



Focus on Fluency:
A Story of Ratios[®]
A Story of Functions[®]
Virtual Engagement Materials

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Focus on Fluency

Directions:

1. Read the following text, and highlight statements that resonate with you.

Why is mathematical fluency so important?

Fluency as a Component of Rigor

The following is reprinted from “Key Shifts in Mathematics” (NGA Center and CSSO, n.d.):

Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity.

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

Conceptual understanding: The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discrete procedures.

Procedural skills and fluency: The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, to access more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.

Application: The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

Choral Counting Exercises

T: Count by ones to 8.

S: 1, 2, 3, 4, 5, 6, 7, 8.

T: Count by eighths to 8 eighths. Start at 0 eighths. (Write as students count.)

S: $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$.

T: 8 eighths is the same as 1 of what unit?

S: 1 one.

T: (Beneath $\frac{8}{8}$, write 1.) Count by eighths again. This time, say 1 one when you arrive at 8 eighths. Start at 0.

S: $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, 1$.

T: Let's count by fourths to 8 fourths. Start at 0 fourths. (Write as students count.)

S: $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$.

T: How many fourths are in 1?

S: 4 fourths.

T: (Beneath $\frac{4}{4}$, write 1.) How many fourths are in 2?

S: 8 fourths.

T: (Beneath $\frac{8}{4}$, write 2.) Let's count by fourths again. This time, when you arrive at 4 fourths and 8 fourths, say the whole number. Start at 0.

S: $0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, 2$.

Continue, counting by halves to 6 halves.

Count by Mixed Numbers

NOTE: This activity builds comfort and fluency with mixed numbers. Change the denominators as students grow in fluency.

T: Let's count by $\frac{1}{2}$ with mixed numbers. Ready? (Rhythmically point up until a change is desired. Show a closed hand, and then point down. Continue, mixing it up.)

S: $\frac{1}{2}, 1, 1\frac{1}{2}, 2$ (stop), $1\frac{1}{2}, 1, \frac{1}{2}, 0$ (stop), $\frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4$ (stop), $3\frac{1}{2}, 3, 2\frac{1}{2}, 2, 1\frac{1}{2}, 1$ (stop), $1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 4\frac{1}{2}, 5$.

T: Excellent. Try it for 30 seconds with your partner. Partner A, you are the teacher today.

S: $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5.

Count by Decimals

NOTE: Practicing skip-counting on the number line, on personal whiteboards, or orally builds a foundation for accessing multiplicative reasoning. Repeat for perhaps four days so students see their fluency grow, and then change the multiple by which you count.

T: Let's count by 9's forward to 108 and then backward to 0.

S: 0, 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, (stop), 99, 90, 81, 72, 63, 54, 45, 36, 27, 18, 9, 0.

T: Now let's count by 9 tenths forward to 108 tenths and then backward to 0 tenths.

S: 0.9, 1.8, 2.7, 3.6, 4.5, 5.4, 6.3, 7.2, 8.1, 9, 9.9, 10.8, (stop), 9.9, 9, 8.1, 7.2, 6.3, 5.4, 4.5, 3.6, 2.7, 1.8, 0.9, 0.

Direct students to count by 4's forward and backward to 48, focusing on the crossing -10 transitions.

Extending Counting Exercises

Skip-Counting on Both Sides of Zero

NOTE: This fluency activity reviews counting sequences from *A Story of Units* but includes multiples to the left of zero on the number line. The visual hand gestures will help students build awareness of the order of numbers to the left of zero.

T: Let's count by 4's starting at 0. (Rhythmically point up until a change is desired. Show a closed hand, and then point down. Change the counting direction in the negative multiples several times. Continue with other multiples, including whole numbers, fractions, and mixed numbers.)

S: 0, 4, 8, 12, 16, 20, 16, 12, 8, 4, 0, -4, -8, -12, -16, -20, -16, -20, -16, -12, -8, -4, 0.

Counting by 10's into Subzero Temperatures

NOTE: This fluency activity reviews counting sequences including negative integers in a real-world context.

T: Let's count by 10 degrees starting at 35 degrees. (Rhythmically point up until a change is desired. Show a closed hand, and then point down. Continue, alternating the starting point.)

