

**Step-by-Step Engineering Design Equations  
for Fiber-reinforced Plastic Beams for Transportation Structures**

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## ABSTRACT

Fiber-reinforced plastic (FRP) shapes (beams and columns) have shown to provide efficient and economical applications in civil engineering structures. However, there are no simplified engineering design procedures for FRP beams, primarily because of the complexity of composite materials, and existing design equations for members of conventional materials can not be readily applied to FRP shapes. Therefore, there is a need to develop design guidelines for practicing engineers and transportation officials, who usually bear the responsibility for making material choices. This report includes the following three major modules: (1) Educational Module, (2) Design Guidelines Modules, and (3) Examples and Application Module. In the design module, simplified step-by-step design equations for FRP beams are presented, and bending, shear, local/global buckling, and material failure are considered. The design equations are developed based on a combined experimental and analytical study of eight representative beams, and are expressed in terms of panel apparent moduli and strengths, and beam stiffness coefficients and geometry. The design parameters are verified by testing these eight sections, and the design equations for bending/shear deflections and bending strains, local and global buckling critical loads, and ultimate bending/shear strengths are validated by the testing data. The guidelines and simplicity of the design equations for FRP beams described can be used in practice by structural engineers concerned with design of FRP composite structures. To promote the use of this report, the results of this study are available through the Internet. Also, a one-time short course is scheduled to be offered to WVDOT-DOH personnel in Charleston, WV. This project is partially sponsored by Creative Pultrusions Inc., who provided technical information and samples for testing.

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## 1. INTRODUCTION

A great need exists for new materials and methods to repair and/or replace deteriorated structures at reasonable costs, and Fiber-Reinforced Plastic (FRP) beams have shown to provide efficient and economical applications in bridges and piers, retaining walls, airport facilities, storage structures exposed to salts and chemicals, and others. FRP materials are lightweight, noncorrosive, nonmagnetic, and nonconductive. In addition, they exhibit excellent energy absorption characteristics, suitable for seismic response-; high strength, fatigue life, and durability; competitive costs based on load-capacity per unit weight; and ease of handling, transportation, and installation. Also, monitoring sensors, such as fiber optics, can be easily integrated into FRP composites during manufacturing. Moreover, FRP composites offer the inherent ability to alleviate or eliminate the following four construction related problems adversely contributing to transportation deterioration worldwide (Head 1996): corrosion of steel, high labor costs, energy consumption and environmental pollution, and devastating effects of earthquakes.

Substantial research on FRP beams has been reported in the U.S. and abroad and has provided significant useful results that can be translated into practice. However, the lack of step-by-step design procedures for FRP shapes presents a problem to builders, government officials, administrators and engineers, who may not be familiar with composites and yet bear the responsibility for making material choices. **This problem is compounded by the lack of**

design guidelines that can be used as references by civil engineers. ...*"The civil engineering community must be educated in the design and application... of new construction materials..."* (Seible and Karbhari 1996).

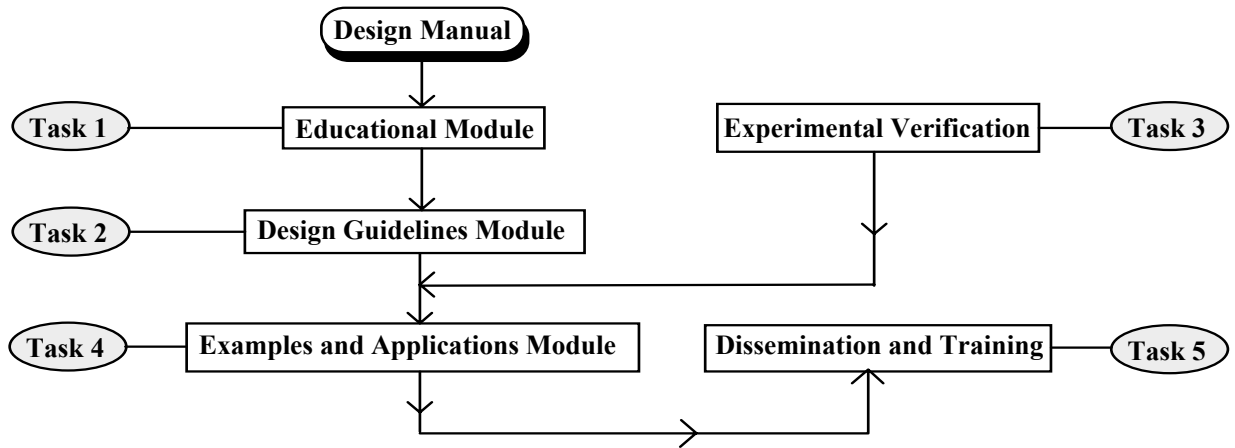
## 2. OBJECTIVE

This report is concerned with the development of step-by-step simplified design equations for FRP beams, accounting for bending, shear, local/global buckling, and material failure. The report is organized in three modules: **(1) Educational Module** to explain in simple terms fundamental concepts, such as constituent materials and processing methods; **(2) Design Guidelines Module** with a commentary which introduces the development of step-by-step design equations and their experimental verifications, and **(3) Examples and Applications Module**. The properties required for design are presented in the form of tables and/or graphs. The illustrative examples are based on data developed for 23 existing sections, and each section is produced with E-glass fiber and Polyester resin by Creative Pultrusions, Inc. of Alum Bank, PA. These sections are representative of the shapes currently used in practice. To verify the accuracy and validity of the design equations, eight representative shapes of the 23 sections considered are experimentally evaluated. The tabulated design parameters presented in the Design Guidelines Module can facilitate the future development of "product-acceptance criteria" for FRP beams produced by any manufacturer by the WVDOT-DOH, similar to the criteria used by CALTRANS for their seismic retrofit program of concrete bridges using FRPs.

## 3. SCOPE OF WORK

The development, verification, and dissemination of the design methodology are accomplished by systematically completing the tasks shown in Figure 1. Based on the research tasks performed in Figure 1, the following major three components are presented in this report: **(1)** development of the **Educational Module**, intended to briefly introduce composite

materials to design engineers (**Section 4**); **(2)** development of the **Design Guidelines Module**, which describes the design considerations for FRP beams; design procedures using simplified equations from tabulated design parameters, graphs, and charts; and experimental evaluation of eight sections to verify the design equations (**Section 5**); **(3)** development of the **Examples and Applications Module**, which presents detailed design problems to illustrate the design equations (**Section 6**). Finally, to disseminate the results, a Web page in the Internet is provided, and a one-time short course will be offered to WVDOT personnel in Charleston, WV.



**Figure 1. Structure of the research program**

#### **4. EDUCATIONAL MODULE**

In this module, a discussion of fiber-reinforced plastic (FRP) composites is presented in terms of constituent materials, consisting of fiber systems and resins, and manufacturing technology, which is limited to the pultrusion process. This section of the manual serves to introduce basic design concepts inherent to composites, such as shear deformation, and common terminology used in practice, such as percent fiber content by volume of composite or "fiber-volume fraction." The following topics are presented in the educational module: (1) introduction to composites and applications in infrastructure; (2) commonly used fiber and



resin systems and their characteristics; (3) influence of fiber architecture, or type of lay up, on the response of a member; (4) protective veils and coatings for specific purposes, such as UV light and fire; (5) problems in use of FRP for structures, such as connections and sensitivity to buckling; (6) advantages of FRP as a structural material; (7) details of the pultrusion manufacturing process; (8) commonly produced standard shapes; (9) factors affecting stiffness and strength; (10) time-dependent behavior of FRP composites, related to creep and load duration concepts; (11) tests and properties of interest to the structural designer, such as bending and shear stiffness and strength; finally (12) important design recommendations and guidelines. Since this module is mainly related to fundamental aspects of composite materials and processes, this information will be provided separately during the seminar to be scheduled for WVDOT-DOH personnel in Charleston, WV.

## **5. DESIGN GUIDELINES MODULE**

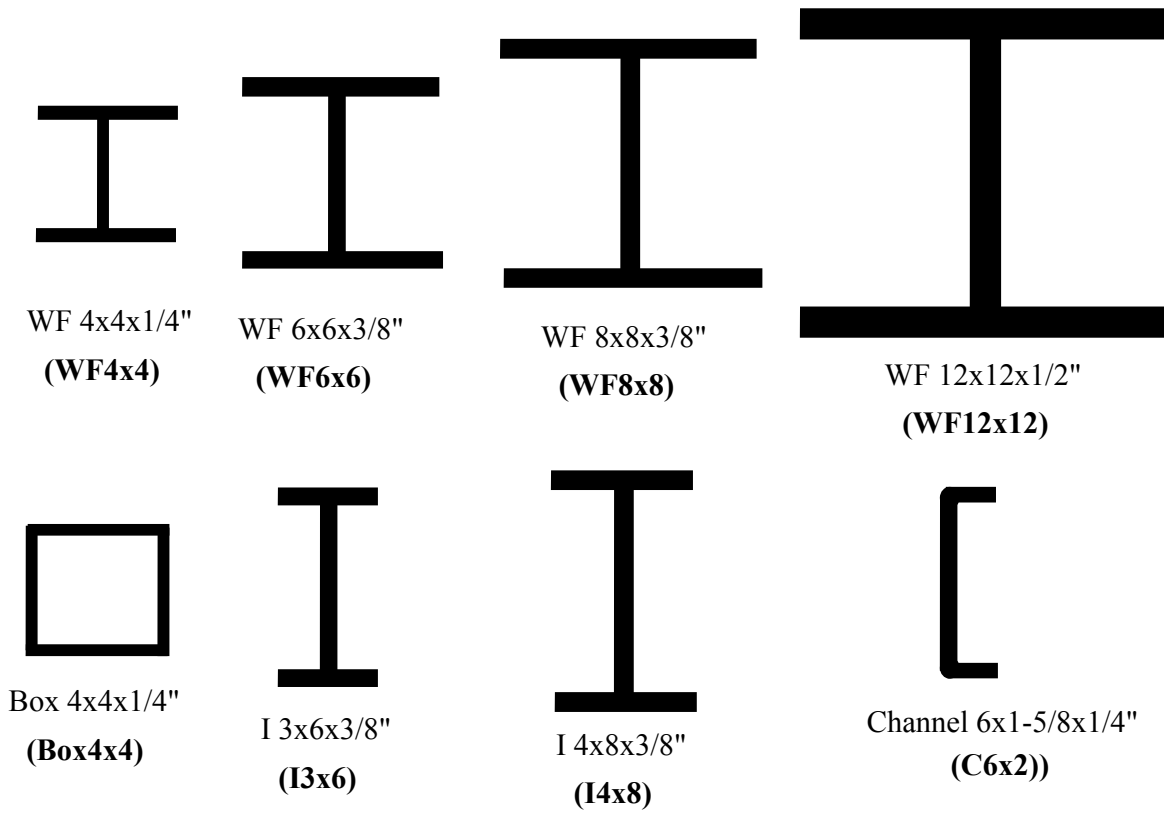
This section is concerned with the development of step-by-step simplified design equations and guidelines for FRP beams, accounting for bending, shear, local/global buckling, and material failure. The design equations are developed based on a combined experimental and analytical study of eight different FRP beams, which are representative of the shapes currently used in practice. The design data is provided for 23 representative "commercial" FRP sections available from Creative Pultrusions, Inc.

