Title: Step-by-Step Engineering Design Equations for FRP Structural Beams

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ABSTRACT

Fiber-reinforced plastic (FRP) shapes (beams and columns) have shown to provide efficient and economical applications in civil engineering structures. This paper presents simplified step-by-step design equations for FRP beams, accounting for bending, shear, local/global buckling, and material failure. The design equations are developed based on a combined experimental and analytical study of eight representative beams, and are expressed in terms of panel apparent moduli and strengths, and beam stiffness coefficients and geometry. The design parameters are verified by testing these eight sections, and the design equations for bending/shear deflections and bending strains, local and global buckling critical loads, and ultimate bending/shear strengths are validated by the testing data. The guidelines and simplicity of the design equations for FRP beams described in this paper can be used in practice by structural engineers concerned with design of FRP composite structures.

INTRODUCTION

Fiber-reinforced plastic (FRP) shapes (beams and columns) have shown to provide efficient and economical applications for bridges, piers, retaining walls, airport facilities, storage structures exposed to salts and chemicals, and others. Substantial research on FRP beams has been reported in the U.S. and abroad and has provided significant useful results that can be translated into practice. However, the lack of step-by-step design procedures for FRP shapes presents a problem to builders, government officials, administrators and engineers, who may not be familiar with composites and yet bear the liability for making material choices.

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This paper is concerned with the development of step-by-step simplified design equations for FRP beams, accounting for bending, shear, local/global buckling, and material failure. The design equations are developed based on a combined experimental and analytical study of eight different FRP beams, which are representative of the shapes currently used in practice.

DESIGN CONSIDERATIONS

Most FRP shapes are thin-walled structures and manufactured by the pultrusion process. The material constituents for low-cost FRP shapes commonly consist of E-glass fiber and polyester or vinylester resins, and due to this choice of materials coupled with complexities of material architecture and geometric shapes, the following primary structural behaviors need to be considered in design:

- relatively large deflections due to the low elastic modulus of resins used;
- considerable shear deformation due to the relatively low shear modulus of the composite;
- critical global and local stability (buckling) due to the thin-walled structure and/or large slenderness ratios of component panels;
- potential material failure due to the relatively low compressive and shear strengths of composites.

To address the above four issues in design, design equations are developed in this study based on design parameters tabulated for 24 FRP representative sections currently produced by Creative Pultrusions Inc., Alum Bank, PA. The following design parameters are considered: panel stiffness and strength properties, beam bending/shear stiffnesses, beam deflections and maximum strains, global critical buckling loads, flange local critical buckling loads, and finally beam bending and shear strengths. The design parameters are obtained using analytical solutions that were developed primarily by the authors [1-4]. To verify various design parameters and equations, eight representative shapes of the 24 selected sections are experimentally tested. The eight beams tested include the following shapes: Wide-Flange (WF) 4"x4"x1/4" (**WF4x4**); WF 6"x6"x3/8" (**WF6x6**); WF 8"x8"x3/8" (**WF8x8**); WF 12"x12"x1/2" (**WF12x12**); Box 4"x4"x1/4" (**Box4x4**); I 3"x6"x3/8" (**I3x6**); I 4"x8"x3/8" (**I4x8**); Channel 6"x1-5/8"x1/4" (**C6x2**).

STEP-BY-STEP DESIGN EQUATIONS

In this section, the design parameters and corresponding simplified step-by-step design equations for FPR beams are presented, and the accuracy of the equations is validated with experimental data.

PANEL STIFFNESS AND STRENGTH

For most pultruded FRP sections, the lay-up of a panel is usually balanced symmetric, and the panel stiffness and strength properties can be obtained either from experimental coupon tests or through theoretical predictions by micro/macromechanics [2, 5]. As shown in Tables 1 and 2, the panel stiffness and strength properties obtained from coupon tests compare well with predicted values. As introduced next, explicit equations, that can be applied in engineering design, for the computation of beam bending and shear stiffness coefficients, deflections, panel strains and stresses, local/global buckling loads, and material failure loads are developed in terms of panel stiffness and strength properties.