S: 35 degrees, 45 degrees, 35 degrees, 25 degrees, 15 degrees, 5 degrees, -5 degrees, -15 degrees, -5 degrees, 5 degrees, 15 degrees.

Counting and Evaluating Multiples of $8x$ with Whiteboards

NOTE: This fluency activity reviews multiplication of rational numbers in an algebraic context.

T: Let's count by $8x$. (Rhythmically point up until a change is desired. Show a closed hand, and then point down. Continue, alternating the starting point.) I'll stop and give you the value of x at intervals. Tell me the value of the expression.

S: $8x$, $16x$, $24x$, $16x$.

T: Stop. Write the value of $16x$ when $x = \frac{3}{4}$ on your personal whiteboard. (Check student responses, 12.) Erase your boards. With which term did we leave off?

S: $16x$.

T: Let's start there. Ready?

S: $16x$, $8x$, $0x$, $-8x$, $-16x$.

T: Stop. Write the value of $-16x$ when $x = \frac{1}{2}$ on your personal whiteboard. (Check student responses, -8.)

Continue with an appropriate sequence of terms.

Counting by a Percent of a Number with Whiteboards

NOTE: This fluency activity reviews percent of a number.

T: Let's count by 20% of 240 on your personal whiteboards. Take a moment to calculate 20% of 240. Then count as high as you can for 1 minute. When I say "Stop," circle the last number you wrote.

S: (Students write, for example, 48, 96, 144, 192, 240, 288.)

T: Stop. Circle the last number you wrote. Share your sequence with your partner, and say what percent of 240 you counted to.

S: (Students write, for example, 288 and share that 288 is 120% of 240.)

T: Okay, let's repeat the same exercise and see if you can go past the number you reached this time. On your mark! Get set! Think!

Continue with a sequence of problems that is appropriate for your students. After students finish, have those who beat their initial score stomp their feet three times.

Protocol for Deliberate Practice: Teach and Repeat by Using Feedback

Time frames may be adjusted.

1. (90 seconds) Teacher A and “students” plan for the instruction and prepare needed materials.
2. (60 seconds) Teacher A stands and delivers instruction to the students.
3. (90 seconds) The students make a positive statement about the teaching strategy and then suggest an improvement. Teacher A “calls the shot” by restating the feedback that he will address.
4. (60 seconds) Teacher A reteaches precisely the same segment of instruction, incorporating the selected feedback.
5. (60 seconds) The students give feedback again.
6. (60 seconds) Teacher A uses the new feedback to teach the segment again.

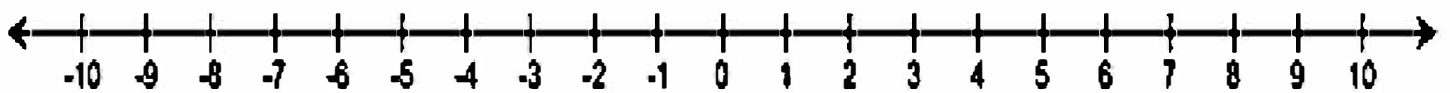
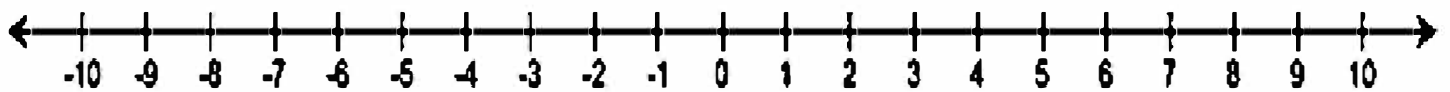
Repeat the cycle so Teacher A teaches the same segment a total of three times and receives three rounds of immediate feedback. The repetition allows success to be encoded. Teacher B goes next.