### **5.1 Design Considerations**

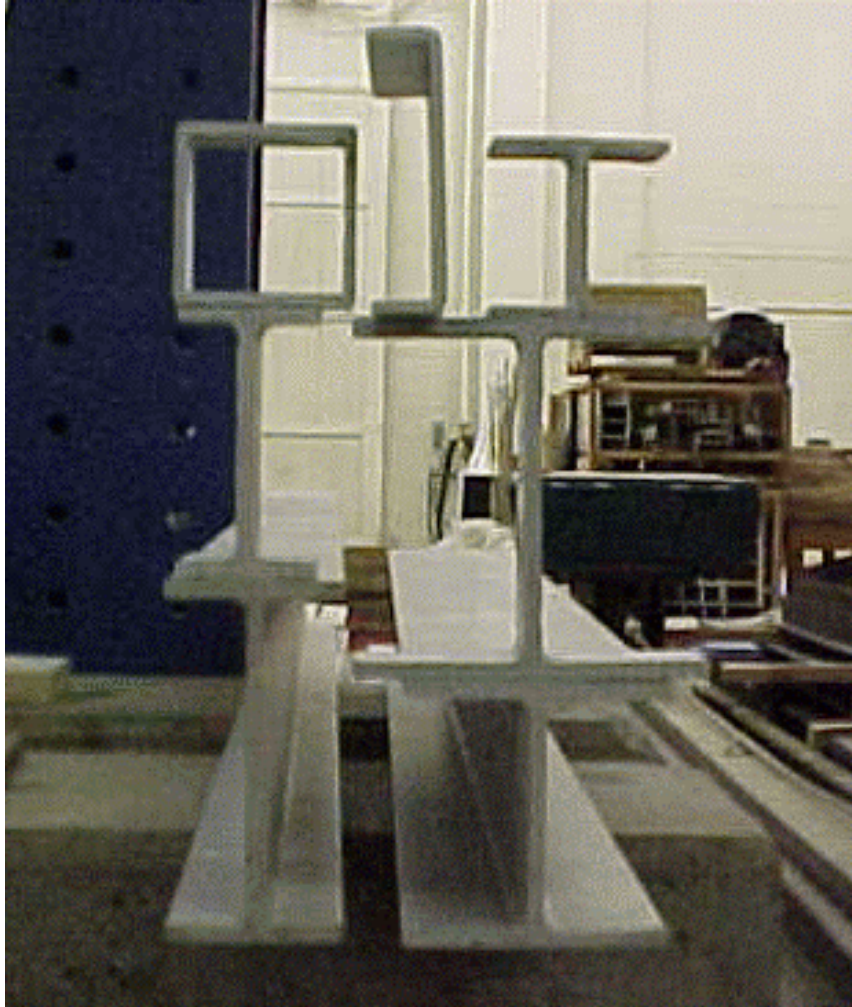
Most FRP shapes are thin-walled structures and manufactured by the pultrusion process. The material constituents for low-cost FRP shapes commonly consist of E-glass fiber and polyester or vinylester resins, and due to this choice of materials coupled with complexities of material architecture and geometric shapes, the following primary structural behaviors need to be considered in design:

- relatively large deflections due to the low elastic modulus of resins used;
- considerable shear deformation due to the relatively low shear modulus of the composite;
- critical global and local stability (buckling) due to the thin-walled structure and/or large slenderness ratios of component panels;
- potential material failure due to the relatively low compressive and shear strengths of composites.

To address the above four issues in design, design equations are developed in this study based on design parameters tabulated for 23 FRP representative sections currently produced by Creative Pultrusions Inc. The following design parameters are considered: panel stiffness and strength properties, beam bending/shear stiffnesses, beam deflections and maximum strains, global critical buckling loads, flange local critical buckling loads, and finally beam bending and shear strengths. The design parameters are obtained using analytical solutions that were developed primarily by the principal investigators (Barbero et al. 1993; Davalos et al. 1996; Davalos and Qiao 1997; Davalos et al. 1997). To verify various design parameters and equations, eight representative shapes of the 23 selected sections are experimentally tested. The eight beams tested include the following shapes (Figure 2): Wide-Flange (WF) 4"x4"x1/4" (**WF4x4**); WF 6"x6"x3/8" (**WF6x6**); WF 8"x8"x3/8" (**WF8x8**); WF 12"x12"x1/2" (**WF12x12**); Box 4"x4"x1/4" (**Box4x4**); I 3"x6"x3/8" (**I3x6**); I 4"x8"x3/8" (**I4x8**); Channel 6"x1-5/8"x1/4" (**C6x2**). A photograph of seven of the above eight shapes is shown in Figure 3.



**Figure 2. Eight representative FRP structural shapes**



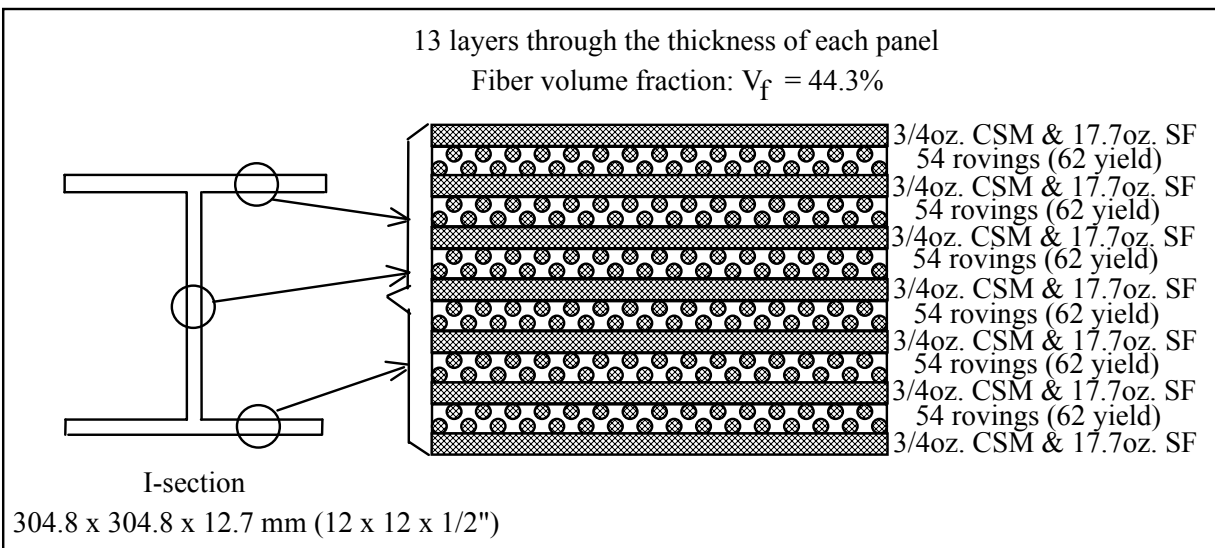
**Figure 3. Seven tested FRP structural shapes (WVU Main Structures Lab)**

## **5.2 Step-by-Step Design Equations**

In this section, the design parameters and corresponding simplified step-by-step design equations for FPR beams are presented, and the accuracy of the equations is validated with experimental data.

### ***5.2.1 Panel stiffness and strength***

For most pultruded FRP sections, the lay-up of a panel is usually balanced symmetric, and the panel stiffness and strength properties can be obtained either from experimental coupon tests or through theoretical predictions by micro/macromechanics (Davalos et al. 1996; Barbero et al. 1999). For example, the material architecture of a wide-flange beam (12"x12"x1/2") is illustrated in Figure 4. Even though the panels of beam are not manufactured by hand lay-up, the pultruded panels can be simulated as a laminated structure (Davalos et al. 1996). The micro/macromechanics approach, which is commonly used for laminates can be efficiently used for modeling of pultruded panels.



**Figure 4. Panel Fiber Architectures of Wide-Flange Beams (WF12x12)**

Most pultruded FRP sections such as wide-flange (WF) beams (Figure 4) consist typically of arrangements of flat panels and mainly include the following three types of layers (Davalos et al. 1996): (1) Continuous Strand Mats (CSM); (2)  $\pm\alpha$  Stitched Fabrics (SF); and (3) rovings or unidirectional fiber bundles. Usually, the reinforcement used is E-glass fibers, and the resin or matrix is either vinyl ester or polyester. Each layer is modeled as a homogeneous, linearly elastic, and generally orthotropic material. Based on the fiber volume fraction and the manufacturer's information, the ply stiffnesses can be computed from micromechanics models

for composites with periodic microstructure (Luciano and Barbero 1994). Then, the stiffnesses of a panel can be computed from classical lamination theory (CLT) (Jones 1976). In CLT, the engineering properties ( $E_x$ ,  $E_y$ ,  $\nu_{xy}$ , and  $G_{xy}$ ) of the panel are computed by assembling the transformed stiffness coefficients into the extensional stiffness matrix  $[A]$ . The engineering properties of the pultruded panel are then computed as (Davalos et al. 1996):

$$E_x = 1 / (t\alpha_{11}), \quad E_y = 1 / (t\alpha_{22}), \quad \nu_{xy} = -\alpha_{12} / \alpha_{11}, \quad G_{xy} = 1 / (t\alpha_{66}) \quad (1)$$

where  $t$  is the panel thickness;  $[\alpha]$  is the compliance matrix, which is the inverse of the extensional stiffness matrix  $[A]$ . Also, the laminate stiffnesses of the panels as indicated in Eq. (1) can be directly obtained from tests of coupon samples, which are cut from the FRP sections and tested in tension, shear (Iosipescu) and torsion. Another alternative method for determination of panel properties is based on *Carpet Plots* (Barbero 1999) for which the panel engineering properties are plotted with respect to the fiber percentages of layers. Once the material architecture is known from the manufacturer, the fiber volume fraction of the panel can be computed and the panel engineering properties can be obtained graphically from *Carpet Plots* (e.g.,  $E_x$  and  $G_{xy}$  in Figure 5 and  $F_{xc}$  and  $F_{xy}$  in Figure 6).

