Journal of Credit Risk **14**(4), 1–35
DOI: 10.21314/JCR.2018.243

Research Paper

A fifty-year retrospective on credit risk models, the Altman Z-score family of models and their applications to financial markets and managerial strategies

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(Received August 8, 2017; revised January 27, 2018; accepted June 8, 2018)

ABSTRACT

Fifty years ago, in 1967, I completed my PhD dissertation, which involved the first multivariate model for predicting the financial health of US manufacturing firms and whether or not they were likely to file for bankruptcy. That work was followed shortly afterward (in 1968) by the publication of the model's specifications. Despite its "old age", the Altman Z-score is still the standard against which most other bankruptcy or default prediction models are measured and is clearly the most used by financial market practitioners and academic scholars for a variety of purposes. The objective of this paper is to reflect upon the evolution of the Altman family of bankruptcy prediction models as well as their extensions and multiple applications in financial markets and managerial decision making.

Keywords: Altman Z-score; bankruptcy prediction; credit risk; probability of default (PD); equity investment strategy; bond strategies.

Print ISSN 1744-6619 | Online ISSN 1755-9723

1 THE EVOLUTION OF CORPORATE CREDIT-SCORING SYSTEMS

Credit scoring systems for identifying the determinants of a firm's repayment likelihood probably go back to the days of the Crusades, when travelers needed "loans" to finance their travels. They were certainly used much later in the United States as companies and entrepreneurs helped to grow the economy, especially in its westward expansion. Primitive financial information was usually evaluated by lending institutions in the 1800s, with the primary types of information required being subjective or qualitative in nature, revolving around ownership and management variables as well as collateral (see Box 1). It was not until the early 1900s that rating agencies and some more financially oriented corporate entities (eg, the DuPont system of corporate ROE growth) introduced univariate accounting measures and industry peer group comparisons with rating designations (see Figure 1). The key aspect of these "revolutionary" techniques was that they enabled the analyst to compare an individual corporate entity's financial performance metrics to a reference database of time series (same entity) and cross-section (industry) data. Then, and even more so today, data and databases were the key elements of meaningful diagnostics. There is no doubt that in the credit-scoring field, data is "king" and models for capturing the probability of default (PD) ultimately succeed, or not, based on whether they can be applied to databases of various sizes and relevance.

The original Altman Z-score model (Altman 1968) was based on a sample of sixty-six manufacturing companies in two groups, bankrupt and nonbankrupt firms, and a holdout sample of fifty companies. In those "primitive" days, there were no electronic databases and the researcher/analyst had to construct their own database from primary (annual report) or secondary (Moody's and Standard & Poor's (S&P) industrial manuals and reports) sources. To this day, instructors and researchers oftentimes ask me for my original sixty-six-firm database, mainly for instructional or reference exercises. It is not unheard of today for researchers to have access to databases of thousands, even millions, of firms (especially in countries where all firms must file their financial statements in a public database, eg, in the United Kingdom). To illustrate the importance of databases, Moody's purchased extensive data on 200 million firms and customer access from Bureau van Dijk Electronic Publishing (EQT) for US$3.3 billion in 2017, while S&P purchased SNL Financial's extensive database, management structure and customer book for US$2.2 billion in 2015. As indicated in Figure 1, the three major rating agencies established a hierarchy of creditworthiness that was descriptive, but not quantified, in its depiction of the likelihood of default. The determination of these ratings was based on a combination of (1) financial statement ratio analytics, usually on a univariate, one-ratio-at-a-time basis; (2) industry health discussions; and (3) qualitative factors evaluating the firm's management plans and capabilities, strategic directions and other, perhaps

BOX 1 Corporate scoring systems over time. [Box continues on next page.]

- Qualitative (subjective) – 1800s
- Univariate (accounting/market measures):
  - rating agency (eg, Moody's (1909), Standard & Poor's (1916)) and corporate (eg, DuPont) systems (early 1900s)
- Multivariate (accounting with market measures) – late 1960s (Z-score) to the present:
  - discriminant, logit, probit models (linear, quadratic)
  - nonlinear and "black box" models (eg, recursive partitioning (Frydman *et al* 1985), neural networks (1990s))
- Discriminant and logit models used for:
  - consumer models – Fair–Isaac (FICO scores)
  - manufacturing public (US) firms (1968) – Z-scores
  - extensions and innovations for specific industries and countries (1970s to the present)
  - ZETA score – industrials (1977)
  - private firm models (eg, Z'-score (1983), Z''-score (1995b))
  - EM score – emerging markets (1995b)
  - bank specialized systems (1990s), Basel II impetus
  - small and medium-sized enterprises, eg, Edmister (1972), Altman and Sabato (2007) and Wiserfunding Ltd (2016; www.wiserfunding.com)
- Option/contingent claims models (1970s to the present):
  - risk of ruin (Wilcox 1971)
  - KMV's credit monitor system model (1993) – extensions of Merton (1974) structural framework
- Artificial intelligence systems (1990s to the present):
  - expert systems
  - neural networks
  - machine learning

"inside", information gleaned from interviews with senior management and experience of the team that was assigned to the rating decision. To this day, the decision process of rating agencies remains essentially the same, with the ultimate rating decision being made based on the firm's likelihood of default or, in some cases, the

BOX 1 Continued.

- Blended ratio/market value models:
  - Altman Z-score (fundamental ratios and market values) – 1968
  - bond score (Credit Sights, 2000; RiskCalc Moody's, 2000)
  - hazard (Shumway 2001)
  - Kamakura's reduced form, term structure model (2002)
  - Z-metrics (Altman *et al* 1995b, RiskMetrics, 2010)
- Reintroduction of qualitative factors and real-time data (FinTech)
  - stand-alone metrics, eg, invoices, payment history
  - multiple factors – data mining (big data payments, governance, time spent on individual firm reports (eg, CreditRiskMonitor's revised FRISK scores, 2017), etc)
  - enhanced blended models (2000s)

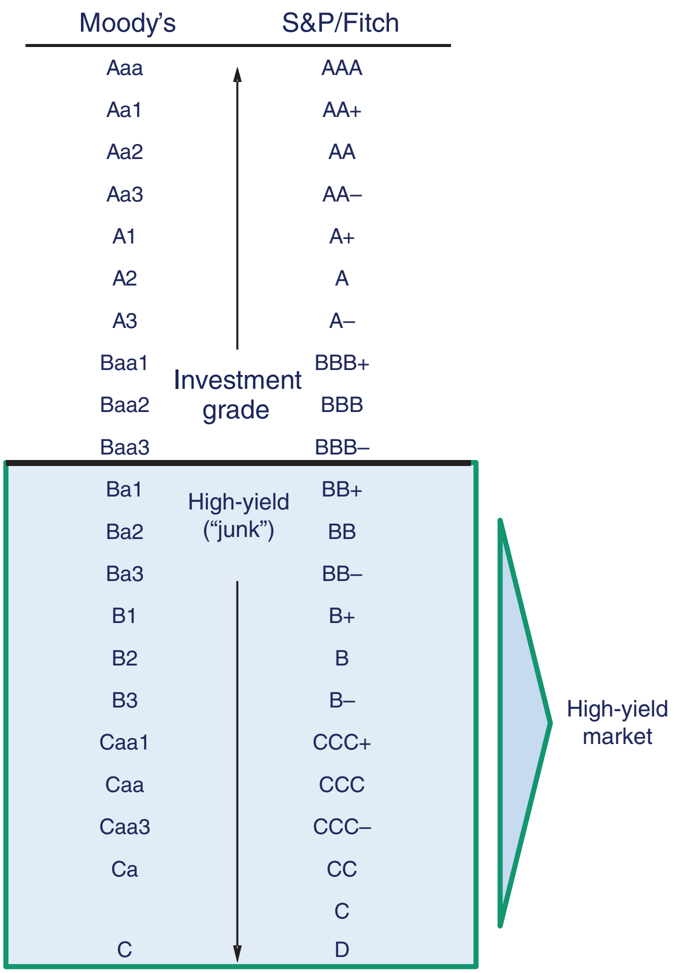
loss given default (LGD) based on expected recovery. These inputs were analyzed on a through-the-business-cycle basis, often based on a "stressed" historical analysis. While the stressed scenario basis for evaluating a firm's solvency is still an important input, rating agencies no longer embrace the business cycle as the key determinant of whether to change a rating.

What can we say about this process and its evaluative results? Here are our opinions.

(1) Since the process has been standardized and carried out fairly consistently over time, it can provide important reference points for the market and is well understood as an "international language of credit". This makes the database of assigned original ratings and rating changes an incredibly important source of data for both researchers and practitioners on an ongoing basis.