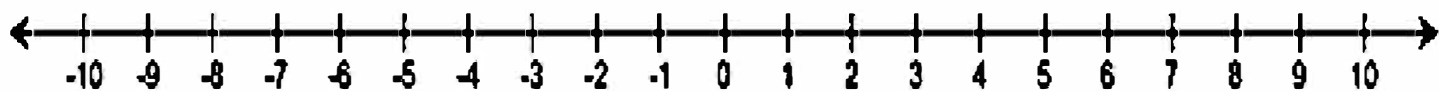
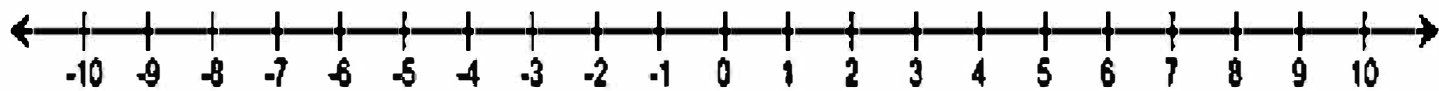
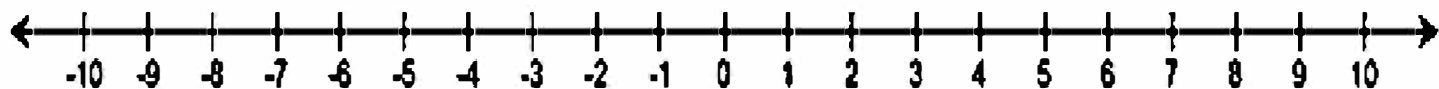
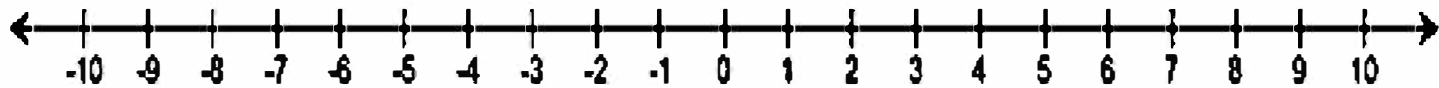
General Pedagogical Stems for “Praises” and “Pushes” in Deliberate Practice

- The wait time after asking _____ (was perfect/could be shortened or lengthened).
- After you asked _____, your cue for a response (was crisp and clear/could be more crisp and clear).
- When checking each personal whiteboard, it was evident (all were checked/some were omitted).
- One (“praise”/”push”) I would give is to _____.
- Next time, you might try _____.
- You (remained/might remain) silent and demonstrate (gesture) as you _____.

Math Stems for “Praises” and “Pushes” in Deliberate Practice

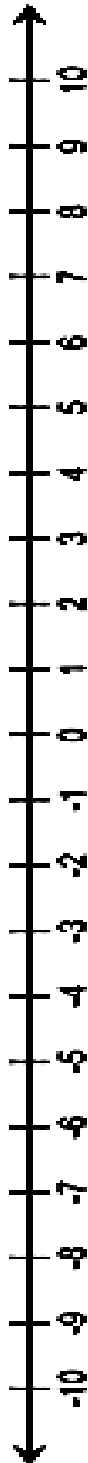
- You (took/did not take) advantage of an opportunity for students to make sense of the problem by _____ (MP.1).
- You (clarified/might clarify) the relationship of the pictorial mode (or context) to the number sentence by _____ (MP.2).
- You (had/did not have) students critique, re-voice, compare, or repeat the mathematical reasoning of their peers when you _____ (MP.3).
- The modeling of the problem was (clear/could have been more clear) because you _____ (MP.4).
- The students (had/did not have) the opportunity to strategically choose a tool when _____ (MP.5).
- Your math vocabulary (was/could have been more) precise when _____ (MP.6).
- You (emphasized/could have emphasized more) the use of structure (or repeated reasoning) when _____ (MP.7 and MP. 8).
- Between Problems ____ and ____, the jump in your teaching sequence was (just the right amount of a jump/perhaps too big a jump).



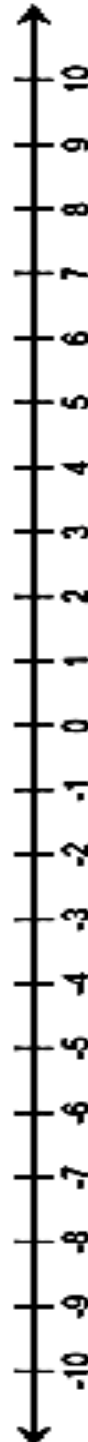




#1



#2



#3

Integer Games

Integer Games Grades 6 and Up

Fishing for Opposites (two to five players) [6.NS.C.6a]