As shown in Tables 1 and 2, the panel stiffness and strength properties obtained from coupon tests compare well with predicted values. As introduced next, explicit equations, that can be applied in engineering design, for the computation of beam bending and shear stiffness coefficients, deflections, panel strains and stresses, local/global buckling loads, and material failure loads are developed in terms of panel stiffness and strength properties.

In summary, the panel engineering properties of FRP shapes can be obtained by the following means: (1) micro/macromechanics approach (material lay-up and fiber volume fraction must be known); (2) experimental coupon tests; and (3) Carpet plots (material fiber percentages must be known). The panel engineering properties by micro/macromechanics for all 23 customized sections are shown in Table 3.

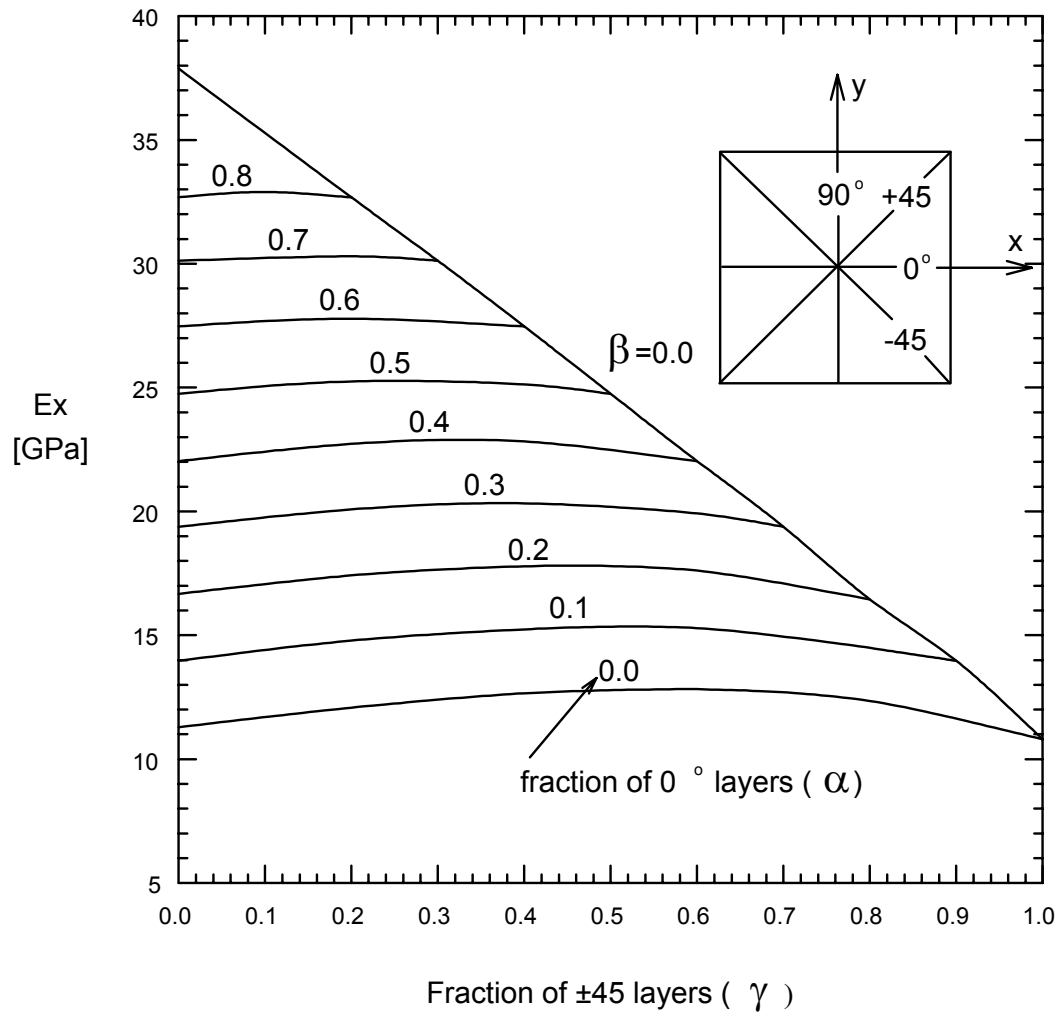
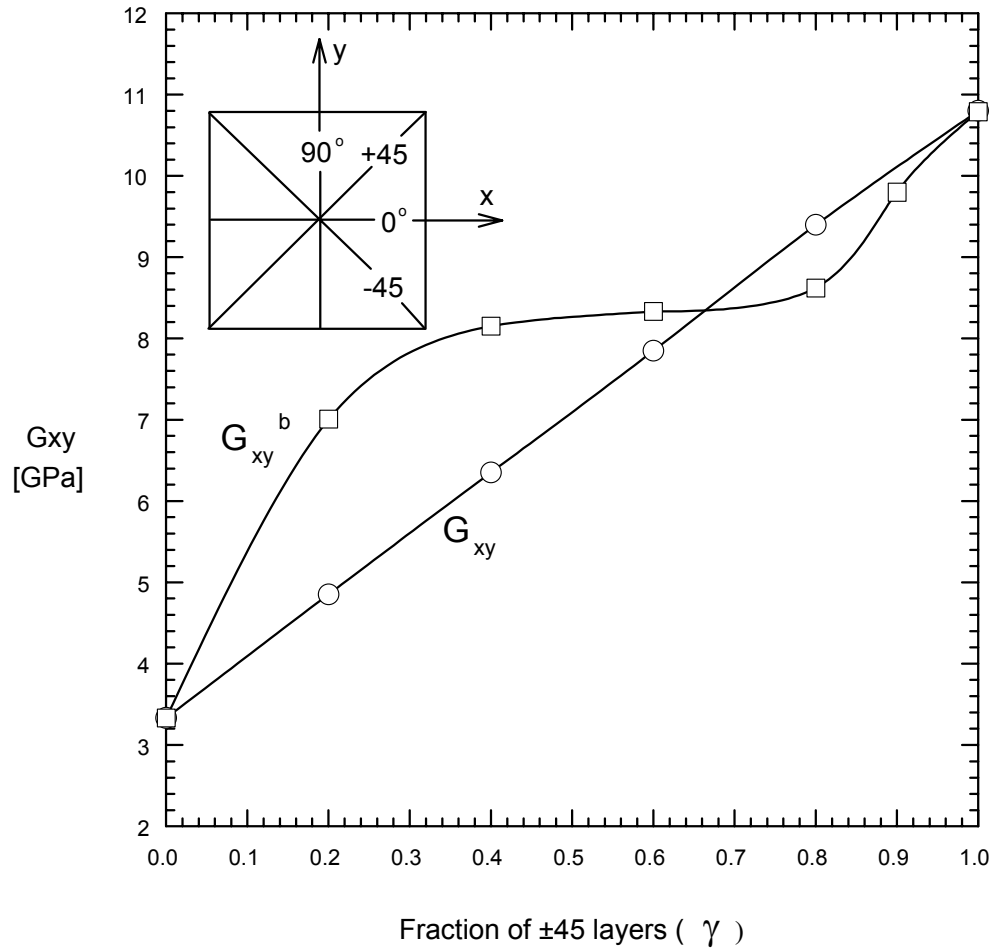
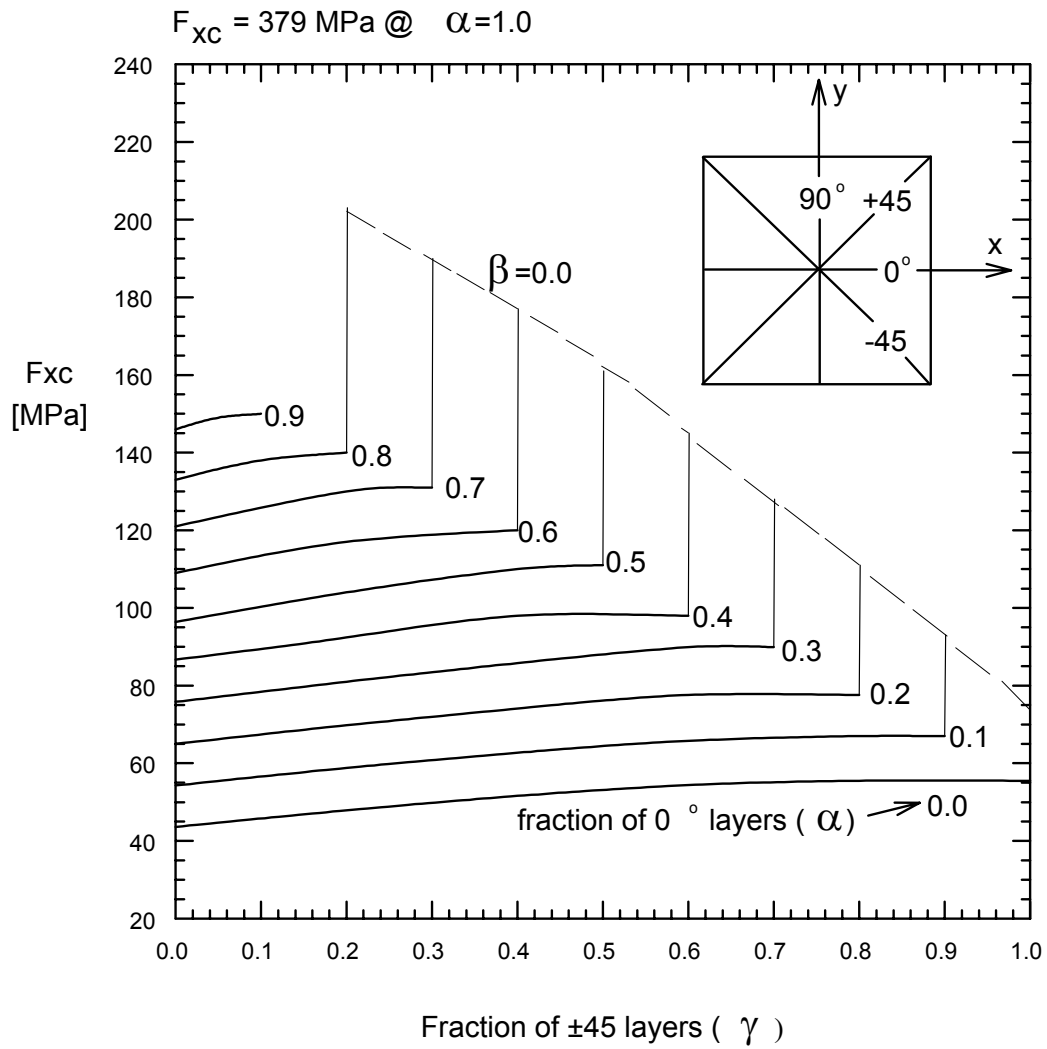


