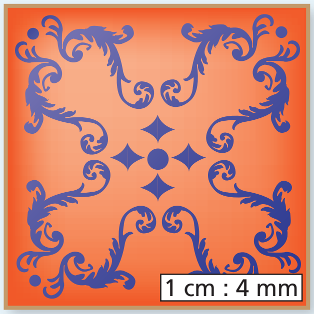
The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

- **Exit Ticket:** A common model train scale is called the HO Scale, where the scale factor is 1 : 87. If the diameter of a wheel on a model train is 0.3 inch, what is the diameter of the actual wheel? 26.1 inches

Extra Example 3

The scale drawing of a miniature glass mosaic tile helps you see the detail on the tile.



4 cm

4 cm

a. Find the perimeter and the area of the tile in the scale drawing. 16 cm; 16 cm^2

b. Find the actual perimeter and area of the tile. 64 mm, 256 mm^2

c. Compare the side lengths of the scale drawing with the actual side lengths of the tile. The side lengths of the scale drawing are 2.5 times the actual side lengths of the mosaic tile.

Self-Assessment
for Problem Solving

7. 291.2 ft, 4515.84 ft^2; 58.24 ft, 180.6336 ft^2

8. Answers will vary.

Learning Target

Solve problems involving scale drawings.

Success Criteria

- Find an actual distance in a scale drawing.
- Explain the meaning of scale and scale factor.
- Use a scale drawing to find the actual lengths and areas of real-life objects.

Learning Target

Construct a polygon with given measures.

Success Criteria

- Use technology to draw polygons.
- Determine whether given measures result in one triangle, many triangles, or no triangle.
- Draw polygons given angle measures or side lengths.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

Teaching Strategy

If you prepare bags of materials for students, label them with the chapter and section number for reuse in following years. Store the prepared bags in a larger bag. Label and file the larger bag with other chapter materials. Make a note in your teaching edition of where the materials are stored.

ELL Support

Explain that a polygon is a figure with three or more angles. Triangles and rectangles are examples of polygons. The word *polygon* comes from the Greek and Latin languages and means "many angles." The prefix *poly-* means "many" and the root *-gon* refers to angles. In this lesson, students will learn how to draw polygons.

Exploration 1

a–c. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
7.G.A.2

Preparing to Teach

- Students should know how to classify two-dimensional figures based on properties, draw polygons, and draw angles. Now they will learn to construct polygons by hand given angle measures or side lengths.
- **MP5 Use Appropriate Tools Strategically:** Students will investigate in the exploration using technology and then later in the lesson they will use other tools. It is important for students to select tools strategically as they develop understanding of mathematical concepts. Discussion of different approaches is essential.

Motivate

- Play a quick game that will help students remember vocabulary relating to triangles. Divide the class into two groups and give each group a vocabulary word. Each group must write the definition on a piece of paper and hand it to you. Definitions must be written in complete sentences. The first team with a correct definition gets a point. The team with the most points at the end wins.
- Some examples: obtuse angle, acute angle, right angle, scalene triangle, isosceles triangle, right triangle, equilateral triangle, equiangular triangle

Exploration 1

- This exploration is best completed using technology. If geometry software is available, let students practice using it before starting the exploration.
- If technology is not available, students can use drinking straws cut to the indicated lengths for polygons (i)–(vii). For polygons (viii)–(xiv), students can use protractors, transparencies, and transparency markers. Students can draw each angle on a separate transparency and then layer the transparencies to form a polygon, if possible. Encourage students to extend the rays of their angles to the edges of the transparencies.
- **Teaching Strategy**: To save time, you could pre-cut straws and prepare reclosable bags with the necessary pieces for each pair of students. Tell students that all of the lengths of straws they need are in the bags.
- Tell students that the given measurements are the only side lengths or angle measures they should use. Students should not introduce other line segments or angles to complete a figure.
- As students complete part (a), they should draw and name the possible figures.
- ? Discuss students' answers in part (a). Go through each set of side lengths and angle measures, asking, "Were you able to form a polygon? If not, why wasn't it working? If so, did all pairs of students draw the same polygon?"
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask volunteers to explain the process they used to determine if a figure was possible. Encourage others to critique their reasoning.
- As you discuss parts (b) and (c), keep in mind that the goal of this exploration is for students to gain an informal understanding of angle sum rules and the Triangle Inequality Theorem.