(2) Original rating assignments are done carefully with adequate resources and a strong desire to assess the repayment potential of the firm on specific issues such as bonds, loans and commercial paper (the so-called plain vanilla issuances of firms) very accurately. The rating assignments do not provide specific quantitative estimates of the PD, but they do provide important benchmarks for not only comparing the actual incidence of default on millions of bond issues for long periods of time but also assessing the bond-rating equivalent (BRE) of nonrated firms and securities in order to eventually provide a PD of corporate debt issuances. Hence, we will show this capability in our

FIGURE 1 Major agencies' bond rating categories.



own "mortality rate" determination – based on our original work (Altman 1989), which has since been updated annually – as well as similar analytics introduced by Moody's and S&P in the early 1990s: the so-called cumulative default rates (CDRs) and rating transitions tables (see, for example, S&P Global 2017).

(3) We have found that the track record of excellent original rating assignments by rating agencies is not matched by their performance regarding the timeliness

of rating changes, ie, transitions as the firm's financial and business performance evolves. Studies such as Altman and Rijken (2004, 2006) clearly show that agency ratings are generally slower to react to changes, primarily deteriorations in performance, than established models based on point-in-time estimations, eg, Z-score-type models or KMV structural estimates. Indeed, rating agencies openly admit that stability of ratings is a very important attribute of their systems and volatile changes are to be avoided. So, it is no surprise that when rating changes do occur, they are slower than those an objective, unemotional model would produce, and these changes, principally downgrades, are typically smaller (ie, have fewer notches) than those a model would have produced. The latter implies that if another rating change were to follow an initial downgrade, it is highly likely that this second change would be in the same direction as the first, ie, there is strong autocorrelation between rating downgrades (see Altman and Kao 1992). We have not encountered much, if any, denial from rating agencies on this observation. After all, the agencies' clients (firms issuing debt) are more comfortable with a system that provides more stable ratings than one that changes, especially negatively, frequently. In addition, those using the service, such as pensions and mutual funds, prefer stability to volatile ratings.

(4) These observations illustrate the ongoing discussions and heated arguments regarding the objectivity and potential bias of ratings based on agencies' business models, namely, that the entity being rated (firms) also pays for the rating. Critics of rating agencies point to this potential conflict of interest and call for other structures, such as the investor-pay model, or for government agencies to provide ratings. These ideas have been floated but do not seem to have resonated well with the protagonist of the rating industry, ie, the users of ratings, primarily investors. Moreover, investors, in some cases, prefer the stability of ratings over short-term volatility, especially if the changes involve a switch from investment grade to noninvestment grade, or vice versa. Hence, despite efforts by regulators to encourage alternative systems for estimating PDs, such as internally generated or vendor models, ratings from the major rating agencies continue to be an important source of third-party assessment for the market. We feel that models such as the Altman Z-score family can still play a very important role in the investment process, despite the continued prominence of agency ratings.

2 MULTIVARIATE ACCOUNTING/MARKET MEASURES

Continuing the evolutionary history of credit-scoring systems beyond univariate systems (such as those followed by rating agencies and prominent scholarly research studies by numerous academics, such as Beaver (1966)), we move to the first multivariate study to attack the bankruptcy prediction subjects: my initial Z-score model (1968). Utilizing one of the first discriminant analysis models applied to the economic–financial social sciences, I (Altman 1968) combined traditional financial statement variables with new and more powerful statistical techniques and, aided by early editions of mainframe computers, constructed the original Z-score model. Consisting of five financial indicators, four of which required only one year of financial statements and one that needed equity market values, the original model (Table 1) demonstrated outstanding original and holdout sample accuracies of type I (predicting bankruptcy) and type II (predicting nonbankruptcy) based on a derived cutoff-score approach (discussed later) and using financial data from one annual statement prior to bankruptcy. The original sample of firms utilized only manufacturing companies that filed for bankruptcy-reorganization under the "old" system, called Chapter X or Chapter XI (now combined under Chapter 11). All firms were publicly held and, given the economy in the United States prior to 1966, all had assets under US$25 million. The sample sizes were small, with only thirty-three firms in each grouping; this is remarkable, as the model is still being used extensively, fifty years after its introduction, on firms of all sizes, including those with billions of dollars in assets. Deakin (1972) compared Beaver's univariate variables with Altman's multivariate structure.

The original Z-score model was linear and did not require more than one set of financial statements. Subsequent to its introduction, similar models utilizing linear and nonlinear variable structures as well as different classification techniques, such as quadratic, logit, probit and hazard model structures, were introduced to attempt not only to classify a firm as bankrupt or not, but also to express the outcome in terms of the PD based on the characteristics of the sample of firms used in the model's development. An alternative approach for developing PDs, based on the Altman BRE method, combined with empirically derived estimates of default incidence for long horizons (eg, 1–10 years) will be discussed shortly.

These discriminant, or logit, models were applied to consumer credit applications (eg, Fair–Isaac (FICO) scores); to nonmanufacturers (eg, ZETA scores (Altman *et al* 1977)); to private and publicly owned firms, in many other countries (built over several decades and continuing to be derived even in current years); to emerging markets (see, for example, Altman *et al* 1995b); to the internal rating systems (IRBs) of banks (starting in the mid-1990s and especially since Basel II was first introduced for discussion in 1999); and to various industries and sizes of firms, including mod-

TABLE 1 Original Z-score component definitions and weightings.

Variable	Definition	Weighting factor
X_1	$\frac{\text{Working capital}}{\text{Total assets}}$	1.2
X_2	$\frac{\text{Retained earnings}}{\text{Total assets}}$	1.4
X_3	$\frac{\text{EBIT}}{\text{Total assets}}$	3.3
X_4	$\frac{\text{Market value of equity}}{\text{Book value of total liabilities}}$	0.6
X_5	$\frac{\text{Sales}}{\text{Total assets}}$	1.0

"EBIT" is earnings before interest and taxes.

els specifically derived for SMEs (see, for example, Edmister (1972), Altman and Sabato (2007), Altman *et al* (2010b) and, most recently for mini-bond issuers in Italy, Altman *et al* (2016)).

Many other exotic statistical and mathematical techniques have been applied to the bankruptcy/default prediction field, including expert systems, neural networks, genetic algorithms, recursive partitioning, and the latest attempts using sophisticated machine-learning methods, motivated by the existence of massive databases and the introduction of nonfinancial data. While these techniques usually surpass the now "primitive" discriminant financial statement-based models in terms of prediction-accuracy tests on original and sometimes holdout samples, the more complex the algorithm and specialized the data sources, the less likely it is the model will be understood by other researchers and practitioners in its real-world applications.

One class of model that boasts both scholarly and practitioner acceptance and usage is the so-called structural models, built after the introduction of Merton's contingent-claims approach (Merton 1974) for valuing risky debt and the later commercialization of this model by KMV's (1993) credit monitor system (KMV Corporation 2000; McQuown 1993). The latter was and still is (marketed by Moody's since 2001) based on a very large sample of defaults derived from companies on a global basis. The result is a PD estimate derived from a distance-to-default calculation, relying primarily on firm market values, historical market volatility measures and levels of debt. Academic researchers and several consultants have replicated the Merton-structural approach and have oftentimes compared it with their own models as well as more traditional models, such as Z-scores and Kamakura's reduced-form approach (see Chava and Jarrow 2004; van Deventer 2012), with results that do not always point to which approach was superior (see, for example, Das *et al* 2009; Bharath and Shumway 2008; Campbell *et al* 2008).

The most recent attempts at building both accurate and practically acceptable models have utilized what we call a blended ratio/market value/macro-variable approach, with some attempts to also include nonfinancial variables, where data exists. These blended models, eg, Z-metrics (Altman *et al* 2010), introduced by Riskmetrics, are probably those that consultants and many financial lenders are either considering or utilizing today, at least compared with more traditionally derived models, sometimes with judgmental adjustments by lending officers. Finally, the latest financial technology (FinTech) innovations explore the use of big data and nontraditional metrics, such as invoice analysis, payable history and governance attributes; "clicks" on negative information events and data (see, for example, CreditRiskMonitor's revised FRISK[T] scoring system (2017)); and social media inputs, in order to capture, on a real-time basis, changes in the credit quality of firms and individuals.