Figure 5a. Carpet Plot for Panel Longitudinal Stiffness,  $E_x$  (Barbero 1999)

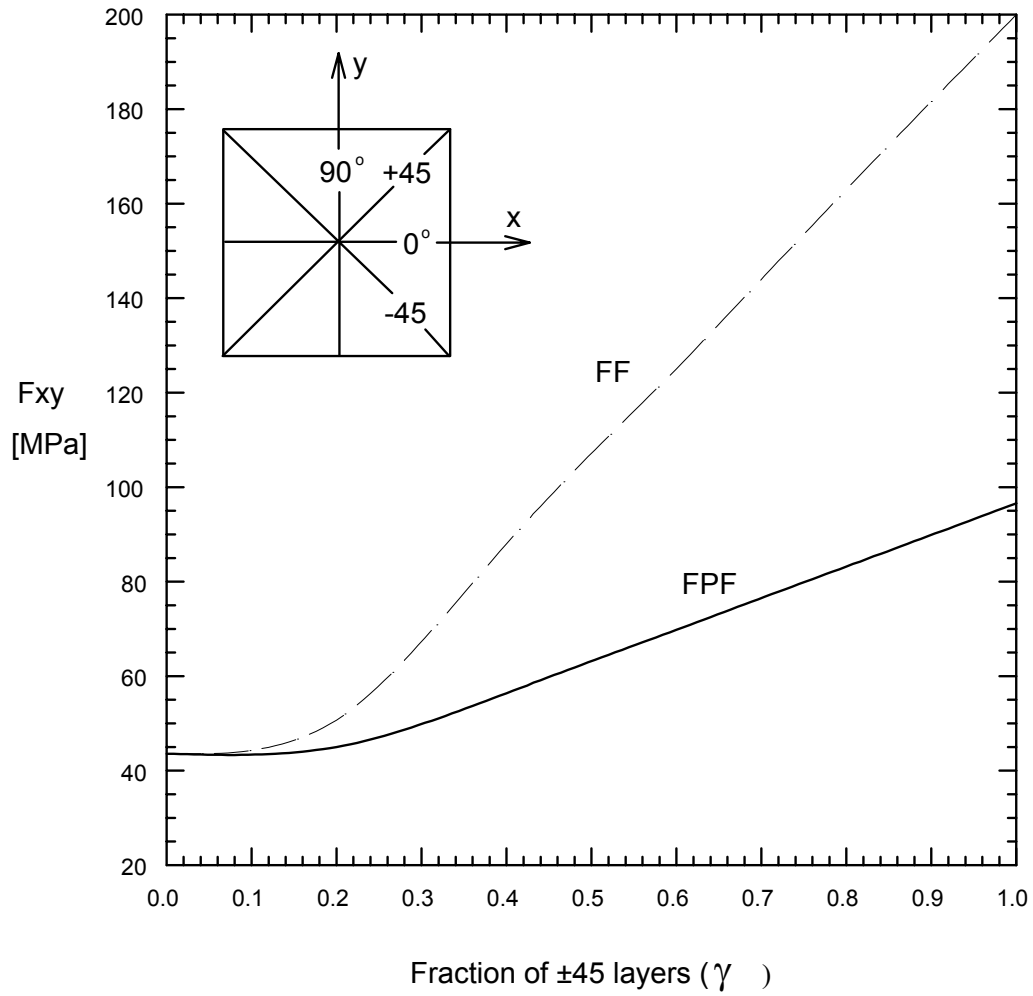


**Figure 5b. Carpet Plot for Panel Shear Stiffness, in plane  $G_{xy}$  and twisting  $G_{xy}^b$  (Barbero 1999)**





**Figure 6a. Carpet Plot for Panel Compressive Strength,  $F_{xc}$  (Barbero 1999)**



**Figure 6b. Carpet Plot for Panel Shear Strength,  $F_{xy}$ , showing both First Ply Failure (FPF) and Fiber Failure (FF) (Barbero 1999)**

**Table 1. Panel Stiffness Properties of FRP Shapes**

FRP Shapes	$E_{xx}$ ( $\times 10^6$ psi)		$G_{xy}$ ( $\times 10^6$ psi)	
	Tension test	Micro/macro-Mechanics	Iosipescu test	Micro/macro-Mechanics
<b>WF6x6</b>	4.155 (COV = 5.28%)	4.206	0.686 (COV = 8.39%)	0.682
<b>I4x8</b>	5.037 (COV = 2.24%)	4.902	0.745 (COV = 9.79%)	0.794
<b>WF4x4</b>	4.391 (COV = 5.55%)	4.167	0.778 (COV = 11.28%)	0.676
<b>Box4x4</b>	4.295 (COV = 10.70%)	3.604	0.548 (COV = 8.39%)	0.550

**Table 2. Panel Strength Properties of FRP Shapes**

FRP Shapes	$F_{cx}$ ( $\times 10^3$ psi)		$F_{xy}$ ( $\times 10^3$ psi)
	Compression test	Strength (Barbero et al. 1999)	Iosipescu test
<b>WF6x6</b>	54.498 (COV = 3.76%)	45.55	12.866 (COV = 2.36%)
<b>I4x8</b>	61.060 (COV = 2.41%)	56.65	13.022 (COV = 8.13%)
<b>WF4x4</b>	57.133 (COV = 3.50%)	53.10	13.167 (COV = 29.17%)
<b>Box4x4</b>	60.657 (COV = 5.33%)	47.20	11.138 (COV = 4.84%)

(NOTE: COV= Coefficient of Variation)

**Table 3. Panel Engineering Properties of 23 FRP shapes**

Section	$E_{xx}$ ( $\times 10^6$ psi)	$E_{yy}$ ( $\times 10^6$ psi)	$G_{xy}$ ( $\times 10^6$ psi)	$\nu_{xy}$	$\nu_{yx}$
Channel 6"x1-5/8"x1/4" ( <b>C6x2</b> )	3.728	1.843	0.651	0.359	0.177
Channel 3"x7/8"x1/4" ( <b>C3x1</b> )	2.941	1.646	0.575	0.372	0.208
Channel 4"x1-1/8"x1/4" ( <b>C4x1</b> )	2.857	1.633	0.568	0.373	0.213
Channel 8"x2-3/16"x3/8" ( <b>C8x2</b> )	3.292	1.616	0.570	0.360	0.177
Channel 10"x2-3/4"x1/2" ( <b>C10x3</b> )	3.704	1.709	0.606	0.355	0.164
WF-Beam 4"x4"x1/4" ( <b>WF4x4</b> )	4.167	1.471	0.676	0.359	0.126
WF-Beam 6"x6"x3/8" ( <b>WF6x6</b> )	4.206	1.482	0.682	0.353	0.127
I-Beam 4"x8"x3/8" ( <b>I4x8</b> )	4.902	1.754	0.794	0.346	0.124
I-Beam 3"x6"x3/8" ( <b>I3x6</b> )	5.012	1.814	0.821	0.342	0.124
I-Beam 2"x4"x1/4" ( <b>I2x4</b> )	4.479	1.575	0.717	0.353	0.124
I-Beam 6"x12"x1/2" ( <b>I6x12</b> )	4.292	1.600	0.758	0.367	0.137
Rectangular Tube 4"x6"x1/4" ( <b>Box4x6</b> )	3.448	1.688	0.595	0.360	0.176
Rectangular Tube 4"x7"x1/4" ( <b>Box4x7</b> )	3.077	1.600	0.561	0.364	0.189
Square Tube 1-3/4"x1-3/4"x1/4" ( <b>Box1x1</b> )	3.773	1.587	0.571	0.342	0.144
Square Tube 2"x2"x1/8" ( <b>Box2x2T8</b> )	3.738	1.587	0.571	0.343	0.146
Square Tube 2"x2"x1/4" ( <b>Box2x2T4</b> )	3.703	1.562	0.563	0.343	0.145
Square Tube 3"x3"x1/4" ( <b>Box3x3</b> )	3.604	1.538	0.552	0.346	0.148
Square Tube 4"x4"x1/4" ( <b>Box4x4</b> )	3.604	1.533	0.550	0.345	0.147
WF 8"x8"x3/8" ( <b>WF8x8</b> )	4.645	1.646	0.749	0.348	0.123
WF Beam 3"x3"x1/4" ( <b>WF3x3</b> )	3.571	1.444	0.522	0.339	0.137
WF Beam 12"x12"x1/2" ( <b>WF12x12A</b> )	3.773	1.905	0.913	0.398	0.201
WF Beam 12"x12"x1/2" ( <b>WF 12x12C</b> )	4.049	2.114	0.760	0.302	0.157
WF Beam 12"x12"x1/2" ( <b>WF12x12T</b> )	4.264	1.587	0.749	0.369	0.137

### 5.2.2 Beam Stiffness Properties

The response of FRP shapes in bending is evaluated using the Mechanics of thin-walled Laminated Beams (MLB) (Barbero et al. 1993; Davalos et al. 1996). After simplifying the MLB formulations, explicit expressions are presented in terms of panel engineering properties for beam bending and shear stiffness coefficients which in return, can be used in simplified equations for prediction of beam deflections, strains and stresses under bending.