1. One player shuffles all *Eureka Math* Integer Cards™ in the deck several times and spreads them facedown on the playing surface. The facedown cards are called *the pond*.
2. Each player selects five cards at random and holds them so only the holder can see their values.
3. If a player has a pair of opposite-valued cards in his hand, he can immediately lay the pair faceup in front of him, stating “_____ is the opposite of _____.”
4. The player who shuffled the cards begins play. She may ask the player to her left if he has the opposite of a card in his hand (e.g., “Do you have the opposite of -5 ?”).
 - a. If he does have the card, he surrenders the card to her and she may lay her pair of opposites in front of her, stating “_____ is the opposite of _____.”
 - b. If he does not have the card, he directs her to “fish for the opposite.” She selects a card of her choosing from the pond.
 - i. If she selects the card she sought, she can lay the pair of opposites, state “_____ is the opposite of _____,” and take another turn.
 - ii. If she does not select the card she sought, she passes her turn to the player to her left, and play continues in a clockwise rotation.
 - iii. If she selects a card whose integer is the opposite of another integer card in her hand, she must wait until her next turn to lay the pair of opposites.
5. The game ends when either a player runs out of cards or the pond runs out of cards (goes dry). The player with the most pairs of opposites wins the game.

Build the Number Line (teams of two students) [6.NS.C.6c]

This challenge requires a large space, such as a long table or the floor of the classroom, and one deck of Integer Cards™ per pair of students.

1. One player on each team shuffles all Integer Cards™ in their deck several times.
2. On the signal “Go,” partners work together to efficiently place each integer card in its proper location along a horizontal or vertical number line. Strategies for building the number line may vary. Players may stack cards containing equal values. Partners indicate that their number line is complete by using a signal such as raising their hands.

The teacher may choose to secretly time the initial round for use as a reference for additional rounds.

3. When finished, the teams discuss strategies players used to place the numbers and any misconceptions they corrected during placement.

If the teacher timed the initial task, she may report the elapsed time that the group needed to complete the task.

4. Next, challenge teams to beat their times and repeat the task, this time by using a more efficient strategy.

Variation: Build the Coordinate Plane (Follow the same steps as in building the number line, but require students to build two number lines that intersect at a right angle.) [6.NS.C.6c]

Order the Integers along the Number Line (one to five players) [6.NS.C.6c]

This game requires a large space such as a long table or the floor of the classroom.

1. One player shuffles all Integer Cards™ in the deck several times and deals 10 random cards to each player, facedown.
2. On the signal “Go,” players place the cards along the number line in their approximate locations according to their integers.
3. The first person to place his integers correctly wins the round.
4. Between rounds, the players discuss strategies for placing the Integer Cards™ more efficiently.
5. Repeat the above steps as desired.

Order the Integers without the Number Line (one to five players) [6.NS.C.7a]

1. One player shuffles all Integer Cards™ in the deck several times and deals 10 random cards to each player, facedown.
2. On the signal “Go,” players order their Integer Cards™ in ascending or descending order. (Order must be preselected).
3. The first person to order his Integer Cards™ correctly wins the round.
4. Between rounds, the players discuss strategies for ordering their Integer Cards™ more efficiently.
5. Repeat the above steps as desired.

War with Integers (two players) [6.NS.C.7a]

1. One player shuffles all Integer Cards™ in the deck several times and then divides them evenly into two piles, facedown (one pile per player).
2. Players reveal the value of the top card on their pile, placing the cards next to each other on the playing surface.
3. In each matchup, players compare the integer values shown. The player whose card shows the higher value collects both cards and places them in a separate collection pile.
4. When players run out of cards, they shuffle their own collection pile and continue play with those Integer Cards™.
5. The winner is the first player to collect all the cards or the player with the most cards at a time limit specified by the teacher.

You may extend the rules by requiring players to indicate how they know their integer has the higher value before they can collect their cards. For example, they might say, “My integer has the higher value because it is farther to the right on the number line than yours.”