2.1 Machine-learning methods

As for machine-learning and big data techniques, I remain somewhat skeptical as to whether practitioners will accept black-box methods for assessing the credit risk of counterparties. Yes, it is undeniable that the current surge in applications of such techniques has captured the interest of many academics and several start-ups in the FinTech space. Indeed, I collaborated with some colleagues (Barboza *et al* 2017) using several machine-learning models (eg, support vector machines (SVMs), boosting, random forest, etc) to predict bankruptcy from one year prior to the event and compared the results to discriminant analysis, logistical regression and neural network methods. Using data from 1985 to 2013, we found a substantial improvement in prediction accuracy (of about 10%) using machine-learning techniques, especially when, in addition to the five Z-score variables, six indicators were included. Our results add one more study to the growing debate of the last few years (2014–17) about the superiority of SVMs versus other machine-learning methods. Almost all of the machine-learning credit models have been published in expert systems and computational journals, with the most prominent being found in *Expert Systems with Applications* (see Barboza *et al* (2017) in our reference list).

3 FROM A SCORING MODEL TO DEFAULT PREDICTION

The construction of a credit-scoring model is relatively straightforward with an adequate and appropriate database of default and nondefault securities, or firms, and accurate predictive variables. In the case of our first model, the Z-score method (named in association with statistical Z-measures and also chosen because it is the

TABLE 2 Classification and prediction accuracy: Z-score (Altman 1968) failure model*.

Year prior to failure	Original sample (33)	Holdout sample (25)	1969–75 predictive sample (86)	1976–95 predictive sample (110)	1997–99 predictive sample (120)
1	94% (88%)	96% (72%)	86% (75%)	85% (78%)	94% (84%)
2	72%	80%	68%	75%	74%
3	48%	—	—	—	—
4	29%	—	—	—	—
5	36%	—	—	—	—

*Using 2.67 as cutoff (1.81 cutoff accuracy in parenthesis).

last letter in the English alphabet), the classification as to whether a corporate entity was likely to go bankrupt or not was determined based on cutoff scores between a "safe" zone and a "distress" zone, with an intermediate "gray" zone. The zones of discrimination from the original Z-score model (Altman 1968) were as follows:

$$Z > 2.99, \quad \text{“safe” zone,}$$
$$1.81 < Z < 2.99, \quad \text{“gray” zone,}$$
$$Z < 1.81, \quad \text{“distress” zone.}$$

These zones were selected based solely on the results of the original, admittedly smallish, samples of thirty-three firms in each of the two groupings (bankrupt and nonbankrupt) from manufacturing firms and their financial statement and equity market values from the 1960s. Any firm whose Z-score was below 1.8 (distressed zone) was classified as "bankrupt" and did, in fact, go bankrupt within one year (100% accuracy); firms whose scores were greater than 2.99 did not go bankrupt (also 100% accuracy), at least until the end of the study period in 1966. There were a few errors in classification for firms with scores between 1.81 and 2.99 (gray zone: three errors out of sixty-six, see Table 2). Keep in mind that these cutoff scores were based solely on the original sample of firms.

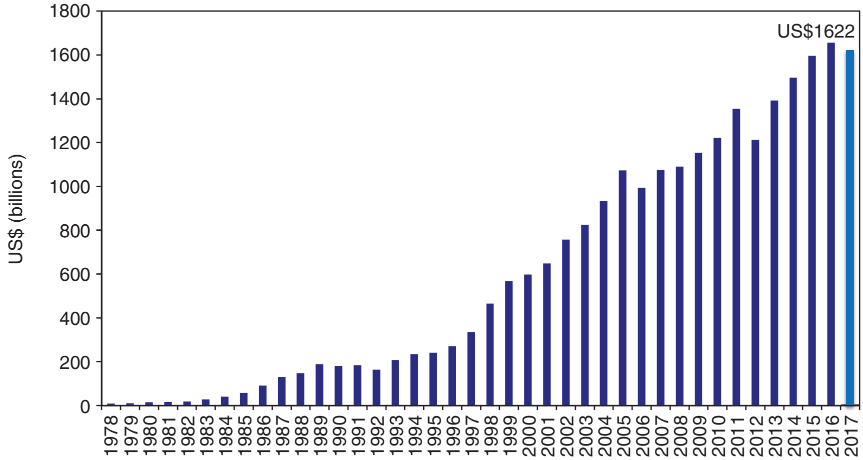
Because the zones were clear, unambiguous and consistent in their subsequent predictions of greater than 85% accuracy, based on data from one year prior to bankruptcy (type I; see Table 2), these designations remain to this day accepted by and useful to market practitioners. While flattering to this writer, this is unfortunate, as it is obvious that the dynamics and trends in creditworthiness have changed significantly over the last fifty years. In addition, classifications of "bankrupt" or "nonbankrupt" are no longer sufficient for many applications of the Z-score model. Indeed, there is very little difference between a firm whose score is 1.81 and one whose score is 1.79; yet, the zones are different. In addition, the "holy-grail" of credit assessment – namely, the PD, and when the default associated with the probability is to take place – is not specified clearly by a certain credit score. We now examine how credit dynamics have changed over our relevant time periods and how we have moved on to precise PD and timing of default estimates.

4 TIME SERIES IMPACT ON CORPORATE Z-SCORES

When we built the original Z-score model in the mid-1960s, financial credit markets were much simpler – some might say primitive – compared with today's highly complex, multistructured environment. Innovations such as high-yield bonds; leverage loans; structured financial products; credit derivatives such as credit default swaps (CDSs); and shadow banking loans were nonexistent then, and riskier companies had few financing alternatives outside of traditional bank loans and trade debt. For example, we see from Figure 2 that the North American High Yield Bond market was not "discovered" until the late 1970s, when the only participants were the so-called fallen angel companies that originally raised debt when they were investment grade (IG). When this paper was written in 2017, the size of the high-yield "junk-bond" market had grown from about US$10 billion of fallen angels in 1978 to about $1.7 billion, with over 85% of this market consisting of "original issue" high-yield issues. In addition, the pace of newly issued high-yield bonds and leverage loans has accelerated since the great financial crisis (GFC) of 2008–9, with more than US$200 billion worth of new bond issues each year since 2010; this has been fueled by a benign credit cycle, which was in its eighth year as of 2017. Not to be outdone, loans to noninvestment grade companies have again become numerous and available at attractive rates as interest rates have fallen, in general, and banks, despite regulatory oversight guidelines (eg, debt/earnings before interest, tax, depreciation and amortization (EBITDA) ratios not to exceed a certain level, say, 6), have competed with public markets in the United States and Europe. These leveraged loans' new issues of several hundred billion US dollars per year have swelled corporate debt ratios to an unprecedented level, as firms have exploited the easy money, low interest rate environment and lenders have sought yield, even for the most risky corporate entities. For example, CCC new issues have averaged 15% of all high-yield bond new issues each year over the period 2010–18 (Q2).

Other factors that have reduced the average creditworthiness of companies as the Z-score model has matured in the 50-year period since its inception are global competitive factors, the enormous power of market dominating firms in certain industries, such as Walmart and Amazon in the retail space, and the amazing susceptibility of larger companies to financial distress and bankruptcy. Indeed, when we built the Z-score model in the 1960s, the largest bankrupt firm in our sample had total assets of less than $25 million (about $125 million, inflation adjusted) compared to an envi-

FIGURE 2 Size of the US high-yield bond market: 1978–2017 (mid-year; US$ billions).



Source: NYU Salomon Center estimates using Credit Suisse, S&P and Chili data.

TABLE 3 Median Z-score by S&P bond rating for US manufacturing firms: 1992–2017.

Rating	2017 (no.)	2013 (no.)	2004–10	1996–2001	1992–5
AAA/AA	4.30 (14)	4.13 (15)	4.18	5.20	5.80*
A	4.01 (47)	4.00 (64)	3.71	4.22	3.87
BBB	3.17 (120)	3.01 (131)	3.26	3.74	2.75
BB	2.48 (136)	2.69 (119)	2.48	2.81	2.25
B	1.65 (79)	1.66 (80)	1.74	1.80	1.87
CCC/CC	0.90 (6)	0.33 (3)	0.46	0.33	0.40
D	−0.10 (9)[a]	0.01 (33)[b]	−0.04	−0.20	0.05

Source: Compustat database, mainly S&P 500 firms, compilation by E. I. Altman, NYU Salomon Center, Stern School of Business. *AAA only. [a]From 01/2014 to 11/2017. [b]From 01/2011 to 12/2013.

ronment with a median annual number of seventeen firms each year since 1998 with liabilities (and assets) of more than $1 billion.