For each laminated wall (e.g., a flange or a web), the stiffness values are obtained either by the micro/macromechanics approach or from coupon tests or from *Carpet Plots*. Incorporating stress resultant assumptions compatible with beam theory without torsion and assuming that the off-axis plies of pultruded panels are balanced symmetric (no extension-shear and bending-twist couplings are present), the extensional, bending, and shear stiffnesses of the  $i$ th panel are expressed as:

$$\bar{A}_i = (E_x)_i t_i, \quad \bar{D}_i = (E_x)_i t_i^3 / 12, \quad \bar{F}_i = (G_{xy})_i t_i \quad (2)$$

Assuming that the beam centroid is the neutral axis of bending (no beam bending-extension coupling), general expressions for the beam bending ( $D$ ) and shear stiffness ( $F$ ) coefficients are computed:

$$D = \sum_{i=1}^n \left[ (E_x)_i t_i \left( h_i^2 + \frac{b_i^2}{12} \sin^2 \phi_i \right) + \frac{(E_x)_i t_i^3}{12} \cos^2 \phi_i \right] b_i, \quad F = \sum_{i=1}^n (G_{xy})_i t_i b_i \sin^2 \phi_i \quad (3)$$

where  $h_i$  is the transverse coordinate of a panel centroid from the neutral axis,  $b_i$  is the panel width,  $t_i$  is the panel thickness and  $\phi_i$  is the cross-sectional orientation of the  $i$ th panel with respect to the bending axis;  $(E_x)_i$  and  $(G_{xy})_i$  are the panel stiffness values obtained either by the micro/macromechanics approach or from coupon tests. Note that  $D$  and  $F$ , respectively, are similar to  $EI$  and  $GA$  for isotropic materials (e.g., steel beams). If the flanges and webs of a section have identical lay-ups and stiffnesses, the beam bending and shear stiffnesses can be expressed simply in terms of panel stiffnesses  $E_x$  and  $G_{xy}$  and geometric properties  $I$  and  $A$ . For example, the beam stiffnesses for two common sections consisting of "I" (about the strong-axis) and box geometries are:

$$\begin{aligned}
\text{"I": } D &= \frac{1}{2}(E_x)_f t_f b_w^2 b_f + \frac{1}{12}(E_x)_w t_w b_w^3 + \frac{1}{6}(E_x)_f t_f^3 b_f, \quad F = (G_{xy})_w t_w b_w \\
\text{Box: } D &= \frac{1}{2}(E_x)_f t_f b_w^2 b_f + \frac{1}{6}(E_x)_w t_w b_w^3 + \frac{1}{6}(E_x)_f t_f^3 b_f, \quad F = 2(G_{xy})_w t_w b_w
\end{aligned} \tag{4}$$

where, subscripts  $f$  and  $w$  identify flange and web components, respectively.

### 5.2.3 Beam deflections and strains and stresses

Displacement and rotation functions can be obtained by solving Timoshenko's beam theory equilibrium equations (Davalos et al. 1996). In particular, available expressions for maximum bending and shear deflections can be used; for example, the maximum deflection for a 3-point loading of a beam of span  $L$  and design load  $P$  is:

$$\delta = \delta_b + \delta_s = \frac{P L^3}{48D} + \frac{PL}{4 K_Y F} \tag{5}$$

where, the bending ( $\delta_b$ ) and shear ( $\delta_s$ ) components of deflection can be independently evaluated; as an approximation in design, the shear correction factor for most FRP sections can be taken as  $K_Y = 1.0$  (Davalos et al. 1996). The maximum top-surface longitudinal strains and in-plane shear strains of the  $i$ th panel are expressed as:

$$\varepsilon_x = \frac{M}{D} y_i \quad \text{and} \quad \gamma_{xy} = \frac{V}{F} \sin \phi_i \tag{6}$$

where  $V$  and  $M$  are, respectively, the resultant internal shear force and bending moment acting on the beam;  $y_i$  is the transverse coordinate of a point from the neutral axis. Based on Eq. (3), Table 4 lists the bending and shear stiffnesses of four selected beams, and Table 5 shows comparisons between predictions from equations (5) and (6) and experimental measurements.

**Table 4. Beam Bending and Shear Stiffness Properties**

FRP Shapes	D = EI (x 10 <sup>8</sup> psi•in. <sup>4</sup> )		F = GA (10 <sup>6</sup> psi•in. <sup>2</sup> )	
	Strong-Axis	Weak-Axis	Strong-Axis	Weak-Axis
<b>WF6x6</b>	1.776	0.570	1.292	3.066
<b>I4x8</b>	2.558	0.199	1.772	2.379
<b>WF4x4</b>	0.334	0.111	0.585	1.351
<b>Box4x4</b>	0.364	0.338	1.100	1.176

**Table 5. Beam Deflections and Strains (L = 12.0 ft)**

FRP Shapes	Axis of Load	Deflection $\delta$ (in/kip)		Strain $\varepsilon_{Top}$ ( $\mu\varepsilon/kip$ )		Strain $\varepsilon_{Bottom}$ ( $\mu\varepsilon/kip$ )	
		Test	Design	Test	Design	Test	Design
<b>WF6x6</b>	Strong	0.388	0.378	-616.9	-608.2	668.6	608.2
	Weak	1.169	1.106	-1902.7	-1900.0	1823.5	1900.0
<b>I4x8</b>	Strong	0.271	0.264	-576.6	-566.0	594.1	566.0
	Weak	3.511	3.152	-3646.8	-3630.0	3557.6	3630.0
<b>WF4x4</b>	Strong	1.833	1.926	-2081.3	-2160.0	2121.8	2160.0
	Weak	5.769	5.627	-5879.6	-6480.0	5913.9	6480.0
<b>Box4x4</b>	Strong	1.886	1.742	-2139.8	-1990.0	2141.7	1990.0
	Weak	1.947	1.873	-1944.4	-2140.0	1903.9	2140.0

### 5.2.4 Beam Local and Global Buckling

A comprehensive analytical approach was developed to study the local buckling behaviors of pultruded FRP shapes (Qiao 1997). The local buckling analysis for discrete laminated plates or panels of FRP shapes was formulated, and the effects of restraint at the flange-web connection

were considered (Figure 7). For the flange panels under compression, simplified expressions for predictions of plate buckling strength are proposed by approximately solving transcendental equations (Qiao 1997):

$$\sigma_x^{cr} = \frac{\pi^2}{12} \left( \frac{t_f}{b} \right)^2 \left[ \sqrt{q} \left( 2 \sqrt{(E_x)_f (E_y)_f} \right) + p \left( (E_y)_f (v_{xy})_f + 2(G_{xy})_f \right) \right] \quad (7)$$

where,  $\sigma_x$  is the critical stress, and  $p$  and  $q$  are constants that are defined by the coefficient of restraint ( $\zeta$ ) at the junction of panels:

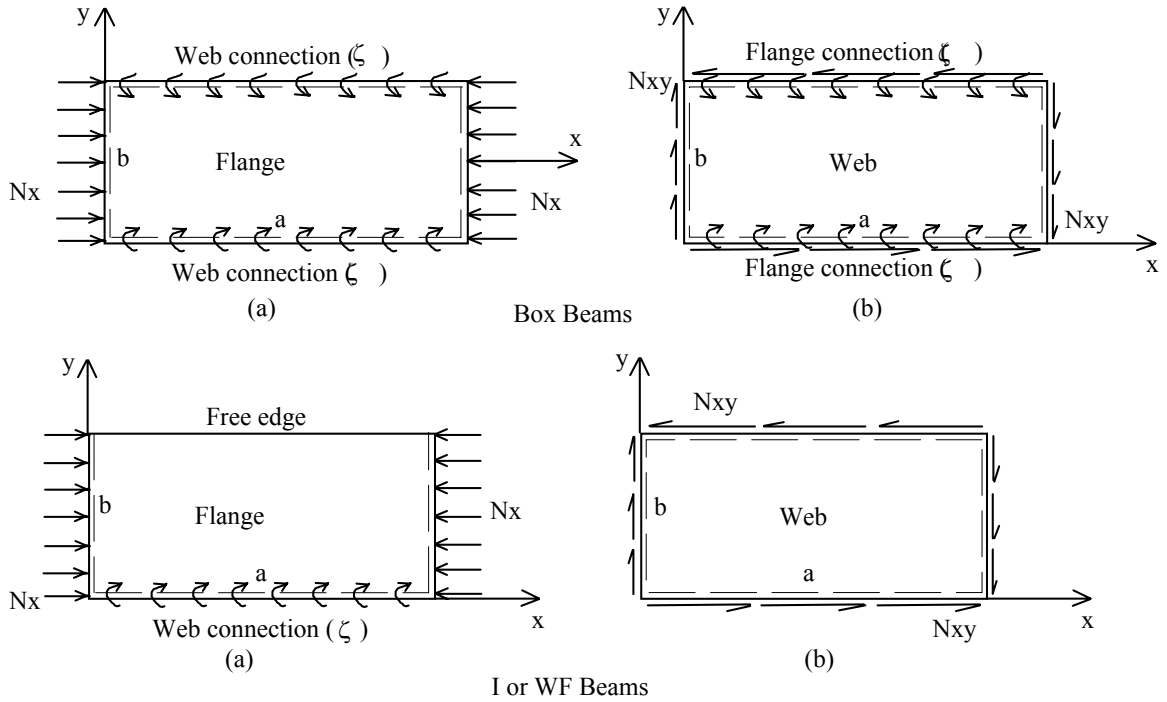
$$\begin{aligned} \text{"I"}: \quad p &= 0.3 + \frac{0.004}{\zeta - 0.5}; \quad q = 0.025 + \frac{0.065}{\zeta + 0.4}; \quad \zeta = \frac{2b_w}{b_f} \frac{(E_y)_f}{(E_y)_w}; \quad b = \frac{b_f}{2} \\ \text{Box}: \quad p &= 2.0 + \frac{0.002}{\zeta - 1.3}; \quad q = 1.0 + \frac{0.08}{\zeta + 0.2}; \quad \zeta = \frac{b_w}{b_f} \frac{(E_y)_f}{(E_y)_w}; \quad b = b_f \end{aligned}$$

For a beam under 3-point bending, the critical local buckling load ( $P_{cr}^{local}$ ) can be obtained in terms of critical stress and beam properties as:

$$P_{cr}^{local} = \frac{8D\sigma_x^{cr}}{(E_x)_f b_w L} \quad (8)$$

An experimental program (Qiao and Davalos 1999) is designed to obtain the local buckling load for beams under bending (Figure 8 for WF12x12 beam), and the Southwell method is used as a data reduction method to obtain the critical buckling loads (Figure 9). As shown in Table 6, the local buckling design values based on Eqs. (7) and (8) compare favorably with testing data for four wide-flange beams.