Variation: War with Absolute Values (Follow the same rules as War with Integers above, except the cards in each matchup are awarded to the player who holds the card with the higher absolute value integer. You may extend the rules by requiring players to indicate how they know their integer has the higher absolute value before they can collect their cards. For example, they might say, “My integer has the higher absolute value because it is farther from zero on the number line than yours.”) [6.NS.C.7c]

Integer Games Grades 7 and UpOpposites Sum to Zero (two to five players) [7.NS.A.1b]

1. One player shuffles all Integer Cards™ in the deck several times and deals seven cards to each player, including herself. She places the remaining cards facedown in a draw pile at the center of the playing area and turns the top card faceup to start a discard pile.
2. Before play begins, if any player holds a pair of opposite integers, she may lay the pair faceup in front of her and say, “Opposites sum to zero.”
3. The player to the left of the dealer begins the game by choosing one of two options:
Option 1: Pick up the card on top of the discard pile.
Option 2: Draw a new card from the draw pile.
 - a. If either card gives the player a pair of opposite Integer Cards™, the player may immediately lay the pair in front of her and say, “Opposites sum to zero,” and then choose one other card to discard.
 - b. If a player does not have a pair of opposite Integer Cards™ after drawing, the player may choose a card to discard, and play resumes with the player to her left. Play continues in a clockwise rotation.
4. The first player to run out of cards wins.

5. If at any time a player lays a pair of opposite Integer Cards™ and does not say, “Opposites sum to zero,” before the next player starts, that player can be asked by any other player to add two cards from the draw pile to her hand.

War with Addition of Integers (two players) [7.NS.A.1b]

1. One player shuffles all Integer Cards™ in the deck several times and divides them evenly into two piles, facedown (one pile per player).
2. Before play begins, players determine who collects matchups representing positive sums and who collects matchups representing negative sums.
3. Players reveal the value of the top card on their pile, placing the cards next to each other on the playing surface.
4. In each matchup, players add the integer values shown (in any order because addition is commutative).
 - a. If the sum of the integers is positive, the player collecting the positive sums takes both cards.
 - b. If the sum of the integers is negative, the player collecting the negative sums takes both cards.
 - c. If the sum of the integers is zero, then the players engage in war.
 - i. Each player lays three more cards facedown and a fourth card faceup.
 - ii. Refer to a through c above until a winner of the matchup is determined. The winner is awarded all the cards played in the matchup.
 - d. The player collects his awarded cards and keeps them in a separate collection pile.
5. When players run out of cards, they shuffle their own collection pile and continue play with those Integer Cards™.
6. The winner is either the first player to collect all the cards or the player with the most cards at a time limit specified by the teacher.

You may extend the rules by requiring players to state the sum of the integers before they can collect their cards. For example, they might say, “The sum of -5 and 2 is -3 .”

War with Subtraction of Integers (two players—order specific) [7.NS.A.1c]

1. One player shuffles all Integer Cards™ in the deck several times and divides them evenly into two piles, facedown (one pile per player).
2. Before play begins, players determine who collects matchups representing positive differences and who collects matchups representing negative differences. Players also determine the order in which to subtract the cards as subtraction is not commutative.
3. Players reveal the value of the top card on their pile, placing the cards next to each other on the playing surface.

4. In each matchup, players subtract the integer values shown in the order they agreed to.
 - a. If the difference of the integers is positive, the player collecting the positive differences takes both cards.
 - b. If the difference of the integers is negative, the player collecting the negative differences takes both cards.
 - c. If the difference of the integers is zero, then the players engage in war.
 - i. Each player lays three more cards facedown and a fourth card faceup.
 - ii. Refer to a through c above until a winner of the matchup is determined. The winner is awarded all the cards played in the matchup.
 - d. The player collects her awarded cards and keeps them in a separate collection pile.
5. When players run out of cards, they shuffle their own collection pile and continue play with those Integer Cards™.
6. The winner is either the first player to collect all the cards or the player with the most cards at a time limit specified by the teacher.

You may extend the rules by requiring players to state the difference of the integers before they can collect their cards. For example, players might say, “The difference of -5 minus 2 is -7 .”

To extend again and reinforce the relationship between addition and subtraction, require that students restate the subtraction problem as an addition problem [7.NS.A.1c] and the corresponding sum before collecting their awarded cards.

War with Multiplication of Integers (two players) [7.NS.A.2a, 2c]

1. One player shuffles all Integer Cards™ in the deck several times and divides them evenly into two piles, facedown (one pile per player).
2. Before play begins, players determine who collects matchups representing positive products and who collects matchups representing negative products.
3. Players reveal the value of the top card on their pile, placing the cards next to each other on the playing surface.
4. In each matchup, players multiply the integer values shown (in any order because multiplication is commutative).
 - a. If the product of the integers is positive, the player collecting the positive products takes both cards.
 - b. If the product of the integers is negative, the player collecting the negative products takes both cards.
 - c. If the product of the integers is zero, then the players engage in war.
 - i. Each player lays three more cards facedown and a fourth card faceup.
 - ii. Refer to a through c above until a winner of the matchup is determined. The winner is awarded all the cards played in the matchup.
 - d. The player collects his awarded cards and keeps them in a separate collection pile.