To demonstrate the implied deterioration in corporate creditworthiness over the last fifty years, one can observe our median Z-score statistics using the S&P credit rating for various sample years shown in Table 3. First, the number of AAA ratings dwindled to just two in 2017 (Microsoft and Johnson & Johnson) from more than fifteen twenty years ago and ninety-eight in the early 1990s. Hence, we now combine AAA- and AA-rated companies to analyze average Z-scores, and that median decreased from a high of 5.20 in the 1996–2001 period to 4.13 in 2013. More important is the steady deterioration of median Z-scores for single-B companies from 1.87 in 1992–5 to 1.70 in 2017. Recall that a score of below 1.8 in 1966 classified a firm as in the distress zone and a bankruptcy threat. However, in the last fifteen or so years, the dominant and largest percentage of issuance in the high-yield market was for single-Bs, and surely all single-Bs do not default! True, a median single-B has a distribution in which 50% of its issues are higher than 1.66, but the probability that all Bs default within five years of issuance is, approximately, "only" 28% (see our mortality rate discussion below). Finally, the median D (default-rated company) had a Z-score of -0.10 in 2017, while the median Z-score in 1966 for bankruptcy entities was $+0.65$ (see Altman 1968; Altman and Hotchkiss 2006). In all time periods of late, the median D firm's Z-score was zero or below (Table 3). Hence, we suggest that a score below zero is consistent with a defaulted company. The cutoff of 1.8, based on our original sample, will place an increasing number, perhaps as much as 25% of all firms, in the old distress zone. Since only a very small percentage of all firms fail each year and an average of about 3.5% of high-yield bond companies default each year, based on data over the last (almost) fifty years (see our default rate calculations in Altman and Kuehne (2017)), the so-called type II error (predicting default when the firm does not) has increased from about 5% in our original analysis to possibly 25–30% in recent periods. Hence, we do *not* recommend that users of our Z-score model make their assessments of a firm's default likelihood based on a cutoff score of 1.8. Instead, we recommend using BREs based on the most recent median Z-scores by bond rating, such as the data listed in Table 3. These BREs can then be converted into more granular PD estimates, as we now discuss.

4.1 PD estimation methods

Box 2 lists two methods that we have used over the years to estimate the PD and LGD of a firm's bond issue at any point in time. The starting point in both methods is a well-constructed and, if possible, intuitively understandable credit-scoring model. For example, in method (1), the Z-score on a new or existing debt issuer is assigned a BRE on a representative sample of bond issues for each of the major rating categories (see Table 3) or, if available, more granular ratings with (+) or (−) "notches" (S&P/Fitch), or 1, 2, 3 (Moody's). See Table 6, later in the paper, for the more granular categorization of another of the Altman Z-score models: Z''-scores.

In addition to the matching of Z-scores by rating category, we can also assess the PD of an issue for various periods of time in the future. The more traditional time-dependent method is called the CDR. Such rates are provided by all of the rating agencies and by several of the investment banks, who provide continuous research on defaults, particularly for the speculative grade or high-yield ("junk bond") mar-

BOX 2 Estimating PD and probability of LGD.

Method (1)

- Credit scores on new or existing debt.
- BREs on new issues (mortality) or existing issues (rating agencies' CDRs).
- Utilizing mortality CDRs to estimate marginal and cumulative defaults.
- Estimating default recoveries and probability of loss.

Or...

Method (2)

- Credit scores on new or existing debt.
- Direct estimation of the PD based on logistic regressions.
- Based on PDs, assign a rating.

ket. This compilation is an empirically derived PD estimate of bonds with a certain rating, eg, "B", at a point in time, and then the default incidence is observed $1, 2, \dots, 10$ years after that point in time. The estimate is for all B-rated bonds, regardless of the age of the bond when it is first tracked. In my opinion, this PD estimate is more appropriate for an existing bond issuer's debt than for bonds when they are first issued. Almost all of the rating agencies, with the exception of Fitch Inc, calculate CDRs based on the number of issuers that default over time compared with the number of issuers in possession of a certain rating at the starting point (regardless of the different ages of the bonds in the basket of, say, B-rated bonds). Therefore, on average, an S&P B-rated bond had about a 5% incidence of default within one year based on a sample of bonds from 1980 to 2016 (see S&P Global 2017).

Before the rating agencies first compiled their CDRs, Altman (1989) created the mortality rate approach for estimating PDs for bonds of all ratings, specifically newly issued bonds, based on the dollar amounts of new issues by bond rating, rather than by issuer. These mortality estimates are based on insurance actuarial techniques for calculating the marginal and CDR, as shown in Box 3. I feel, as with human mortality, that there are certain characteristics of bonds, or loans, at birth that are critical in determining the likelihood of default up to/over ten years after issuance (the usual maturity of newly issued bonds). In addition, those characteristics can be summarized into an issue's (but not an issuer's) bond rating at birth. Implicit in these PD estimates is the aging effect of a bond issue, whereby the mortality rate of the first year after issuance is relatively low compared with that of the second year; similarly, the marginal rate of the second year is usually lower than that of the third, as shown in Table 4. Note that the mortality rates in Table 4 are based on the incidence of

BOX 3 Marginal and cumulative mortality rate actuarial approach.

$$\text{MMR}_{(r,t)} = \frac{\text{total value of defaulting debt from rating } r \text{ in year } t}{\text{total value of the population at the start of year } t},$$

where MMR is the marginal mortality rate.

One can measure the cumulative mortality rate (CMR) over a specific time period ($1, 2, \ldots, T$ years) by subtracting the product of the surviving populations of each of the previous years from one (1.0), that is,

$$\text{CMR}_{(r,t)} = 1 - \Pi\text{SR}_{(r,t)}, \quad t = 1, \ldots, N,\ r = \text{AAA}, \ldots, \text{CCC}.$$

Here, $\text{CMR}_{(r,t)}$ is the cumulative mortality rate of r in t, and $\text{SR}_{(r,t)}$ is the survival rate in (r, t), $1 - \text{MMR}_{(r,t)}$.

TABLE 4 Mortality rates by original rating: all rated corporate bonds*, 1971–2016 (all values are percentages).

		Years after issuance									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
AAA	Marginal	0.00	0.00	0.00	0.00	0.01	0.02	0.01	0.00	0.00	0.00
	Cumulative	0.00	0.00	0.00	0.00	0.01	0.03	0.04	0.04	0.04	0.04
AA	Marginal	0.00	0.00	0.20	0.06	0.02	0.01	0.01	0.01	0.02	0.01
	Cumulative	0.00	0.00	0.20	0.26	0.28	0.29	0.30	0.31	0.33	0.34
A	Marginal	0.01	0.03	0.11	0.12	0.09	0.05	0.02	0.24	0.07	0.04
	Cumulative	0.01	0.04	0.15	0.27	0.36	0.41	0.43	0.67	0.74	0.78
BBB	Marginal	0.32	2.34	1.24	0.98	0.49	0.22	0.25	0.16	0.17	0.33
	Cumulative	0.32	2.65	3.86	4.80	5.27	5.48	5.71	5.86	6.02	6.33
BB	Marginal	0.92	2.04	3.85	1.95	2.42	1.56	1.44	1.10	1.41	3.11
	Cumulative	0.92	2.94	6.68	8.50	10.71	12.11	13.37	14.32	15.53	18.16
B	Marginal	2.86	7.67	7.78	7.75	5.74	4.46	3.60	2.05	1.73	0.75
	Cumulative	2.86	10.31	17.29	23.70	28.08	31.29	33.76	35.12	36.24	36.72
CCC	Marginal	8.11	12.40	17.75	16.25	4.90	11.62	5.40	4.75	0.64	4.26
	Cumulative	8.11	19.50	33.79	44.55	47.27	53.40	55.91	58.01	58.28	60.05

Source: S&P (New York) and author's compilation. *Rated by S&P at issuance. Based on 3280 defaulted issues.

default for a forty-six-year period, 1971–2016. For example, the marginal default (or mortality) rate of a BB-rated issue for years 1, 2 and 3 after issuance is 0.92%, 2.04% and 3.85%, respectively. After three years, the marginal rates seem to flatten out at between 1.5% and 2.5% per year.

Method (1)'s PD estimate is derived from Box 3's equations. When these have

TABLE 5 Mortality losses by original rating: all rated corporate bonds*, 1971–2016 (all values are percentages).