TABLE 1. PANEL STIFFNESS PROPERTIES OF FRP SHAPES

FRP Shapes	E_{xx} (x10^6 psi)		G_{xy} (x10^6 psi)	
	Tension test	Micro/macro-Mechanics	Iosipescu test	Micro/macro Mechanics
WF6x6	4.155 (COV = 5.28%)	4.206	0.686 (COV = 8.39%)	0.682
I4x8	5.037 (COV = 2.24%)	4.902	0.745 (COV = 9.79%)	0.794
WF4x4	4.391 (COV = 5.55%)	4.167	0.778 (COV = 11.28%)	0.676
Box4x4	4.295 (COV = 10.70%)	3.604	0.548 (COV = 8.39%)	0.550

TABLE 2. PANEL STRENGTH PROPERTIES OF FRP SHAPES

FRP Shapes	F_c (x10^3 psi)		F_{xy} (x10^3 psi)
	Compression test	Strength [5]	Iosipescu test
WF6x6	54.498 (COV = 3.76%)	45.55	12.866 (COV = 2.36%)
I4x8	61.060 (COV = 2.41%)	56.65	13.022 (COV = 8.13%)
WF4x4	57.133 (COV = 3.50%)	53.10	13.167 (COV = 29.17%)
Box4x4	60.657 (COV = 5.33%)	47.20	11.138 (COV = 4.84%)

BEAM STIFFNESS PROPERTIES

The response of FRP shapes in bending is evaluated using the Mechanics of thin-walled Laminated Beams (MLB) [2, 3]. We simplify the MLB formulations and present explicit expressions in terms of panel engineering properties for beam bending and shear stiffness coefficients, which in return can be used in simplified equations for prediction of beam deflections, strains and stresses under bending. Assuming that the beam centroid is the neutral axis of bending (no beam bending-extension coupling), general expressions for the beam bending *(D)* and shear stiffness *(F)* coefficients are computed:

$$D = \sum_{i=1}^{n}\left[(E_x)_i\, t_i \left(h_i^2 + \frac{b_i^2}{12}\sin^2\phi_i \right) + \frac{(E_x)_i\, t_i^3}{12}\cos^2\phi_i \right] b_i\,, \quad F = \sum_{i=1}^{n} (G_{xy})_i\, t_i\, b_i \sin^2\phi_i \tag{1}$$

where b_i is the panel width, t_i is the panel thickness and ϕ_i is the cross-sectional orientation of the *ith* panel with respect to the bending axis; $(E_x)_i$ and $(G_{xy})_i$ are the panel stiffness values obtained either by the micro/macromechanics approach or from coupon tests. Note that D and F, respectively, are similar to EI and GA for isotropic materials (e.g., steel beams). If the flanges and webs of a section have identical lay-ups and stiffnesses, the beam bending and shear stiffnesses can be expressed simply in terms of panel stiffnesses E_x and G_{xy} and geometric properties I and A. For example, the beam stiffnesses for two common sections of "I" (in strong-axis) and box geometries are:

$$\text{"}I\text{"}:\quad D=\frac{1}{2}(E_x)_f t_f b_w^2 b_f+\frac{1}{12}(E_x)_w t_w b_w^3+\frac{1}{6}(E_x)_f t_f^3 b_f,\quad F=(G_{xy})_w t_w b_w$$

$$Box:\quad D=\frac{1}{2}(E_x)_f t_f b_w^2 b_f+\frac{1}{6}(E_x)_w t_w b_w^3+\frac{1}{6}(E_x)_f t_f^3 b_f,\quad F=2(G_{xy})_w t_w b_w \tag{2}$$

where, subscripts f and w identify flange and web components.