Laurie's Notes

Scaffolding Instruction

- In the exploration, students used technology to investigate constructing polygons given side lengths and angle measures. Now they will learn to construct polygons by hand, with the focus on triangles.
- **Emerging:** Students may struggle with using a protractor and may not understand the angle sum rules and the Triangle Inequality Theorem. They will benefit from guided instruction for the examples.
- **Proficient:** Students understand how to construct polygons using technology and have a good sense of the angle sum rules and the Triangle Inequality Theorem. They should review the examples to gain understanding of constructing polygons by hand. Then have students complete the Try It exercises before proceeding to the Self-Assessment exercises.

EXAMPLE 1

- No constraints are given for the side lengths. So, one way to draw the triangle is to draw the first given angle at one end of a segment, draw the second given angle at the other end, and extend the rays for the two angles until they intersect.
- **Teaching Tip:** Encourage students not to make tiny drawings. It can be difficult to measure angles when the side lengths are shorter than the radius of the protractor.

? **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** "Does the order in which you draw the angles matter? Explain." No, after you draw two angles, the third angle should have the measure of the remaining angle. Have half the students start with the 30° angle and the other half start with the 60° angle. Students should measure to verify that the third angle has the desired measure.

? "How do you classify this triangle?" right scalene

- Students should compare their triangles to those of their neighbors. They should realize that many different-sized triangles can have angles of 30°, 60°, and 90°.They should understand that every neighbor's triangle should be a right scalene triangle. Point this out if they do not make this conclusion.

Try It

- Have students draw the triangles and then check with a neighbor.

EXAMPLE 2

- You may want to have students draw the sides in inches instead of centimeters to make the triangle a little larger and easier to draw.
- Students should compare their triangles to those of their neighbors. This time, each student should have the same sized triangle. Discuss the push-pin note.
- Repeat the construction but with the 4-centimeter leg horizontal (instead of the 3-centimeter leg). Have them "match up" this triangle with the original triangle. No matter the orientation, the triangles will be the same.

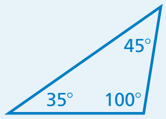
? "How do you classify this triangle?" obtuse scalene

Try It

- Have students draw the triangle and then check with a neighbor.

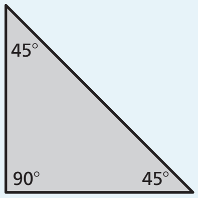
Extra Example 1

Draw a triangle with angle measures of 35°, 45°, and 100°, if possible.

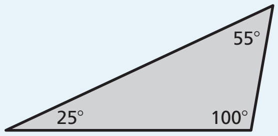


Try It

1.



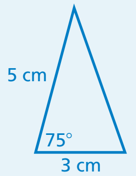
2.



3. not possible

Extra Example 2

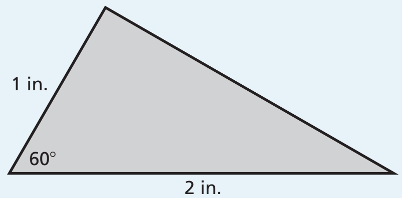
Draw a triangle with side lengths of 3 centimeters and 5 centimeters that meet at a 75° angle.



Not actual size

Try It

4.



Learning Target

Understand absolute values and ordering of rational numbers.

Success Criteria

- Graph rational numbers on a number line.
- Find the absolute value of a rational number.
- Use a number line to compare rational numbers.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL Support

Students may know the word *rational* as meaning "reasonable or sensible" in everyday language. In math, a rational number is a number that can be written as a ratio. The ratio must be written with integers in both the numerator and the denominator, and the denominator cannot be zero. Be aware that students from foreign countries may use atypical phrases to express fractions, such as "two by three" for $\frac{2}{3}$. Explain that in American English "two by three" would signal multiplication, not division.

Exploration 1

a. *Sample answer:* 3 units; the absolute value

b. Answers will vary. The number to the right on the number line is greater.

c. Check students' work.