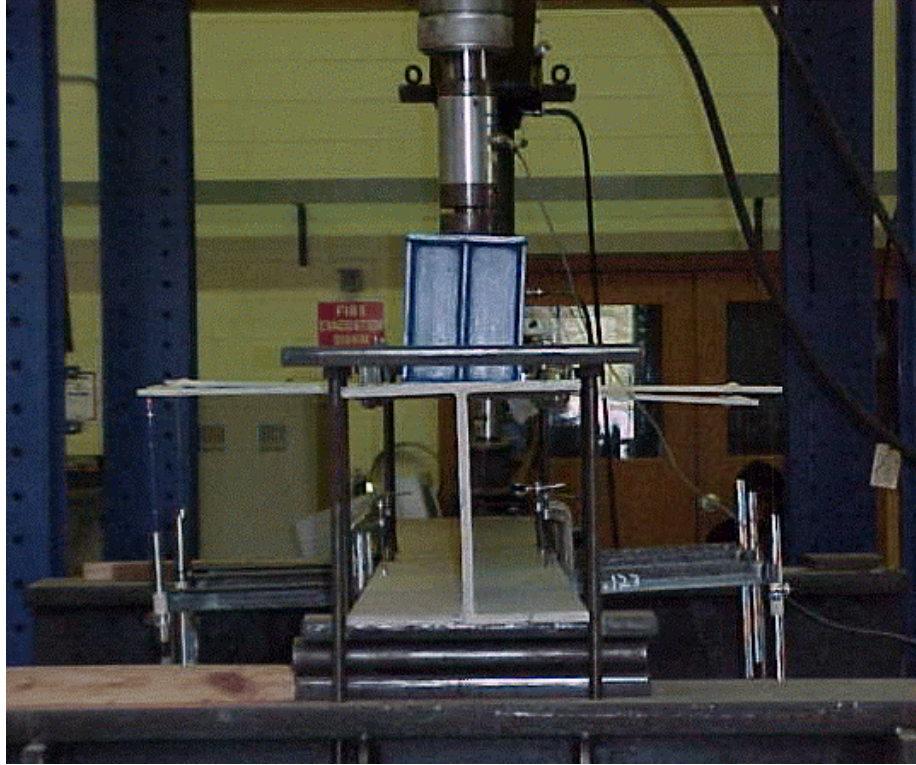




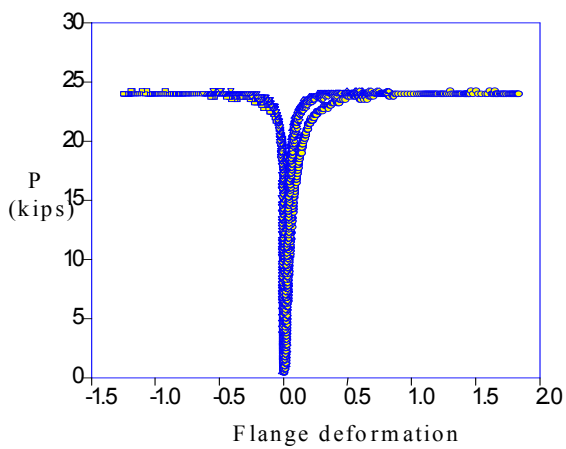
**Figure 7. Modeling of Local Buckling of FRP Shapes**

**Table 6. Beam Critical Local and Global Buckling Loads**

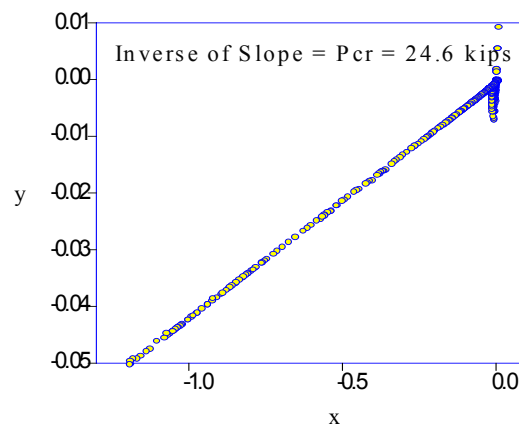
FRP Shapes	Local Buckling, $P_{cr}^{local}$			Global Buckling, $P_{cr}^{global}$		
	Span (ft)	Test (kips)	Design (kips)	Span (ft)	Test (kips)	Design (kips)
<b>WF4x4</b>	6.0	8.83	6.46	-	-	-
<b>WF6x6</b>	6.0	23.13	20.62	12	5.60	4.24
<b>WF8x8</b>	6.0	24.53	22.04	12	12.93	12.78
<b>WF12x12</b>	6.0	29.27	33.33	14.5	30.01	38.25



**Figure 8. Experimental Testing of Wide-Flange Beam for Local Buckling**



(a) Relative top flange displacement



(b) Southwell method to determine the critical load

**Figure 9. Data Reduction Technique for Local Buckling Load**

For long-span FRP beams without lateral supports and with large slenderness ratios, global buckling is prone to happen. Based on Vlasov's theory (Pandey et al. 1995), a simplified engineering equation for flexural-torsional buckling of an "I" section under a concentrated mid-span load is expressed as:

$$P_{cr}^{global} = \frac{17.17}{L^2} \sqrt{D \cdot JG} \sqrt{1 + \frac{\pi^2}{L^2} \frac{I_{ww}}{JG}} \quad (9)$$

where,

$$JG = \frac{2(G_{xy})_f t_f^3 b_f}{3} + \frac{(G_{xy})_w t_w^3 b_w}{3};$$

$$I_{ww} = \frac{(E_x)_f t_f b_w^2 b_f^3}{24} + \frac{(E_x)_f t_f^3 b_f^3}{36} + \frac{(E_x)_w t_w^3 b_w^3}{144}$$

Again, the predicted design values [Eq. (9)] are compared with experimental test values in Table 6.

### 5.2.5 Beam Ultimate Bending and Shear Failure

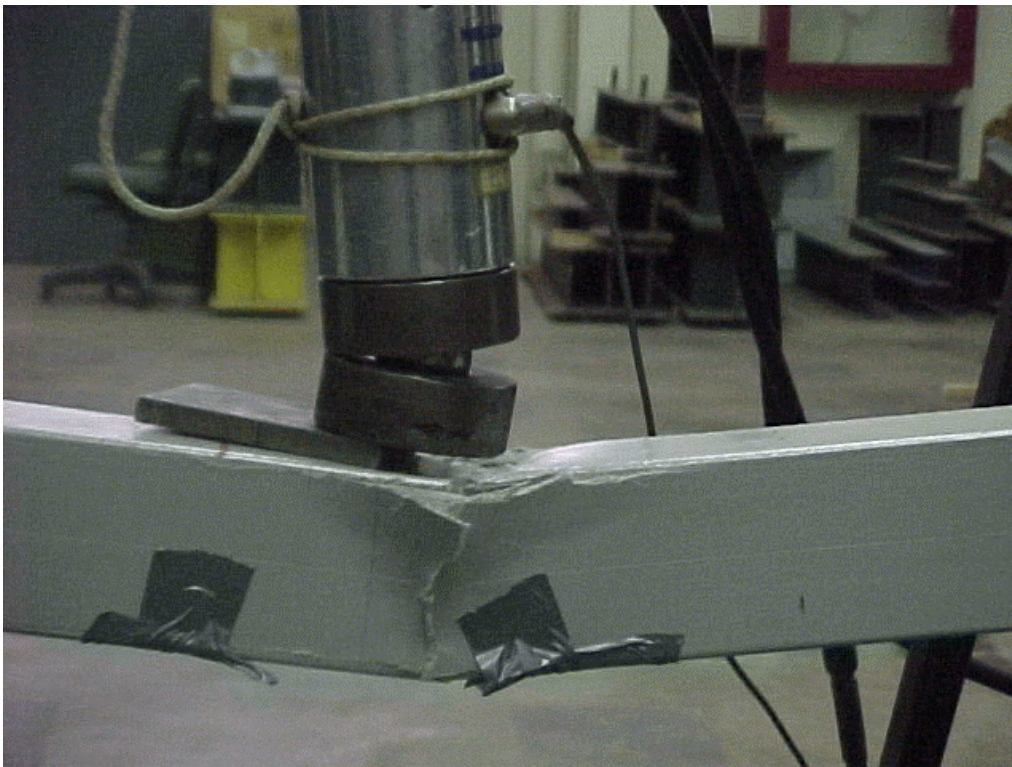
Due to the relatively low compressive and shear strength properties of FRP composites, the material failure needs to be evaluated in design. Similar to beam deflection and buckling, the beam bending and shear strengths (ultimate failure loads) for 3-point bending can be expressed in terms of panel strength properties as:

$$Bending: P_{fail}^{bending} = \frac{8F_c D}{(E_x)_f (b_w - t)L}; \quad Shear: P_{fail}^{shear} = 2F_{xy} b_w t_w \quad (10)$$

where,  $F_c$  and  $F_{xy}$  are given in Table 2. The design values and testing results are given in Table 7. Figure 10 shows a box beam (Box4x4) under compressive failure.

**Table 7. Beam Ultimate Bending and Shear Loads**

FRP Shapes	Bending, $P_{fail}^{bending}$			Shear, $P_{fail}^{shear}$		
	Span (ft)	Test (kips)	Design (kips)	Span (ft)	Test (kips)	Design (kips)
<b>I3x6</b>	9.5	11.52	12.10	2.0	22.00	28.95
<b>I4x8</b>	9.5	20.28	24.50	-	-	-
<b>WF4x4</b>	9.5	5.25	7.52	2.0	12.80	13.11
<b>Box4x4</b>	9.5	5.79	7.98	2.0	17.40	22.27



**Figure 10. A Box Beam (Box4x4) under Compressive Failure**

### **5.3 Design Procedures for FRP Beams**

To facilitate the design of FRP beams under bending, the following design guidelines are recommended:

- 1) Characterize the beam panel material properties (stiffness and strength) from either coupon tests or micro/macromechanics and empirical formulas or *Carpet Plots*.
- 2) From Eq. (3), obtain the beam bending and shear stiffness coefficients, which in turn can be used to predict the beam deflection and strains and stresses.
- 3) Determine the local and global buckling resistance of beam sections by Eqs. (8) and (9), respectively.
- 4) Predict the beam failure (bending and shear) loads based on the panel strength data and Eq. (10).

Following the above design procedures, two examples of beam analysis are illustrated in the next section of Example and Application Module.

## 6. EXAMPLE AND APPLICATION MODULE

According to the guidelines listed in Section 5, two FRP shapes: an I-beam and a box beam, are analyzed to illustrate the design procedure and application of step-by-step beam design equations.