- When players run out of cards, they shuffle their own collection pile and continue play with those Integer Cards™.
- The winner is either the first player to collect all the cards or the player with the most cards at a time limit specified by the teacher.

You may extend the rules by requiring players to state the product of the integers before they can collect their cards. For example, they might say, “The product of -5 times 2 is -10 .”

War with Division of Integers (two players—order specific) [7.NS.A.2b, 2c]

- One player shuffles all Integer Cards™ in the deck several times and divides them evenly into two piles, facedown (one pile per player).
- Before play begins, players determine who collects matchups representing positive quotients and who collects matchups representing negative quotients. Players also determine the order in which to divide the cards as division is not commutative.
- Players reveal the value of the top card on their pile, placing the cards next to each other on the playing surface.
- In each matchup, players divide the integer values shown in the order players agreed to.
 - If the quotient of the integers is positive, the player collecting the positive quotients takes both cards.
 - If the quotient of the integers is negative, the player collecting the negative quotients takes both cards.
 - If the quotient of the integers is zero or undefined, then the players engage in war.
 - Each player lays three more cards facedown and a fourth card faceup.
 - Refer to a through c above until a winner of the matchup is determined. The winner is awarded all the cards played in the matchup.
 - The player collects her awarded cards and keeps them in a separate collection pile.
- When players run out of cards, they shuffle their own collection pile and continue play with those Integer Cards™.
- The winner is either the first player to collect all the cards or the player with the most cards at a time limit specified by the teacher.

You may extend the rules by requiring players to state the quotient of the integers before they can collect their cards. For example, they might say, “The quotient of -5 divided by 2 is -5 halves.”

Target Number (two to four players)

The object of this game is for players to get cards that, by using a chosen operation, are close to or equal to the target number determined by the teacher. Before play begins, the teacher selects the target number and the operation(s) players are to use to reach the target. Start the first game with a target of zero and addition as the only operation. As students build fluency with the game, new

complexities may be built in, including changing the target number, changing the operation, and/or using more than one operation to hit the target.

1. One player shuffles all Integer Cards™ in the deck several times and deals four cards facedown to each player, including herself. The dealer then places the remaining cards in the center of the playing area as a draw pile, turns the top card faceup and places it next to the draw pile to start a discard pile.
2. The player to the left of the dealer begins play by picking up either a card from the draw pile or the top card from the discard pile. He then chooses any one of the five cards from his hand to discard onto the discard pile.
 - a. If the cards he holds hit the target number, he may announce this and show his cards to the other players, describing how his cards hit the target number. The player wins the current round, and a new round begins.
 - b. If the cards he holds do not hit the target number, he surrenders his turn and play continues with the player to his left.
3. Play continues until either a player hits the target number or a designated time has been reached, in which case the player whose score is closest to the target number wins the round.

High Hand or Low Hand (two to four players) [7.NS.A.1d]

The object of the game is to have the highest value of four cards or the lowest value of four cards when the game ends.

1. One player shuffles all Integer Cards™ in the deck several times and then deals four cards to each player. The dealer places the remaining cards in the center of the table and turns the top card faceup to start a discard pile.
2. The player to the left of the dealer begins play by picking up the top card from the draw pile or discard pile and then placing one card from her hand on the discard pile.
3. Play continues until a player claims to have the highest or lowest hand or until time is called. If a player claims to have the highest or lowest hand, all players must show their hands and agree on the winner.

NOTE: Students should notice that when attempting to gain the highest hand, discarding a negative value card increases their score (subtracting a negative), while discarding a positive value card decreases their score (subtracting a positive). Conversely, picking up a positive card increases their score (adding a positive), while picking up a negative card decreases their score (adding a negative). The strategy is reversed when attempting to achieve the lowest hand.