Laurie's Notes

STATE STANDARDS
Preparing for 7.NS.A.1, 7.NS.A.3

Preparing to Teach

- In this lesson, students will use integers, absolute values, and number lines, which were all introduced in a previous course. It is important that students review these foundational skills because they are necessary for adding and subtracting rational numbers.
- **MP6 Attend to Precision:** Students may incorrectly say, "Absolute value just makes the number positive." Be sure that students correctly refer to the absolute value of a number as the distance the number is from zero.

Motivate

- Tell students that positive and negative signs help describe a relationship between amounts. For example, a negative number can represent the time before an event and a positive number can represent the time after the event.
  - 3 hours before a rocket launch can be represented by T − 3 and 3 hours after the rocket launch can be represented by T + 3.

? "Why is the letter T used?" It represents the time of the rocket launch.

- Solicit other examples of real-life situations involving **integers**.
- Review and discuss the definitions for integers and a **rational number**.

Exploration 1

- Have students use index cards or pieces of paper to label the points on their floor number lines. Check that the tick marks on their number lines are equally spaced and labeled properly. If space is an issue, students can make number lines at their desks using counters or tiles for tick marks.
- As students complete part (a), listen for understanding that the distance between a number and 0 on a number line is the **absolute value**.
- Have students share their reasoning for part (b). Their language may not be precise, but tell students that they will focus on using precise language in this course and it will improve!
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Have students share their reasoning for part (c) and ask other students to critique it. For the third bullet, a student might say, "It's not possible. If your number is positive and your partner's number is less than yours, then your partner is between 0 and you. So, you can't be the same distance from 0." Expect other students to listen carefully and critique the reasoning by saying, "Your partner can be at the opposite of your number. For example, if you are standing at 4, your partner can stand at −4." Do not do all the thinking for students.

The reasoning students use in part (c) can help you assess students' initial understanding of comparing rational numbers on a number line.

? "Is the opposite of 2 on the number line?" yes

? "Are integers the only numbers on the number line? Explain." No, rational numbers are also on the number line.

? "Is 5 the only number 4 units from 1 on the number line? Explain." No, −3 is also 4 units from 1 on the number line.

Laurie's Notes

Key Idea

- Write the Key Idea on the board.
- This Key Idea connects subtraction to finding distances on a number line, an idea that was investigated in the Motivate.
- ? "When finding the distance between two points, does the order in which you subtract matter?" Listen for students to state that subtraction is *not* commutative, however, because you take the absolute value, you can subtract in either order. The differences will be opposites and the absolute values of opposites are the same value.
- Demonstrate a few examples by replacing p and q with integers and then fractions.

EXAMPLE 4

- Work through the problem, referring to the number line as you work.
- Remind students that the order of the numbers does not matter because you find the absolute value of the difference. Demonstrate the alternate method: $\left|-\frac{1}{3}-(-2)\right| = \left|-\frac{1}{3}+2\right| = \left|1\frac{2}{3}\right| = 1\frac{2}{3}$ and encourage students to think about the order in which they will subtract the numbers. Some calculations are more efficient than others.

Try It

- Have students complete the exercises on whiteboards.

Self-Assessment for Concepts & Skills

These exercises will help students assess their understanding of all three success criteria.

? "How does absolute value help in finding the distance between two numbers on a number line?" *Sample answer:* Absolute value represents the distance between a number and 0 on a number line, so finding the absolute value of the difference of two numbers represents the distance between the two numbers.

ELL Support

Have students work in pairs to complete Exercises 12–15 and display their answers to Exercises 13–15 for your review. For Exercise 12, have two pairs discuss their ideas and reach an agreement for the answer. Have groups present their answers to the class.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com.*

Extra Example 4

Find the distance between $-3\frac{1}{4}$ and $2\frac{1}{4}$ on a number line. $5\frac{1}{2}$

Try It

9. 12

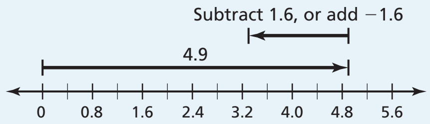
10. 7.8

11. $2\frac{1}{6}$

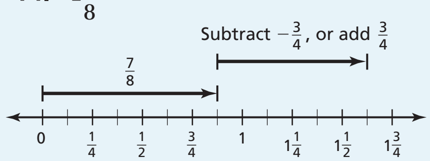
Self-Assessment for Concepts & Skills

12. Draw an arrow from zero to the first number. Draw a second arrow from the first number the length of the absolute value of the second number toward the left if it is positive or toward the right if it is negative.