### 6.1 Example 1: Design of a FRP I-Beam

**Problem Statement:** Analyze a thin-walled composite material I-beam based on the following data and the Carpet Plots shown in Figures 5 and 6:

**Beam Dimensions:** 0.2 m x 0.4 m x 0.02 m (**Figure 11**)

**Panel Material and Lay-up:** E-glass Fiber with Polyester ( $V_f = 50.0\%$ )

Unidirectional fiber layer ( $0^\circ$ ):  $\alpha = 60\%$

$90^\circ$  fiber layer:  $\beta = 10\%$

$\pm 45^\circ$  fiber layer:  $\gamma = 30\%$

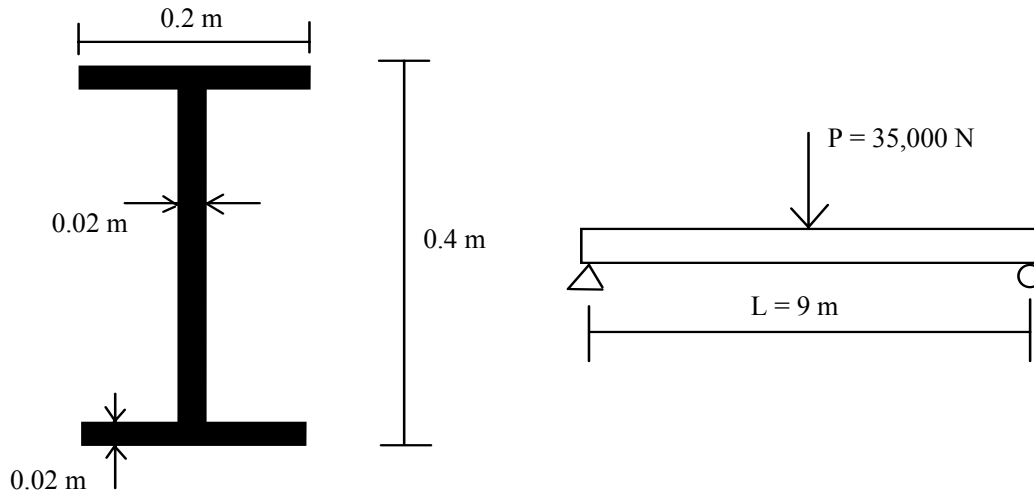
Uniform panel thickness and material lay-up throughout the beam.

**Beam Span:**

$L = 9.0 \text{ m}$

**Loading type:**

Beam under 3-point bending with central concentrated load  
 $P = 35,000 \text{ N}$  (**Figure 11**)



**Figure 11. FRP I-beam Dimensions and Loading**

**Requirements:** Evaluate this section as follows:

- 1) Based on the *Carpet Plots*, obtain the panel equivalent moduli and strength values.
- 2) Compute the beam bending ( $D = EI$ ) and shear ( $F = GA$ ) stiffness values.
- 3) Compute the beam maximum deflection, and also bending and shear deflection components.
- 4) Compute the beam maximum bending and shear stresses and corresponding Factors of Safety (F.S.).
- 5) Compute the local buckling strength ( $\sigma_x^{cr}$ ) of the beam top flange and the beam local buckling load. Also check the corresponding F.S.
- 6) Compute the critical global buckling load ( $P_{cr}^{global}$ ) and obtain the corresponding F.S.

**Solution:** Based on the Carpet Plots (Figures 5 and 6) and design guidelines given in Section 5, the following solutions are obtained:

**1) Panel Engineering Properties:**

Carpet Plots:

$$\begin{aligned} \text{Unidirectional Fiber (0}^\circ\text{) Layer:} & \quad \alpha = 60\%; \\ \text{+/-45}^\circ\text{ Fabrics:} & \quad \gamma = 30\%; \\ \text{Transverse Fiber (90}^\circ\text{) Layer:} & \quad \beta = 10\%. \end{aligned}$$

Panel Stiffness Properties:

$$E_x = 26.8 \text{ GPa (Figure 5a) and } G_{xy} = 5.6 \text{ GPa (Figure 5b)}$$

Panel Strength Properties:

$$F_{xc} = 120 \text{ MPa (Figure 6a) and } F_{xy} = 50 \text{ MPa (Figure 6b)}$$

**2) Beam Stiffness Coefficients:**

From Eq. (4), the beam bending and shear stiffnesses:

$$"I": \quad D = \frac{1}{2}(E_x)_f t_f b_w^2 b_f + \frac{1}{12}(E_x)_w t_w b_w^3 + \frac{1}{6}(E_x)_f t_f^3 b_f, \quad F = (G_{xy})_w t_w b_w$$

$$\text{where: } (E_x)_f = (E_x)_w = E_x = 26.8 \text{ GPa; } (G_{xy})_f = (G_{xy})_w = G_{xy} = 5.6 \text{ GPa;}$$

$$t_f = t_w = 0.02 \text{ m; } b_f = 0.2 \text{ m; } b_w = 0.4 \text{ m;}$$

Beam Bending Stiffness:

$$D = EI = 11.442 \text{ MPa} \cdot \text{m}^4.$$

Beam Shear Stiffness:

$$F = GA_w = 44.80 \text{ MPa} \cdot \text{m}^2.$$

**3) Beam Deflections:**

Beam Maximum Deflection [Eq. (5)]:

$$\delta_{\max} = \delta_b + \delta_s = \frac{P L^3}{48D} + \frac{PL}{4 K_Y F}; \quad (\text{where, assume } K_Y \approx 1.0)$$

$$\delta_{\max} = 0.04820 \text{ m} = 48.220 \text{ mm.}$$

Beam Deflection due to Bending:

$$\delta_b = \frac{P L^3}{48D};$$

$$\delta_b = 0.04247 \text{ m} = 42.480 \text{ mm.}$$

Beam Deflection due to Shear:

$$\delta_s = \frac{PL}{4 K_Y F};$$

$$\delta_s = 0.00161 \text{ m} = 1.607 \text{ mm.}$$

#### 4) Beam Stresses:

Maximum Resultant Moment and Shear Force:

$$\text{For a beam under 3-point bending: } M_{\max} = \frac{PL}{4} \text{ and } V_{\max} = \frac{P}{2};$$

where:  $P = 35,000 \text{ N}$  and  $L = 9.0 \text{ m}$ .

$$M_{\max} = 78,750 \text{ N}\cdot\text{m} \text{ and } V_{\max} = 17,500 \text{ N.}$$

Maximum Bending and Shear Strains [Eq.(6)]:

$$\varepsilon_x = \frac{M}{D} y_i \quad \text{and} \quad \gamma_{xy} = \frac{V}{F} \sin \phi_i ;$$

where:  $y_i = b_w/2 = 0.2 \text{ m}$  (maximum compressive bending strain occurs on the top flange);  $\phi_i = 90^\circ$  (maximum shear strain in the web panel)

$$\varepsilon_x = 1,376 \text{ } \mu\text{e} \text{ and } \gamma_{xy} = 390.6 \text{ } \mu\text{e};$$

Approximate Maximum Bending and Shear Stresses:

$$\sigma_x \approx E_x \varepsilon_x \text{ and } \tau_{xy} \approx G_{xy} \gamma_{xy} ;$$

$$\sigma_x \approx 36.877 \text{ MPa}; \tau_{xy} \approx 2.187 \text{ MPa.}$$

Factors of Safety (F.S.) for Design:

$$(F.S.)_{\text{bending}} = \frac{F_{xc}}{\sigma_x} \quad \text{and} \quad (F.S.)_{\text{shear}} = \frac{F_{xy}}{\tau_{xy}} ;$$

$$(F.S.)_b = 3.25 \text{ and } (F.S.)_s = 22.86 \Rightarrow \text{Safe Design}$$

#### 5) Beam Local Buckling Load:

Critical Local Buckling Stresses [(Eq.(7))]:



$$\sigma_x^{cr} = \frac{\pi^2}{12} \left( \frac{t_f}{b} \right)^2 \left[ \sqrt{q} \left( 2 \sqrt{(E_x)_f (E_y)_f} \right) + p \left( (E_y)_f (\nu_{xy})_f + 2(G_{xy})_f \right) \right]$$

where: for I-beam:  $\zeta = \frac{2b_w (E_y)_f}{b_f (E_y)_w} = 4.0$ ;  $b = \frac{b_f}{2} = 0.1 \text{ m}$ ;

"I":  $p = 0.3 + \frac{0.004}{\zeta - 0.5} = 0.301$ ;  $q = 0.025 + \frac{0.065}{\zeta + 0.4} = 0.0397$ .

$(E_y)_f \approx (E_x)_f/6 = 4.467 \text{ GPa}$  (The unidirectional fiber along the  $y$ -axis is about one-sixth of one along the  $x$ -axis) and  $(\nu_{xy})_f = 0.35$  (assumed, which also can be obtained from Carpet Plots).

$\sigma_x^{cr} = 269.83 \text{ MPa}$ .

Critical Beam Buckling Load [Eq.(8)]:

$$P_{cr}^{local} = \frac{8D\sigma_x^{cr}}{(E_x)_f b_w L}$$

$P_{cr}^{local} = 256,005 \text{ N} > P = 35,000 \text{ N}$

Factor of Safety for Local Buckling:

$$(F.S.)_{Local} = \frac{P_{cr}^{local}}{P} = 7.31 \Rightarrow \text{Safe Design for Local Buckling.}$$

## 6) Beam Global Buckling Load:

Critical Beam Global Buckling Load [Eq.(9)]:

$$P_{cr}^{global} = \frac{17.17}{L^2} \sqrt{D \cdot JG} \sqrt{1 + \frac{\pi^2}{L^2} \frac{I_{ww}}{JG}}$$

where:  $JG = \frac{2(G_{xy})_f t_f^3 b_f}{3} + \frac{(G_{xy})_w t_w^3 b_w}{3} = 11947 \text{ Pa} \cdot \text{m}^4$ ;

$$I_{ww} = \frac{(E_x)_f t_f b_w^2 b_f^3}{24} + \frac{(E_x)_f t_f^3 b_f^3}{36} + \frac{(E_x)_w t_w^3 b_w^3}{144} = 28730 \text{ Pa} \cdot \text{m}^6.$$

$P_{cr}^{global} = 83,370 \text{ N}$

Factor of Safety for Global Buckling:

$$(F.S.)_{\text{global}} = \frac{P_{cr}^{\text{global}}}{P} = 2.44 \Rightarrow \text{Safe Design for Global Buckling}$$

## 7) Summary

Based on the step-by-step procedures shown above for the I-beam performance under a mid-span design load, it is concluded that the beam satisfies the design requirements.