BEAM DEFLECTIONS AND STRAINS AND STRESSES

Displacement and rotation functions can be obtained by solving Timoshenko's beam theory equilibrium equations [2]. In particular, available expressions for maximum bending and shear deflections can be used; for example, the maximum deflection for a 3-point loading of a beam of span L and design load P is:

$$\delta = \delta_b + \delta_s = \frac{PL^3}{48D} + \frac{PL}{4K_Y F} \tag{3}$$

where, the bending (δ_b) and shear (δ_s) components of deflection can be independently evaluated; as an approximation in design, the shear correction factor for most FRP sections can be taken as K_Y = 1.0 [2]. The maximum top-surface longitudinal strains and in-plane shear strains of the *ith* panel are expressed as:

$$\varepsilon_x=\frac{M}{D}h_i \quad and \quad \gamma_{xy}=\frac{V}{F}\sin\phi_i \tag{4}$$

where V and M are, respectively, the resultant internal shear force and bending moment acting on the beam; h_i is the transverse coordinate of a point from the neutral axis. Based on Eq. (2), Table 3 lists the bending and shear stiffnesses of four selected beams, and Table 4 shows comparisons between predictions from equations (3) and (4) and experimental measurements.

TABLE 3. BEAM BENDING AND SHEAR STIFFNESS PROPERTIES

FRP Shapes	D = EI (x 10^8 psi • in.^{4})		F = GA (10^6 psi • in.^{2})	
	Strong-Axis	Weak-Axis	Strong-Axis	Weak-Axis
WF6x6	1.776	0.570	1.292	3.066
I4x8	2.558	0.199	1.772	2.379
WF4x4	0.334	0.111	0.585	1.351
Box4x4	0.364	0.338	1.100	1.176

BEAM LOCAL AND GLOBAL BUCKLING

A comprehensive analytical approach was developed to study the local buckling behaviors of pultruded FRP shapes [6]. The local buckling analysis for discrete laminated plates or panels of FRP shapes was formulated, and the effects of restraint

TABLE 4. BEAM DEFLECTIONS AND STRAINS (L = 12.0 FT)

FRP Shapes	Axis of Load	Deflection δ (in/kip)		Strain ε_{Top} (με/kip)		Strain ε_{Bottom} (με/kip)	
		Test	Design	Test	Design	Test	Design
WF6x6	Strong	0.388	0.378	-616.9	-608.2	668.6	608.2
	Weak	1.169	1.106	-1902.7	-1900.0	1823.5	1900.0
I4x8	Strong	0.271	0.264	-576.6	-566.0	594.1	566.0
	Weak	3.511	3.152	-3646.8	-3630.0	3557.6	3630.0
WF4x4	Strong	1.833	1.926	-2081.3	-2160.0	2121.8	2160.0
	Weak	5.769	5.627	-5879.6	-6480.0	5913.9	6480.0
Box4x4	Strong	1.886	1.742	-2139.8	-1990.0	2141.7	1990.0
	Weak	1.947	1.873	-1944.4	-2140.0	1903.9	2140.0

at the flange-web connection were considered. For the flange panels under compression, simplified expressions for predictions of plate buckling strength are proposed by approximately solving transcendental equations [6]:

$$\sigma_x^{cr} = \frac{\pi^2}{12}\left(\frac{t_f}{b}\right)^2 \left[\sqrt{q}\left(2\sqrt{(E_x)_f (E_y)_f}\right) + p\left((E_y)_f (v_{xy})_f + 2(G_{xy})_f\right)\right] \tag{5}$$

where, σ_x is the critical stress, and p and q are constants that are defined by the coefficient of restraint (ζ) at the junction of panels:

$$"I":\ p = 0.3 + \frac{0.004}{\zeta - 0.5};\ q = 0.025 + \frac{0.065}{\zeta + 0.4};\ \zeta = \frac{2b_w}{b_f}\frac{(E_y)_f}{(E_y)_w};\ b = \frac{b_f}{2}$$

$$Box:\ p = 2.0 + \frac{0.002}{\zeta - 1.3};\ q = 1.0 + \frac{0.08}{\zeta + 0.2};\ \zeta = \frac{b_w}{b_f}\frac{(E_y)_f}{(E_y)_w};\ b = b_f$$

For a beam under 3-point bending, the critical local buckling load (P_{cr}^{local}) can be obtained in terms of critical stress and beam properties as:

$$P_{cr}^{local} = \frac{8D\sigma_x^{cr}}{(E_x)_f b_w L} \tag{6}$$

As shown in Table 5, design equations based on Eqs. (5) and (6) compare favorably with testing data for four wide-flange beams.