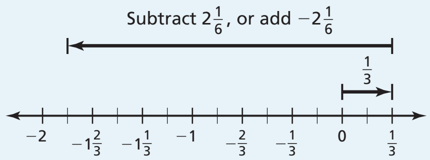
13. 3.3



14. $1\frac{5}{8}$



15. $-1\frac{5}{6}$



Extra Example 2

a. Find $(-8)^2$. 64

b. Find -9^2. -81

c. Find $-3 \cdot (-12) \cdot 4$. 144

d. Find $4(5 - 11) + 2$. -22

Try It

7. 0 **8.** -3

9. 24

Formative Assessment Tip

Always-Sometimes-Never True (AT-ST-NT)
This strategy is useful in assessing whether students overgeneralize or under generalize a particular concept. When answering, a student should be asked to justify his or her answer and other students listening should critique the reasoning.
AT-ST-NT statements help students practice the habit of checking validity when a statement (or conjecture) is made. Are there different cases that need to be checked? Is there a counterexample that would show the conjecture to be false?
To develop these statements for a lesson, consider the common errors or misconceptions that students have relating to the success criteria of the lesson. Allow private think time before students share their thinking with partners or the whole class.

Self-Assessment
for Concepts & Skills

10. a. They have the same sign.
b. They have different signs.

11. -32 **12.** 35

13. 30

14–15. See Additional Answers.

Laurie's Notes

Discuss

- Students should know the meaning of exponents. Write the expression 5^2 on the board and ask students to tell you what it means.
- **Vocabulary Review:** 5 is the *base* and 2 is the *exponent*. The exponent of a power indicates the number of times the base is used as a factor. So, $5^2 = 5 \times 5 = 25$ and it is read as "5 raised to the second power" or "5 squared."
- Review the order of operations and properties of multiplication with students.

EXAMPLE 2

- Part (a) shows how to raise a negative integer to a power.
- **Common Error:** When a negative number is raised to a power, the number must be written within parentheses. In part (b), the expression is read as "the opposite of 2 squared." If you wanted to raise -2 to the second power, it would be written as $(-2)^2$. For part (b), the order of operations says to square the number and then take its opposite.
- **Extension:** "When you raise a negative number to a power is the answer always positive?" No, if the exponent is odd the answer is negative.
- In part (c), make sure students recognize that using the Commutative Property of Multiplication allows them to multiply only positive numbers in the last step.

Indicator 2g.ii - The Teaching Edition encourages teachers to ask probing questions to engage students in constructing arguments and analyzing the arguments of others. The note for Example 2 encourages teachers to ask an extension question about the example to get students to construct arguments about negative numbers raised to a power.

Try It

- Have stud

Self-

- Exer
succ
- Have students work in pairs for Exercises 10–13. Ask volunteers to write their answers on the board.
- Use *Always-Sometimes-Never True* for Exercises 14 and 15. Students should explain their reasoning and give examples.

ELL Support

Proceed as described in Laurie's Notes for the exercises and allow students to work in pairs. Have each pair display their answers for Exercises 11–13 on a whiteboard for your review. Have two pairs form a group to discuss Exercises 10, 14, and 15. Monitor discussions and provide support as needed. Then use *Always-Sometimes-Never True* for Exercises 14 and 15.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Laurie's Notes

EXAMPLE 3

- Point out the steps of problem-solving plan as you solve this problem.
- Ask two different students to read the problem to the class. Use *Popsicle Sticks* to select students interpret the problem.
- Discuss the verbal model. Ask students to explain each part.
- ? Stop at $n \geq 150$ and ask, "Is this the answer to the question?" Remind students to answer the question with a sentence.