## 6.2 Example 2: Design of a FRP Box Beam

**Problem Statement:** Analyze an existing pultruded FRP structural box-beam (Box4x4) based on the following data and the panel engineering properties given in Tables 1 and 2:

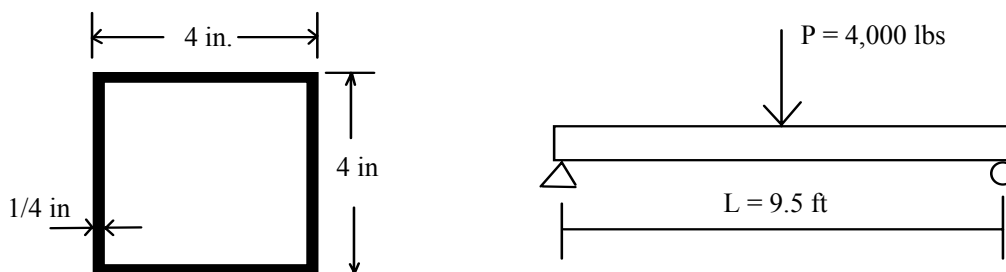
**Beam Dimensions:** 4 in x 4 in x 1/4 in (**Figure 12**)

**Panel Material and Lay-up:** E-glass Fiber with Polyester

Uniform panel thickness and material lay-up throughout the beam, and the experimental coupon test values shown in Tables 1 and 2 will be used in the design analysis

**Beam Span:** L = 9.5 ft

**Loading type:** Beam under 3-point bending with central concentrated load  
P = 4,000 lbs (**Figure 12**)



**Figure 12. FRP Box-beam (Box4x4) Dimensions and Loading**

**Requirements:** Based on the design procedure given in Section 5.3, perform the following design analysis:

- 1) Based on the experimental coupon test results of panels, summarize the panel engineering properties.

- 2) Compute the beam bending ( $D = EI$ ) and shear ( $F = GA$ ) stiffnesses values.
- 3) Compute the beam maximum deflection.
- 4) Predict the beam maximum bending and shear strains.
- 5) Predict the local buckling strength ( $\sigma_x^{cr}$ ) of the beam top flange and the beam local buckling load. Also check the corresponding F.S.
- 6) Predict the beam failure (bending and shear) loads.

**Design Solution:** Based on the panel experimental test data (Tables 1 and 2) and design guidelines given in Section 5, the following design analysis for the box-beam is performed:

**1) Panel Engineering Properties:**

From Panel Coupon Test Data:

Box Panel Stiffness Properties (Table 1):

$$E_x = 4.295 \times 10^6 \text{ psi and } G_{xy} = 0.548 \times 10^6 \text{ psi.}$$

Box Panel Strength Properties:

$$F_{xc} = 60.657 \times 10^3 \text{ psi and } F_{xy} = 11.138 \times 10^3 \text{ psi.}$$

**2) Beam Stiffness Coefficients:**

From Eq. (4), the beam bending and shear stiffnesses:

$$\text{"Box": } D = \frac{1}{2}(E_x)_f t_f b_w^2 b_f + \frac{1}{6}(E_x)_w t_w b_w^3 + \frac{1}{6}(E_x)_f t_f^3 b_f, \quad F = 2(G_{xy})_w t_w b_w$$

$$\text{where: } (E_x)_f = (E_x)_w = E_x = 4.295 \times 10^6 \text{ psi; } (G_{xy})_f = (G_{xy})_w = G_{xy} = 0.548 \times 10^6 \text{ psi;}$$

$$t_f = t_w = 0.25 \text{ in; } b_f = b_w = 4.0 \text{ in;}$$

Beam Bending Stiffness:

$$D = EI = 4.586 \times 10^7 \text{ psi} \cdot \text{in}^4.$$

Beam Shear Stiffness:

$$F = GA_w = 1.096 \times 10^6 \text{ psi} \cdot \text{in}^2.$$

**3) Beam Deflections:**

Beam Maximum Deflection [Eq. (5)]:

$$\delta_{\max} = \delta_b + \delta_s = \frac{PL^3}{48D} + \frac{PL}{4K_Y F}; \text{ (where, assume } K_Y \approx 1.0)$$

$$\delta_{\max} = 2.692 + 0.104 = 2.796 \text{ in.}$$

#### 4) Beam Maximum Strains and Stresses:

##### Maximum Resultant Moment and Shear Force:

$$\text{For a beam under 3-point bending: } M_{\max} = \frac{PL}{4} \text{ and } V_{\max} = \frac{P}{2};$$

where:  $P = 4,000 \text{ lb}$  and  $L = 9.5 \text{ ft}$ .

$$M_{\max} = 114,000 \text{ lb}\cdot\text{in} \text{ and } V_{\max} = 2,000 \text{ lb.}$$

##### Maximum Bending and Shear Strains [Eq.(6)]:

$$\varepsilon_x = \frac{M}{D} y_i \quad \text{and} \quad \gamma_{xy} = \frac{V}{F} \sin \phi_i;$$

where:  $y_i = b_w/2 = 2.0 \text{ in.}$  (maximum compressive bending strain occurs on the top flange);  $\phi_i = 90^\circ$  (maximum shear strain in the web panel)

$$\varepsilon_x = 4,972 \mu\varepsilon \text{ and } \gamma_{xy} = 1,825 \mu\varepsilon;$$

##### Approximate Maximum Bending and Shear Stresses:

$$\sigma_x \approx E_x \varepsilon_x \text{ and } \tau_{xy} \approx G_{xy} \gamma_{xy};$$

$$\sigma_x \approx 21,354.7 \text{ psi; } \tau_{xy} \approx 1,000.1 \text{ psi.}$$

##### Factors of Safety (F.S.) for Design:

$$(F.S.)_{\text{bending}} = \frac{F_{xc}}{\sigma_x} \quad \text{and} \quad (F.S.)_{\text{shear}} = \frac{F_{xy}}{\tau_{xy}};$$

$$(F.S.)_b = 2.841 \text{ and } (F.S.)_s = 11.127 \Rightarrow \text{Safe Design}$$

#### 5) Beam Local Buckling Load:

##### Critical Local Buckling Stresses [(Eq.(7)]:

$$\sigma_x^{cr} = \frac{\pi^2}{12} \left( \frac{t_f}{b} \right)^2 \left[ \sqrt{q} \left( 2 \sqrt{(E_x)_f (E_y)_f} \right) + p \left( (E_y)_f (v_{xy})_f + 2(G_{xy})_f \right) \right]$$

$$\text{where: for box-beam: } \zeta = \frac{2b_w (E_y)_f}{b_f (E_y)_w} = 2.0; \quad b = b_f = 4 \text{ in;}$$

$$\text{Box: } p = 2.0 + \frac{0.002}{\zeta - 1.3} = 2.0029; \quad q = 1.0 + \frac{0.08}{\zeta + 0.2} = 1.0364.$$

$(E_y)_f \approx (E_x)_f/10 = 0.430 \times 10^6$  psi. [For this box beam, all of the roving (unidirectional) fiber is oriented along the  $x$ -axis and there are a few CSM layers which contribute to the stiffness along the  $y$ -axis; the transverse stiffness is assumed about one-tenth of the longitudinal stiffness].  $\nu_{xy} \approx 0.3$  (assumed).

$$\sigma_x^{cr} = 16,773 \text{ psi.}$$

Critical Beam Buckling Load [Eq.(8)]:

$$P_{cr}^{local} = \frac{8D\sigma_x^{cr}}{(E_x)_f b_w L}$$

$$P_{cr}^{local} = 3,137.9 \text{ lbs} < P = 4,000 \text{ lbs} \Rightarrow \text{Local Buckling occurs on the Top}$$

Flange

Factor of Safety for Local Buckling:

$$(F.S.)_{Local} = \frac{P_{cr}^{local}}{P} = 0.784 < 1.0 \Rightarrow \text{Not Safe Design for Local Buckling.}$$

## 6) Beam Failure Loads:

Beam Ultimate Bending Failure Load [Eq. (10)]:

$$\text{Bending: } P_{fail}^{bending} = \frac{8F_c D}{(E_x)_f (b_w - t)L}$$

where:  $F_c = 60.657 \times 10^3$  psi.

$$P_{fail}^{bending} = 12,120 \text{ lbs} > P = 4,000 \text{ lbs}$$

Factor of Safety for Material Failure in Bending:

$$(F.S.)_{bending} = \frac{P_{fail}^{bending}}{P} = 3.03 \Rightarrow \text{Safe Design for material failure in bending}$$

Beam Ultimate Shear Failure Load [Eq. (10)]:

$$\text{Shear: } P_{fail}^{shear} = F_{xy} b_w t_w$$

where:  $F_{xy} = 11.138 \times 10^3$  psi.

$$P_{fail}^{shear} = 11,138 \text{ psi.}$$

Factor of Safety for Material Failure in Shear:

$$(F.S.)_{\text{shear}} = \frac{P_{\text{fail}}^{\text{bending}}}{P/2} = 5.57 \Rightarrow \text{Safe Design for material failure in shear}$$

Note: each web is assumed to carry half of the load.

## 7) Summary

In summary, the following two critical issues in design for this box beam should be considered: (1) large deflection,  $\delta = 2.796$  in under  $P = 4,000$  lbs, and (2) beam local buckling load,  $P_{cr}^{\text{local}} = 3,138$  lbs, which is smaller than the design load,  $P = 4,000$  lbs.

## 7. DISSEMINATION OF RESEARCH

Finally, to disseminate the results, a Web page in the Internet has designed and is available at the address: <http://coel.ecgf.uakron.edu/~civil/fclty/qiao/qiao.html> (link to *FRP Structural Shapes*). Also, a one-time course is scheduled to be offered to WVDOT personnel in Charleston, WV.

## 8. CONCLUSIONS

In this report, simplified design parameters and equations for FRP beams are developed, and the accuracy and validity of the design equations are verified by testing eight representative shapes out of 23 sections. A design procedure that accounts for most critical issues in FRP beam design is presented, which can be used in the future to develop general design guidelines and also "*product-acceptance criteria*" for FRP beams produced by any manufacturer. Besides the Design module presented in this report, an Education module has been developed to

conduct seminars for practicing engineers and government officials. Also, an Example and Application module is given to illustrate the usefulness of the proposed step-by-step design equations in engineering practice.

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