		Years after issuance									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
AAA	Marginal	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00
	Cumulative	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.03	0.03
AA	Marginal	0.00	0.00	0.03	0.02	0.01	0.01	0.00	0.01	0.01	0.01
	Cumulative	0.00	0.00	0.03	0.05	0.06	0.07	0.07	0.08	0.09	0.10
A	Marginal	0.00	0.01	0.04	0.05	0.05	0.04	0.02	0.02	0.05	0.03
	Cumulative	0.00	0.01	0.05	0.10	0.15	0.19	0.21	0.23	0.28	0.31
BBB	Marginal	0.23	1.53	0.70	0.58	0.26	0.16	0.10	0.09	0.10	0.18
	Cumulative	0.23	1.76	2.44	3.01	3.26	3.42	3.51	3.60	3.70	3.87
BB	Marginal	0.55	1.18	2.30	1.11	1.38	0.74	0.78	0.48	0.73	1.09
	Cumulative	0.55	1.72	3.98	5.05	6.36	7.05	7.78	8.22	8.89	9.88
B	Marginal	1.92	5.38	5.32	5.20	3.79	2.45	2.34	1.13	0.91	0.53
	Cumulative	1.92	7.20	12.13	16.70	19.86	21.82	23.65	24.52	25.20	25.60
CCC	Marginal	5.37	8.68	12.49	11.45	3.42	8.61	2.32	3.34	0.40	2.72
	Cumulative	5.37	13.58	24.38	33.04	35.33	40.89	42.27	44.19	44.42	45.93

Source: S&P (New York) and author's compilation. *Rated by S&P at issuance. Based on 2714 issues.

been adjusted for recoveries on the defaulted issue, we can derive estimates for the LGD in Table 5. This critical LGD estimate can be utilized to estimate expected losses in a bank's Basel II or III capital requirements, or for an investor's expected loss on a portfolio of bonds categorized by bond rating. (See our discussion of lender applications later in Table 8). The earliest measures of LGD that I am aware of are from Altman (1977), Altman *et al* (1977) and Altman (1980). Later, it was found that the key variable in estimating LGD is the concurrent PD (Altman *et al* 2005).

Method (2) utilizes a different approach to estimate PDs. Instead of using empirical estimates of defaults by bond rating, companies are analyzed with a logistic regression methodology, whereby the company is assigned a "0" or "1" dependent variable based on whether it has defaulted or not at a specific point in time. Then, a number of independent explanatory variables are analyzed in the regression format to arrive at a PD estimate of between 0 and 1. The resulting PDs are then assigned a rating equivalent based on, for example, the percentage of bond issues that are AAA, AA, A, ..., CCC in the real world. This logistic structure is used widely in the academic literature and has been a standard technique ever since the early work of Ohlson (1980). We (see, for example, Altman and Rijken (2010) on Z-metrics) have also used it for our hybrid model estimations.

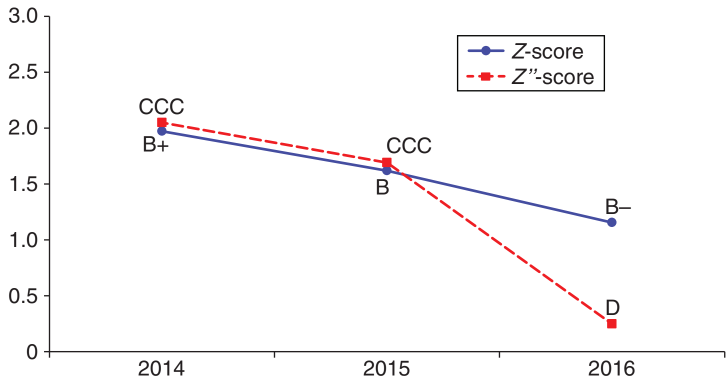
So, which is the superior technique for estimating PD: method (1) or method (2)? I favor the BRE approach for newly issued debt (the mortality rate approach), but for existing issues the CDR method seems to be more appropriate. The reasons are as follows. The mapping of PDs to BREs using mortality rates, or CDRs, is based on over one million issues and about 3500 defaults over the last forty-five years. Logistical regression models' PDs are solely a function of the sample characteristics used to build the model, and the results are based on the logistic structure, which may not be representative of large sample properties. The beauty of logistical regression estimates, however, is that the analyst can access PDs directly from the results and avoid the mapping of scores as an intermediate step. Tests of type I and type II accuracies are available for both methods, along with statistical area under the curve (AUC) accuracy measures on both original and holdout samples. The latter is very important in helping to validate the empirical results from samples over time and from different industrial groups. From my experience, both methods have yielded very impressive type I accuracies in numerous empirical tests.

5 Z-SCORE MODEL FOR INDUSTRIALS AND PRIVATE FIRMS

As noted earlier, the original 1968 Z-score model was based on a sample of manufacturing, publicly held firms, whose assets and liability size amounted to no greater than US$25 million. The fact that this model has retained its high type I accuracy on subsequent samples of manufacturing firms (Table 2) and is still used extensively by analysts and scholars, even for nonmanufacturers, is quite surprising given that it was developed fifty years ago. It is evident, however, that nonmanufacturing firms, such as retailers and service firms, have very different asset and liability structures as well as income statement relationships with asset levels, eg, the sales/total assets ratio, which is considerably greater on average for retail companies than for manufacturers, perhaps even twice as high. And, given the 1.0 weighting for the variable (X_5) in the Z-model (see Table 1), most retail companies have a higher Z-score than manufacturers. Even the beleaguered Sears, Roebuck and Company's latest Z-score (see Figure 3) was 1.3 in 2016: that is equivalent to a BRE of B−, or a D rating if we are using the Z''-score (discussed next). The latter model does not contain the sales/ total asset ratio and was developed for a broad cross-section of industrial sector firms as well as firms outside of the United States (see Altman *et al* 1995b).

To adjust for any industrial sector impact, we have built second-generation models for a more diverse industrial grouping, eg, the ZETA model (Altman *et al* 1977), and for diverse firms in emerging markets (Altman *et al* 1995b). Additional Altman Z-score models have been developed over the last fifty years for various organizational structured firms, eg, private firms (Z'), developed at the same time as the original Z-model (1968); textile firms in France (Altman *et al* 1974); industrials in

FIGURE 3 Z- and Z''-score models applied to Sears, Roebuck and Company: BREs and scores from 2014 to 2016.



Source: S&P Capital IQ and NYU Salomon Center calculations.

the United States (ZETA, Altman *et al* 1977), Brazil (Altman *et al* 1979), Canada (Altman and Lavallee 1981), Australia (Altman and Izan 1982), China (Altman *et al* 2010a) and South Korea (Altman *et al* 1995b); and non-US emerging market (Z'') firms (Altman *et al* 1995b). SME models have been produced for the United States (Altman and Sabato 2007) and the United Kingdom (Altman *et al* 2010b), for Italian SMEs and minibonds (Altman *et al* 2016) and for sovereign default risk assessment (Altman and Rijken 2011).

6 PRIVATE FIRM MODELS

It has been most convenient to build credit-scoring models for publicly owned, listed companies in the United States and abroad due to data availability. Models for private firms can be built, indirectly, by using only those variables related to private firms but based on publicly owned firm data, or by accessing databases that are populated by both publicly owned and private companies. The latter source of data is especially available in several European countries via tax reporting and government credit bureau sources (eg, the United Kingdom) as well as firms' private databases, eg, from Bureau van Dijk (now owned by Moody's).

I used the indirect method in the Z'-score models, which is discussed in Altman (1983) and shown in Box 4. The only difference between this and the original Z-score model is the substitution of the book value of equity for the market value in X_4. Note that all of the coefficients are now different, but only slightly so, and the zones (safe, gray and distress) are slightly different as well. There is some loss in accuracy due to this model adjustment, but, over the years, this private firm model has retained its accuracy based on applications to individual private firm bankruptcies. These results have never been published, however.

I have also built numerous models for firms in non-US countries, generally following the pattern of first trying the original model on a sample of local firm bankruptcies and nonbankruptcies before adding or subtracting the variables thought to be helpful in those countries for a more accurate prediction. In some cases, different criteria for the distressed firm sample had to be used due to a lack of formal bankruptcies. One example is our China model (Altman *et al* 2010a), which utilized firms classified as "ST" (special treatment) due to their consistent losses and their book equity dropping below par value. In others, such as in Australia (Altman and Izan 1982), the explanatory variables were all adjusted for industry averages, so the model was thought to be more appropriate and accurate across a wide spectrum of industrial sectors. For the sovereign risk assessment model (Altman and Rijken 2011), in addition to traditional financial ratios, market value levels and volatility measures, the authors added macroeconomic variables, such as yield spreads and inflation indicators, to their Z-metrics model, which they applied to all nonfinancial, listed firms in order to assess the sovereign's private sector health. This modeling approach is applicable to any country in the world as long as data on listed or nonlisted private sector companies is available. The issue of sovereign risk application will be discussed again later, following Table 8.