Self-Assessment for Problem Solving

- Students may benefit from trying the exercises independently and then working with peers to refine their work. It is important to provide time in class for problem solving, so that students become comfortable with the problem-solving plan.
- Remind students to follow the steps of the problem-solving plan. Students may complain about writing the steps, but it will make solving more complicated problems easier.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Formative Assessment Tip

Sentence Summary

This technique asks students to write a single sentence to describe what they have learned about a topic. You may ask students to summarize new information, make a comparison, or describe a problem and solution. Give students time to reflect before writing. Discourage responses like, "I learned how to subtract." *Sentence Summary* can give you a quick glimpse into each student's level of understanding of the material.

Closure

Sentence Summary: Give each student an index card. Ask students to write about what they have learned about solving two-step inequalities. Allow time for students to reflect and discourage responses like, "I learned how to find the solution." Use these responses to plan your instruction for the next day.

Extra Example 3

A cheerleading squad orders T-shirts that cost $18 each. The price per T-shirt decreases $0.15 for each T-shirt that is ordered. How many T-shirts should the squad order for the price per T-shirt to be no greater than $16.35? $18 - 0.15n \leq 16.35$; at least 11 T-shirts

Self-Assessment for Problem Solving

13. $4x - 3000 \geq 750$; $x \geq 938$ tokens

14. $p \leq \$70$

Learning Target

Write and solve two-step inequalities.

Success Criteria

- Apply properties of inequality to generate equivalent inequalities.
- Solve two-step inequalities using the basic operations.
- Apply two-step inequalities to solve real-life problems.

Laurie's Notes

Scaffolding Instruction

- Students should have a basic understanding of the success criteria from the exploration. Now they will formalize their understanding and use **cross products** to tell whether ratios form a proportion.
- **Emerging:** Students have a general understanding of how equivalent ratios can be used to identify proportional relationships, but they need more practice with the processes. These students will benefit from guided instruction for the examples.
- **Proficient:** Students understand the relationship between equivalent ratios and proportions. They should review the Key Ideas and vocabulary before completing Try It Exercises 5–8. Then have students check their understanding with the Self-Assessment exercises.

Key Idea

- Write the definition for a **proportion** on the board.
- ? Ask, "How can you determine whether two ratios form a proportion?" *Sample answer:* If the ratios are equivalent, they form a proportion.
- **FYI:** Without units associated with the numeric values, students think of proportions as fractions.
- If students are comfortable with writing equivalent fractions and simplifying fractions, they will generally have a good sense about working with proportions.

EXAMPLE 1

- The strategy shown in this example is to write the ratios as fractions in simplest form.
- ? "What is the relationship between $\frac{2}{3}$ and $\frac{3}{2}$?" They are reciprocals.
- After completing part (b), you may want to write the proportion $\frac{10}{40} = \frac{2.5}{10}$ on the board.
- Discuss some alternate strategies for part (b). Students may not realize that they could have divided $\frac{2.5}{10}$ by 2.5 to obtain $\frac{1}{4}$. Point out that they could have multiplied the quantities in the second ratio by 4 to obtain the other ratio, which means the ratios are equivalent.

Try It

- Students should work independently and then check their answers with a neighbor.

Extra Example 1

Tell whether the ratios form a proportion.

a. 10 : 18 and 45 : 81 yes

b. 15 : 12 and 6 : 3 no

Try It

1. yes
2. no
3. no
4. yes

Laurie's Notes

EXAMPLE 3

- Work through the problem as shown. Encourage students to use mental math to find 20% of \$70: 10% of 70 is 7, so 20% of 70 is 2(7), or 14.
- Two steps are used in Method 1: Find 20% of \$70 and then add this amount to the original amount of \$70.
- "Can this problem be solved in one step? Explain." Yes, 120% of \$70 = \$84. "Explain why 120% makes sense." You pay 100% of the store's cost plus an additional 20% markup for a total of 120%.
- Method 2 uses the fact that the selling price is 120% of what the store paid. Make sure students understand this. The ratio table shows the division and multiplication to get to 120% and that this is also the sum of the first two rows, which is comparable to the procedure in Method 1.
- Also show students how to use a proportion, $\frac{a}{70} = \frac{120}{100}$ and a one-step percent equation, $a = 1.2(70)$.
- Discuss the percent bar model in the Check note.
- **Common Error:** Students find the markup (\$14) instead of the selling price (\$84).
- **MP2 Reason Abstractly and Quantitatively:** Now go back and ask students what other ways they can solve the previous examples. Deep understanding results from considering different ways in which to solve problems.