High Hand or Low Hand Version 2 [7.NS.A.3]

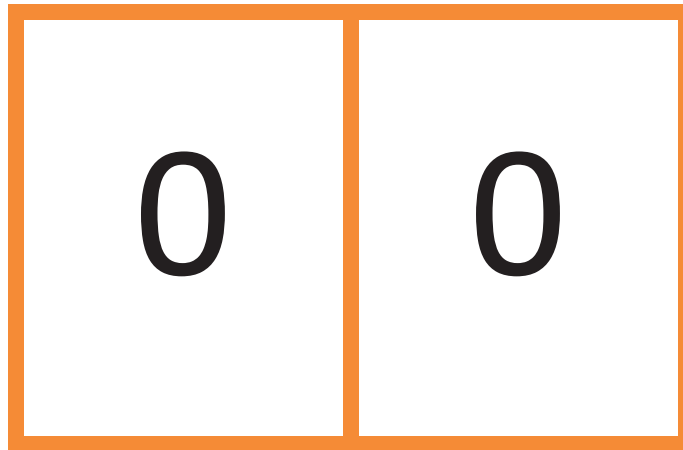
The object of the game is to make the highest value or the lowest value with four cards and by using any operation in any order.

1. The teacher displays four cards by using a document camera and decides on a high or low target.
2. Students are given a specific amount of time, possibly 1 minute, to make the highest or lowest value possible (as instructed) by using the cards in any order and with any of the four basic operations. Each card can only be used once.
3. In groups of four, students determine who has the highest or lowest value. The winner of the group of four then shares the winning value with the whole class and a class winner is determined.
4. Shuffle the deck of Integer Cards™, and repeat as many times as desired.

The Integer Cards

1	2	3	4
5	<u>6</u>	7	8
<u>9</u>	10	11	12

-1	-2	-3	-4
-5	-6	-7	-8
-9	-10	-11	-12



Rapid White Board Exchanges

Directions:

1. Read the following text, and highlight statements that resonate with you.

Implementing an RWBE requires that each student be provided with a personal white board, a white board marker, and a means of erasing his or her work. An economic choice for these materials is to place sheets of card stock inside sheet protectors to use as the personal white boards and to cut sheets of felt into small squares to use as erasers.

An RWBE consists of a sequence of 10 to 20 problems on a specific topic or skill that starts out with a relatively simple problem and progressively gets more difficult. The teacher should prepare the problems in a way that allows him or her to reveal them to the class one at a time. A flip chart or PowerPoint presentation can be used, or the teacher can write the problems on the board and either cover some with paper or simply write only one problem on the board at a time.

The teacher reveals, and possibly reads aloud, the first problem in the list and announces, “Go.” Students work the problem on their personal white boards as quickly as possible and hold their work up for their teacher to see their answers as soon as they have the answer ready. The teacher gives immediate feedback to each student, pointing and/or making eye contact with the student and responding with an affirmation for correct work, such as “Good job!”, “Yes!”, or “Correct!”, or responding with guidance for incorrect work such as “Look again,” “Try again,” “Check your work,” and so on. In the case of the RWBE, it is not recommended that the feedback include the name of the student receiving the feedback.

If many students have struggled to get the answer correct, go through the solution of that problem as a class before moving on to the next problem in the sequence. Fluency in the skill has been established when the class is able to go through each problem in quick succession without pausing to go through the solution of each problem individually. If only one or two students have not been able to successfully complete a problem, it is appropriate to move the class forward to the next problem without further delay; in this case, find a time to provide remediation to those students before the next fluency exercise on this skill is given.

Architecture of a Sprint

Sprints are designed in sets.

Directions:

- Analyze how the mathematical complexity in the sample Sprint below increases within and across sets.
- Read the following text, and highlight statements that resonate with you.

Set 1: Problems 1–11 are very easy. Every student can complete the set successfully.