7 THE Z''-SCORE MODEL

As noted in Figure 3, we built a model (Z''-score) for all industrials, manufacturers and nonmanufacturers, in 1995, first applying it to Mexican companies and then to other Latin American firms. It has since been successfully applied in the United States and many other countries, usually with superior accuracy compared with the original Z-score model if the data includes nonmanufacturers. This Z''-score model is also applicable to privately owned firms, since X_4 is denominated in book equity to total liabilities, not market values. This substitution is particularly important for environments where the stock market is not considered a good valuation measure due to its size, scope, liquidity or trading factors. In addition, note that the original fifth variable, sales/total assets, is no longer in this model. We found that the X_5 variable was particularly sensitive to industrial sector differences, eg, retail or service firms versus manufacturing companies, and in countries where the capital for investment in fixed assets was inadequate. Finally, this version of the Altman family of models that use discriminant analysis also has a constant term (3.25). This constant standardizes the results such that scores slightly above or below zero are in the D-rated BRE (see

BOX 4 Z'-score private firm model.

$$Z' = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.998X_5,$$
$$X_1 = \frac{\text{current assets} - \text{current liabilities}}{\text{total assets}},$$
$$X_2 = \frac{\text{retained earnings}}{\text{total assets}},$$
$$X_3 = \frac{\text{earnings before interest and taxes}}{\text{total assets}},$$
$$X_4 = \frac{\text{book value equity}}{\text{total liabilities}},$$
$$X_5 = \frac{\text{sales}}{\text{total assets}}.$$

Source: author's calculations.

BOX 5 Z''-score model for manufacturers and nonmanufacturer industrials as well as developed and emerging market credits (1995).

$$Z'' = 3.25 + 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4,$$
$$X_1 = \frac{\text{current assets} - \text{current liabilities}}{\text{total assets}},$$
$$X_2 = \frac{\text{retained earnings}}{\text{total assets}},$$
$$X_3 = \frac{\text{earnings before interest and taxes}}{\text{total assets}},$$
$$X_4 = \frac{\text{book value of equity}}{\text{total liabilities}}.$$

Source: author's calculations from Altman *et al* (1995b).

Table 6 for BREs that are even more granular than the major rating categories). The type I accuracy of the Z''-score model over time is shown in Table 7.

8 SCHOLARLY IMPACT

Perhaps because of their simplicity, transparency and consistent accuracy over the years, the Z-score models have been referenced and used as benchmarks in a large number of academic and practitioner studies in finance and accounting. These references and comparisons have taken at least three forms. The first involves constructing alternative models and frameworks to predict bankruptcy or defaults. The original model and its success using a combination of financial and market valuation data with robust statistical analysis made the task of default risk assessment

TABLE 6 US BREs based on Z''-score model: $Z'' = 3.25 + 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4$.

Rating	Median 1996 Z''-score[a]	Median 2006 Z''-score[a]	Median 2013 Z''-score[a]
AAA/AA+	8.15 (8)	7.51 (14)	8.80 (15)
AA/AA−	7.16 (33)	7.78 (20)	8.40 (17)
A+	6.85 (24)	7.76 (26)	8.22 (23)
A	6.65 (42)	7.53 (61)	6.94 (48)
A−	6.40 (38)	7.10 (65)	6.12 (52)
BBB+	6.25 (38)	6.47 (74)	5.80 (70)
BBB	5.85 (59)	6.41 (99)	5.75 (127)
BBB−	5.65 (52)	6.36 (76)	5.70 (96)
BB+	5.25 (34)	6.25 (68)	5.65 (71)
BB	4.95 (25)	6.17 (114)	5.52 (100)
BB−	4.75 (65)	5.65 (173)	5.07 (121)
B+	4.50 (78)	5.05 (164)	4.81 (93)
B	4.15 (115)	4.29 (139)	4.03 (100)
B−	3.75 (95)	3.68 (62)	3.74 (37)
CCC+	3.20 (23)	2.98 (16)	2.84 (13)
CCC	2.50 (10)	2.20 (8)	2.57 (3)
CCC−	1.75 (6)	1.62 (5)[b]	1.72 (4)[b]
CC/D	0 (14)	0.84 (120)	0.05 (94)[c]

[a]Sample sizes in parentheses. [b]Interpolated between CCC and CC/D. [c]Based on ninety-four Chapter 11 bankruptcy filings, 2010–13. *Sources*: Compustat, company filings and S&P.

more attractive to scientific researchers in many disciplines. It opened the door for not only finance and accounting scholars but also statisticians and mathematicians to find better and more efficient indexes and to examine new indicators and techniques, especially as more expansive and easily accessible databases became available.

New frameworks have involved seemingly more powerful statistical and mathematical techniques, such as logit, probit or quadratic nonlinear regressions; artificial intelligence; neural networks; genetic algorithms; recursive partitioning; machine learning; and structural, distance-to-default or hazard models, among others. Since the Z-score model is easily replicable, it was chosen by many researchers to be compared in terms of accuracy of classification and prediction. These studies are too numerous to list individually, but they probably number in the hundreds, including several by this author with numerous coauthors (see the bibliography).[1] This combi-

[1] Indeed, Bellovary *et al* (2007) review 165 bankruptcy prediction models from 1965 to 2006, including many by this author, and a large number of similar articles. They conclude that multiple discriminant analysis and neural networks are the most promising methods for bankruptcy

nation of simple, but theoretically well-grounded, empirical analysis provided new and attractive avenues in bankruptcy research, laying the foundation for an expanded modern understanding of bankruptcy prediction. For example, recent studies (by Altman, Iwanicz-Drozdowska, Laitinen and Suvas in 2016 and 2017) have looked at several dimensions of bankruptcy prediction research for (a) long distance (ten years) time series accuracy, (b) a number (five) of different statistical techniques, and (c) numerous (thirty-four) different country databases and environments. The results, covering thirty-one European countries and three others (China, Colombia and the United States), showed that, while models built specifically for individual countries usually outperformed the original Z-models, the added value of new country-specific variables and data as well as numerous frameworks was not dramatic. Despite somewhat higher accuracies using the Z''-score variables on data specific to each country, we found that the original weightings continued to exhibit remarkable performances, despite their being determined more than two decades earlier. Duffie *et al* (2007) also explored the multiperiod aspects of a bankruptcy prediction model.

Studies using accounting data, among other variables, potentially suffer when that data either is not very reliable, eg, from emerging markets, or is subject to earnings management manipulations. A recent study by Cho *et al* (2012) reconstructed Z-scores for this manipulation with the resulting accuracy improved.

The second dimension of the Z-score's scholarly impact is its international "reach". Since the original model and its derivatives (eg, Z''-score) have stood the test of time, the model has been widely applied in multiple settings, including applications across all domains, with its sharp focus on a few key variables. Also important are its robust empirical stability over long periods of time and its global applicability and understandability. We are familiar with Z-score-type models being built

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prediction, but caution that higher model accuracy is not guaranteed by using a greater number of factors. According to the authors, "since Altman's study, the number and complexity of bankruptcy prediction models have dramatically increased". Keasey and Watson (1991) describe discriminant analysis as the main technique used in this knowledge field. Willer do Prado *et al* (2016) found that logistic regression and neural networks became popular after the 1990s, with logistic regression and discriminant analysis being the most-used techniques up to the end date of their article's sample period in 2014. Willer do Prado *et al* used bibliometric evaluation (see Pinto *et al* 2014) to evaluate research about credit risk and bankruptcy using Reuters "Web of Science" database from 1968 to 2014. They found, through their exhaustive investigation, that the bankruptcy prediction field appeared to be multidisciplinary, spanning not only finance and accounting but also operations research, management, mathematics, data processing, engineering and a broad range of statistical fields. Unsurprisingly, they discovered an increased number of bankruptcy studies after the 2008 crisis. Finally, Willer do Prado *et al* (2016) listed the ten most-cited articles in the bankruptcy prediction field (Table 3 in their study), with Altman (1968) registering 1483 Web of Science cites. The next most-cited article was Huang *et al* (2004) with 250, and Hillegeist *et al* (2004) came in third with 165 cites.

TABLE 7 Classification and prediction accuracy (type I): Z-score bankruptcy model*.

No. of months prior to bankruptcy filing	Original sample (33)	Holdout sample (25)	2011–14 predictive sample (69)
6	94%	96%	93%
18	72%	80%	87%

*See Altman *et al* (1995c); see also summary in Altman and Hotchkiss (2006).

and tested in at least thirty different countries, based on at least seventy individual articles, and even more in studies analyzing at least that many countries in a single study. Indeed, I helped assemble two special journal issues devoted to a large number of specific country models.[2] Those studies, and more, are also listed and described in Altman and Hotchkiss (2006, Appendix to Chapter 11). More recent studies can be found in our earlier discussion on scholarly impact and in Choi (1997).