Self-Assessment for Problem Solving

- Encourage students to use a Four Square to complete these exercises. Until students become comfortable with the problem-solving plan, they may only be ready to complete the first square.
- **Think-Pair-Share:** Students should work independently before working with a partner. Then have each pair compare their answers with another pair.

Have students use *Thumbs Up* to indicate their understanding of solving percent problems involving discounts and markups.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

- **Entry Ticket:** "The original price of a book is \$10. The book is on sale for 50% off. If the store is offering an additional 50% discount, will they give you the book for free? Explain." No, the sale price for a book discounted 50% followed by another 50% discount would be 0.5(0.5) = 0.25, or 25% of the original price (\$2.50).

Have extra tickets available for students who forget their *Entry Tickets* the next day. Those students can complete their *Entry Tickets* at the back of the room before going to their seats.

Extra Example 3

A store pays \$15 for a baseball cap. What is the selling price when the markup is 60%? \$24

Self-Assessment for Problem Solving

8. *Sample answer:* Use the \$15 coupon for any item under \$150. Use the 10% coupon for any item above \$150.
9. **a.** \$1000
   b. 5%
   c. \$69.75

Formative Assessment Tip

Entry Ticket
This technique is similar to *Exit Ticket*. You pose a short problem, or ask a question, at the end of class. Students write their responses on tickets as part of their homework. Students give you their *Entry Tickets* as they enter your class the next day. Because they are short responses, you can read them quickly to decide if you need to answer any questions or clear up any misconceptions.

Learning Target

Solve percent problems involving discounts and markups.

Success Criteria

- Use percent models to solve problems involving discounts and markups.
- Write and solve equations to solve problems involving discounts and markups.

Laurie's Notes

Scaffolding Instruction

- As students continue to conduct experiments, they will begin to understand how outcomes are affected by events that are *not* equally likely.
- **Emerging:** After the exploration, students may not be able to make predictions nor draw conclusions. The examples will help them master the success criteria.
- **Proficient:** Students have an understanding of **probability** and possible outcomes. After students review the Key Ideas, they should work through Examples 2 and 3 before completing the Try It and Self-Assessment exercises.

Key Idea

- Discuss the vocabulary words: **experiment**, **outcomes**, **event**, and **favorable outcomes**. You can relate the vocabulary to the exploration and to rolling two number cubes.
- ? "What does it mean to perform an experiment at *random*?" All of the possible outcomes are equally likely.
- Ask students to identify the favorable outcomes for the events of choosing each color of marble. green (2), blue (1), red (1), yellow (1), purple (1)
- Be sure students understand that there can be more than one favorable outcome.
- ? "What are some other examples of experiments and events? What are the favorable outcomes for these events?" *Sample answer:* An experiment is rolling a number cube with the numbers 1–6. An event is rolling a number greater than 4, with favorable outcomes of 5 and 6.

EXAMPLE 1

- In part (a), students may say there are four possible outcomes (1, 2, 3, or 4). Remind them to count each section of the spinner as a possible outcome.
- Work through parts (b) and (c) as shown.
- ? "Would it make a difference if the sections with the 1s were in consecutive locations on the spinner? Explain." No, the answers would be the same. Only the size of the sections matter, not the locations.
- ? "What are the favorable outcomes of spinning a prime number?" 2 and 3
- ? "In how many ways can spinning a number greater than 4 occur?" 0 ways

Try It

- Have students work in pairs to complete the exercise.

Extra Example 1

You roll a number cube.

a. How many possible outcomes are there? 6

b. What are the favorable outcomes of rolling an odd number? 1, 3, and 5

c. In how many ways can rolling a number greater than 4 occur? 2 ways

Try It

1. a. 8

b. A, E, A

c. 5