For long-span FRP beams without lateral supports and with large slenderness ratios, a global buckling is prone to happen. Based on a Vlasov theory [7], a simplified engineering equation for flexural-torsional buckling of an "I" section is expressed as:

$$P_{cr}^{global} = \frac{17.17}{L^2}\sqrt{D \cdot JG}\sqrt{1 + \frac{\pi^2}{L^2}\frac{I_{ww}}{JG}} \tag{7}$$

where, $$JG = \frac{2(G_{xy})_f t_f^3 b_f}{3} + \frac{(G_{xy})_w t_w^3 b_w}{3};$$

$$I_{ww} = \frac{(E_x)_f t_f b_w^2 b_f^3}{24} + \frac{(E_x)_f t_f^3 b_f^3}{36} + \frac{(E_x)_w t_w^3 b_w^3}{144}$$

Again, the design equation [Eq. (7)] is correlated with experimental tests in Table 5.

TABLE 5. BEAM CRITICAL LOCAL AND GLOBAL BUCKLING LOADS

FRP Shapes	Local Bucking, P_{cr}^{local}			Global Buckilng, P_{cr}^{global}		
	Span (ft)	Test (kips)	Design (kips)	Span (ft)	Test (kips)	Design (kips)
WF4x4	6.0	8.83	6.46	-	-	-
WF6x6	6.0	23.13	20.62	12	5.60	4.24
WF8x8	6.0	24.53	22.04	12	12.93	12.78
WF12x12	6.0	29.27	33.33	14.5	30.01	38.25

BEAM ULTIMATE BENDING AND SHEAR FAILURE

Due to relatively low compressive and shear strength of FRP composites, the material failure needs to be evaluated. Similar to beam deflection and buckling, the beam bending and shear strength (ultimate failure loads) can be expressed in terms of panel strength properties as:

$$\text{Bending: } P_{fail}^{bending} = \frac{8F_c D}{(E_x)_f (b_w - t)L}; \text{ Shear: } P_{fail}^{shear} = F_{xy} b_w t_w \qquad (8)$$

where, F_c and F_{xy} are given in Table 2. The comparisons between the design and testing results are shown in Table 6.

TABLE 6. BEAM ULTIMATE BENDING AND SHEAR LOADS

FRP Shapes	Bending, $P_{fail}^{bending}$			Shear, P_{fail}^{shear}		
	Span (ft)	Test (kips)	Design (kips)	Span (ft)	Test (kips)	Design (kips)
I3x6	9.5	11.52	12.10	2.0	22.00	28.95
I4x8	9.5	20.28	24.50	-	-	-
WF4x4	9.5	5.25	7.52	2.0	12.80	13.11
Box4x4	9.5	5.79	7.98	2.0	17.40	22.27

DESIGN PROCEDURES FOR FRP BEAMS

To facilitate the design of FRP beams under bending, the following design guidelines are recommended:

- Characterize the beam panel material properties (stiffness and strength) from either coupon tests or micro/macromechanics and empirical formulas.
- From Eq. (1), obtain the beam bending and shear stiffness coefficients, which in turn can be used to predict the beam deflection and strains and stresses.

- Determine the local and global buckling resistance of beam sections by Eqs. (6) and (7), respectively.
- Predict the beam failure (bending and shear) loads based on the panel strength data and Eq. (8).

CONCLUSIONS

In this paper, simplified design parameters and equations for FRP beams are developed, and the accuracy and validity of the design equations are verified by testing eight representative shapes out of 24 sections. A design procedure that accounts for most critical issues in FRP beam design is presented, which can be used in the future to develop general design guidelines and also *"product-acceptance criteria"* for FRP beams produced by any manufacturer.

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