The third dimension is related to its impact on corporate financial management, especially the important subject of optimal capital structure and the trade-off between the tax advantage of debt financing and expected bankruptcy and other distress costs. My contribution to this question (Altman 1984) discussed and measured empirically, for the first time, the so-called indirect bankruptcy costs. In addition, since both tax benefits and bankruptcy costs are based on expected values, contingent upon the probability of bankruptcy, an important aspect of the trade-off debate is that probability. We selected the Z-score model's expected default probability algorithm, albeit an early version of the probability estimation technique, to complete the empirical measures for firms that went bankrupt in three different industrial sectors. Our findings were cited directly by an in-depth study from *The Economist* (Emmott 1991), which highlighted Modigliani/Miller's irrelevance theories compared with traditional optimal capital structure arguments. Perhaps the main differences between the two theories are the existence and magnitude of expected bankruptcy costs. These arguments are still among the most important fundamental and hotly debated issues in modern corporate financial management, and references to the bankruptcy cost measure can be found in countless corporate finance articles as well as just about every relevant basic or advanced textbook.

[2] The two special issues of *Journal of Banking & Finance* on international bankruptcy prediction models that I edited were published in 1984 (Volume 8, Issue 2) and 1988 (Volume 12, Issue 7).

9 FINANCIAL DISTRESS PREDICTION APPLICATIONS

Over the last fifty years, we have gleaned numerous insights and ideas from many helpful, interested financial market practitioners and academic colleagues with respect to applications of the Z-score models.[3] I will be forever grateful for these insights, because it means so much to a researcher to see his or her scholarly contributions make their way into the real world and be applied in a constructive way.[4] Table 8 provides lists of those applications whereby I, and others, have utilized the Altman Z-score family of models for both external-to-the-firm (left column) and internal-to-the-firm (right column) and research (right column) analytics and applications. There is no time or space in this paper to discuss all of these applications, so I defer a more comprehensive discussion to our revised text (Altman *et al* 2019). In this paper, however, we will discuss just those listed in italics in Table 8.

10 LENDER APPLICATIONS

Throughout this paper, I have discussed a number of important applications of credit risk models, such as Z-scores for lending institutions. These include the accept/reject decision (Altman 1970), estimates of the PD and LGD (Altman 1989), and costs of errors in default loss estimation (see, for example, Altman *et al* 1977). In addition to these generalized applications, the introduction of Basel II in 1999 drew upon Z-scores and the structure proposed in CreditMetrics (Gupton *et al* 1977). Later, Gordy (2000, 2003), among others, discussed the anatomy of credit risk models and capital allocation under Basel II.

11 TO FILE CHAPTER 11 OR NOT

One of the most interesting and rewarding applications of the Z-score model, at least for me, was the essence of my testimony on December 5, 2008 before the US House of Representatives Finance Committee's deliberation on whether to continue to bail out General Motors, Inc (GM) and Chrysler Corporation, or to "suggest" that these firms file for the "privilege" or "right" to reorganize under the protective confines of Chapter 11 of the US Bankruptcy Code. In the history of the US financial and legal systems, this choice had been given to very few firms that had the opportunity to qualify for bailout with taxpayers' monies. However, countless distressed firms

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[3] I apologize to the many authors whose published studies are not specifically cited; I also appreciate immensely their interest in and the attention paid to our models' extensions and tests over time.

[4] Indeed, the Z-score model has even made its way into a novel written by a bestselling author: see Thomas Pynchon's *Bleeding Edge* (2013, p. 63).

TABLE 8 Z-score's financial distress prediction applications.

External (to the firm) analytics	Internal (to the firm) and research analytics
• *Lenders (eg, pricing, Basel capital allocation)*	• *To file or not (eg, GM)*
• Bond investors (eg, quality junk portfolio)	• *Comparative risk profiles over time*
• Long/short investment strategy on stocks (eg, baskets of strong balance sheet companies and indexes: STOXX, Goldman, Nomura, Morgan Stanley, for example)	• *Industrial sector assessment (eg, energy)*
• Security analysts and rating agencies	• *Sovereign default risk assessment*
• Regulators and government agencies	• Purchasers, suppliers assessment
• Auditors (audit risk, going concern model)	• Accounts receivable management
• Advisors (eg, assessing clients' health)	• Researchers – scholarly studies
• M&A (eg, bottom fishing)	• Chapter 22 assessment
	• *Managers – managing a financial turnaround*

Source: Altman, NYU Salomon Center.

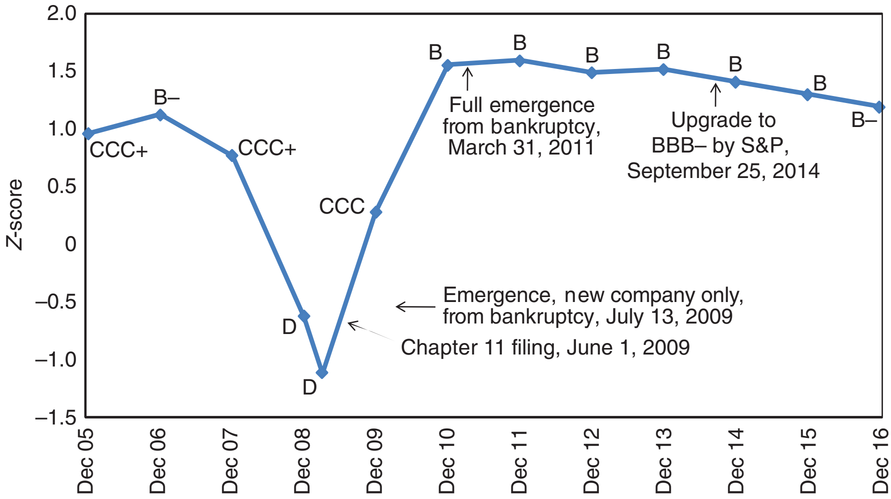
consider whether to file or not when they or their creditors face the prospect of the very survival of their company as a going concern.

The House invited a panel of academics and practitioners to discuss two issues: (1) whether the CEOs of the "Big Three" US auto-makers, in presenting their restructuring plans and strategies, should be granted an additional loan subsidy from Troubled Asset Relief Program (TARP) funds (Ford did not apply); and (2) whether these large auto-dealers should be bailed out or made to file for bankruptcy reorganization like the vast majority of ailing companies. For my testimony, we presented arguments on the benefits of Chapter 11, such as the ability to borrow monies with debtor-in-possession (DIP) financing and the automatic stay on nonessential interest and principal on existing loan obligations.[5] We also presented analyses of the then-current, and historical, Z-scores and their BREs. The data shown in Figure 4 was one of the primary determinants for my conclusion that GM was destined to go bankrupt, even with a temporary bailout, and should thus file for bankruptcy reorganization as soon as it was feasible to do so.

Note that GM's Z-score was in the CCC BRE, a highly risky zone, for several years before the crisis in 2008, even when it was still considered investment grade by all of the rating agencies (eg, in 2005; see Figure 4). In addition, at the time of my testimony in December 2008, GM's score was −0.63, deep into the D-rated BRE

[5] The complete testimony can be found on YouTube: "Altman testimony before the US House of Representatives Finance Committee", December 5, 2008.

FIGURE 4 Z-score model applied to GM (consolidated data): BREs and scores from 2005 to 2016.



zone. Hence, I strongly suggested a Chapter 11 filing and that GM should petition the bankruptcy court for a US$50 billion DIP loan, most likely from the Federal Government since none of the major banks at that time were in sufficient financial shape to offer a loan of that size. The House, and particularly its Finance Committee members, voted to continue the bailout, despite my arguments. The US Senate, however, voted not to continue the bailout. Nevertheless, before leaving office, President George W. Bush provided, by executive order, the bailout to give GM and Chrysler more time to restructure. It was now "Obama's problem". GM's Z-score continued to crater in the early months of 2009 and, despite management changes and the bailout, the company finally filed for bankruptcy under Chapter 11 on June 1, 2009. To assist the reorganization, Congress and the bankruptcy court provided a US$50 billion DIP loan: the exact amount I had suggested six months earlier!

In a remarkably short period – just forty-three days – GM emerged from bankruptcy and was once again on its way to being a going concern. Figure 4 shows the firm's improvement from deep in the D-rated BRE zone to a B-rated BRE in about twelve to eighteenth months. The DIP loan was first exchanged for new equity, and that equity was subsequently sold in the open market, whereby not only did the government not lose any of its "investment", it actually made a profit! GM today is a solid, thriving global auto-competitor with, again, an investment-grade rating (BBB), which it achieved in 2014. However, note that the Z-score model placed GM in the

TABLE 9 Comparing the financial strength of high-yield bond issuers in 2007 and 2012, 2014 and 2016 (3Q).

Year	Number of firms		Average Z-score/ (BRE)*	Median Z-score/ (BRE)*	Median Z'-score/ (BRE)*	Median Z''-score/ (BRE)*
	Z-score	Z''-score				
2007	294	378	1.95 (B+)	1.84 (B+)	4.68 (B+)	4.82 (B+)
2012	396	486	1.76 (B)	1.73 (B)	4.54 (B)	4.63 (B)
2014	577	741	2.03 (B+)	1.85 (B+)	4.66 (B+)	4.74 (B+)
2016 (3Q)	581	742	1.97 (B+)	1.70 (B)	4.44 (B)	4.63 (B)

Source: author's calculations, and data from Altman and Hotchkiss (2006) and S&P Capital IQ/Compustat. *BRE.

single-B BRE at the end of 2014, not at investment grade. This low BRE continued through the end of 2016. So, while GM has indeed improved considerably since its bankruptcy, it was still, to me, of a noninvestment grade.

12 COMPARATIVE RISK PROFILE OVER TIME

Students of history often ask to compare a current situation with that of some past period(s). This query is particularly relevant in financial markets, when a benchmark period in the past is related to some financial crisis, and it is discussed whether we can learn from the environment that existed then. Such is the case of the financial crisis of 2008–9 and whether credit conditions today are similar, or not, to those just prior to that crisis. One metric that I have found useful in comparing credit market conditions over time is our Z-score models. Was the average (or median) firm creditworthiness better, worse or about the same in, say, 2016 compared with 2007? One might have some priors based on related macro- or micro-observations, such as cash on the balance sheet, interest rates or GDP growth. A more holistic, objective measure, in my opinion, is one based on default probabilities that consider multiple attributes, such as the Z-scores of relevant samples of firms in the two periods.

Table 9 shows the average and median Z- and Z''-scores for a large sample of high-yield bond issuers in 2007 and 2016, with some intermediate years between the two periods. We find these comparisons extremely helpful in clarifying certain conclusions based on the qualitative or quantitative opinions of some experts. The average Z-score was 1.95 (B+ BRE) in 2007 and 1.97 (also B+ BRE) in 2016 (3Q), and the median score was actually higher in 2007 (1.84) than in 2016 (1.70). The average and median Z''-scores, a measure that is probably more appropriate given that the high-yield firms in both periods came from many different industrial sectors (see our discussion earlier about Z versus Z''), were both higher in 2007 than in 2016. Tests of the means between 2007 and 2016 showed that they were insignificantly different, so our conclusion is that the average credit profile of risky debt-issuing firms was about the same in 2007 and 2016. I leave it up to the reader to determine if this was good or bad news for default estimates in 2017 and beyond.

13 PREDICTING DEFAULTS IN SPECIFIC SECTORS

Over time, default cycles have produced carnage in one or more industrial sectors. If this has persisted for several years, these sectors have tended to draw the particular attention of researchers and practitioners. Hence, the textile industry model (Altman *et al* 1974), the broker–dealer model (Altman and Lorris (1976), US airlines (Altman and Gritta 1984) and most recently the US energy and mining sectors have motivated specific analyses and tests.

A recent empirical test (Altman and Kuehne 2017 (updated)) of Z-score models analyzed this score's accuracy in the energy and mining sectors. Rather than build a model based on energy firm data only, we decided to assess both the Z- and Z''-score models on a sample of bankruptcies in 2015, 2016 and 2017, a period in which energy-related firms accounted for more than half of the total defaults. Table 10 shows the results of just the bankruptcies, ie, type I accuracy, for two periods prior to the filing of the thirty-one firms with data available for a Z-score test as well as the larger number of firms (fifty-four) with data available for the Z''-score test. Our results were quite impressive, especially for the Z-score model, which we built, as noted earlier, based only on manufacturing firm data. Indeed, 87% of the energy and mining companies had Z-scores in the D-rated zone (defaulted BRE) based on data from one or two quarters prior to the filing, and the remaining six firms in the sample had a CCC or B− BRE. For data from five or six quarters prior to filing, the results were still impressive, with 56% in the D-rated BRE zone and most of the remaining firms at CCC, ie, only two out of the thirty-one firms had a B-rated BRE. While the results for the Z''-score model were not as accurate – 76% had a D-rated BRE and the remaining firms had at least a B-rated BRE, based on data from the last quarter prior to filing for bankruptcy – they were still impressive and quite accurate.[6]

So, it appears that our original Z- and Z''-score models retain their high accuracy level for distress prediction, even for some industries that were not included in our original tests. However, we are not able to generalize our results to cover all nonmanufacturers, especially service firms.

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[6] We have also tested our models for the type II error. Results show a reasonably high type II error, although the overall accuracy is still impressive. Note that the sample sizes are different for the comparison of Z and Z'' models.

TABLE 10 Applying the Z-score models to recent energy and mining company bankruptcies.

	Z-score				Z''-score			
	$t-1$*		$t-2$**		$t-1$*		$t-2$**	
BREs	**#**	**%**	**#**	**%**	**#**	**%**	**#**	**%**
A								
BBB+								
BBB								
BBB−								
BB+							1	2
BB							0	0
BB−							3	5
B+			2	6	1	2	1	2
B					3	5	13	24
B−					3	5	6	11
CCC+	5	16	12	39	1	2	8	15
CCC					2	4	8	15
CCC−					4	7	9	16
D	26	84	17	55	41	75	6	11
Total	31	100	31	100	55	100	55	100

13.1 Sovereign default risk

An intriguing application of the Z-score model is to use it to assess the default risk of sovereign nations' debt. We (Altman and Rijken 2011) were inspired by the World Bank's study (Pomerleano 1998), which analyzed the causes of the financial crisis in Southeast and East Asia in 1997–8. It found that the original Z-score model clearly demonstrated that the country that was most vulnerable to private sector defaults prior to the crisis was South Korea. Indeed, South Korea had the lowest average Z-score for listed firms out of all the Asian countries, but it was given a high investment grade by all of the rating agencies in December 1996. It needed to be bailed out by the International Monetary Fund shortly thereafter! This illustration inspired us, more than a decade later, to analyze sovereign default risk in a unique way.

The aggregation of Z-scores, or, in the case of Altman and Rijken (2011), a more up-to-date version called Z-metrics$^{\mathrm{T}}$, proved to be exceptionally accurate in predicting which European countries had the most serious financial problems post the 2008 crisis. Their bottom-up approach added a new micro-economic element to the arsenal of predictive measures for sovereign risk assessment that had never before been studied.

14 MANAGING A FINANCIAL TURNAROUND

One of the most interesting and important applications of the Z-score model, from an internal and active perspective rather than the passive standpoint of a distressed firm, is using it to guide the successful turnaround of a firm. I suggested this application and wrote up a case study on the GTI Corporation (see Altman and LaFleur (1981); it can also be found in Altman and Hotchkiss (2006) and Altman *et al* (2019)). The idea is a simple one: if a model is effective in predicting bankruptcy, why would it not be helpful in the management of distressed firms, in identifying strategies and their impact on performance metrics? In the case of the GTI Corporation, the new CEO, James LaFleur, strategically simulated the impact of his management changes on the resulting Z-scores and only made those changes that resulted in an improved Z-score. His strategy did result in a remarkably successful turnaround. Here, again, was an application of the Z-score model that I had never considered until a practitioner suggested its use.

15 CONCLUSION

This paper has assessed the statistical and fundamental characteristics of the Altman (1968) Z-score model over the fifty years since its creation. In addition, I have listed a large number of proposed and experienced applications of the original Z-model as well as several subsequent ones, with a more detailed discussion on the specifics and importance of several of these applications. This fifty-year-old model has demonstrated an impressive resilience over the years and, notwithstanding massive growth in the size and complexity of global debt markets and corporate balance sheets, has not only exhibited longevity as an accurate predictor of corporate distress, but also shown that it can be successfully modified for a number of applications beyond its original focus. The list, shown in Table 8, is almost assuredly incomplete, especially in view of the large number of scholarly works that have cited Z-score models for a wide range of empirical research investigations. While I am surprised at the longevity of the Z-score models' usefulness, I cannot help but wonder what some analysts might conclude in the year 2068 about its 100-year track record.

DECLARATION OF INTEREST

The author reports no conflicts of interest. The author alone is responsible for the content and writing of the paper.

ACKNOWLEDGEMENTS

The author thanks Robert Benhenni, Alessandro Danovi, Sanjiv Das, Martin Fridson, Michael Gordy, Malgorzata Iwanicz-Drozdowski, Tony Kao, Erkki Laitinen, Stuart Kovensky, Frederick Lundtofte, Herbert Rijken and Arto Suvas for their helpful comments and edits of this paper, and Lourdes Tanglao and Brenda Kuehne of the NYU Salomon Center for their technical assistance. The resulting work is still very much the responsibility of the author.

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