1.	$6 = 2 \times \underline{\quad}$	
2.	$6 = 3 \times \underline{\quad}$	
3.	$9 = 3 \times \underline{\quad}$	
4.	$8 = 4 \times \underline{\quad}$	
5.	$10 = 5 \times \underline{\quad}$	
6.	$10 = 2 \times \underline{\quad}$	
7.	$20 = 10 \times \underline{\quad}$	
8.	$20 = 5 \times 2 \times \underline{\quad}$	
9.	$12 = 6 \times \underline{\quad}$	
10.	$12 = 3 \times \underline{\quad}$	
11.	$12 = 4 \times \underline{\quad}$	
12.	$12 = 2 \times 2 \times \underline{\quad}$	
13.	$12 = 3 \times 2 \times \underline{\quad}$	
14.	$24 = 8 \times \underline{\quad}$	
15.	$24 = 4 \times 2 \times \underline{\quad}$	
16.	$24 = 4 \times \underline{\quad} \times 2$	
17.	$24 = 3 \times 2 \times \underline{\quad}$	
18.	$24 = 3 \times \underline{\quad} \times 2$	
19.	$16 = 8 \times \underline{\quad}$	
20.	$16 = 4 \times 2 \times \underline{\quad}$	
21.	$8 \times 2 = 4 \times \underline{\quad}$	
22.	$8 \times 2 = 2 \times 2 \times \underline{\quad}$	

Set 2: Problems 12–22 are easy. About 65 percent of students will complete sets 1 and 2 successfully.

23.	$28 = 4 \times \underline{\quad}$	
24.	$28 = 2 \times 2 \times \underline{\quad}$	
25.	$28 = 2 \times \underline{\quad} \times 2$	
26.	$28 = \underline{\quad} \times 2 \times 2$	
27.	$36 = 2 \times 2 \times \underline{\quad}$	
28.	$9 \times 4 = 2 \times 2 \times \underline{\quad}$	
29.	$9 \times 4 = 6 \times \underline{\quad}$	
30.	$9 \times 4 = 2 \times 3 \times \underline{\quad}$	
31.	$8 \times 6 = 4 \times \underline{\quad} \times 2$	
32.	$8 \times 8 = 4 \times \underline{\quad} \times 2$	
33.	$9 \times 9 = \underline{\quad} \times 9$	
34.	$6 \times 6 = \underline{\quad} \times 6$	
35.	$6 \times 4 = \underline{\quad} \times 8$	
36.	$16 \times 2 = \underline{\quad} \times 8$	
37.	$2 \times 18 = \underline{\quad} \times 4$	
38.	$28 \times 2 = \underline{\quad} \times 7$	
39.	$24 \times 3 = \underline{\quad} \times 8$	
40.	$8 \times 6 = \underline{\quad} \times 4$	
41.	$12 \times 6 = \underline{\quad} \times 9$	
42.	$27 \times 3 = \underline{\quad} \times 9$	
43.	$54 \times 2 = \underline{\quad} \times 9$	
44.	$8 \times 13 = \underline{\quad} \times 26$	

Set 3: Problems 23–33 are at grade level of instruction. About 25 percent of students will complete sets 1, 2, and 3 successfully.

Set 4: Problems 34–44 are above grade level of instruction. In general, 0 percent of students will complete all four sets successfully.

Reading: Sprint History and Details

Directions:

1. Read read the following text, and highlight statements that resonate with you.

Background Information

Dr. Yoram Sagher invented Sprints after visiting high school mathematics classes in South Chicago. His subsequent invention, Sprints, are designed to engage students heartily, to give them something to look forward to academically as much as they look forward to what happens in the hallway or on the yard because a Sprint gets students' adrenaline flowing and helps them build fluency. Fluency is critical so that students do not use mental energy to address lower-level skills when solving complex or multi-step problems. The playful, embedded patterns in a Sprint help students think flexibly about structures and relationships rather than getting stuck calculating.

Problems in the Sprint are designed with patterns (see, for example, the use of the associative and commutative properties as in the Grade 5 Sprint) or structures that address Standards for Mathematical Practice 8* by encouraging students to “look for and express regularity in repeated reasoning” (NGA and CCSSO 2010, 8). Therefore, students must not skip around while working a Sprint, seeking the easiest problems to work. Doing so would divert students' concentration from recognizing the properties and relationships that will help them unlock the ability to solve more problems.

Differentiation Techniques

To make Sprints enjoyable and meaningful for all students from Kindergarten through Grade 8, you may need to use one or more of the following differentiation techniques.

- For lower-performing students, provide the Sprint the night before you administer it in class, casually and without any directives. Without pressure, students often practice on their own, before class, to feel more successful.
- Use Sprints as extra practice without following the Directions for Administration of Sprints. However, Sprints should never be graded, no matter how they are delivered.
- Always consider finding Sprints in *Eureka Math* that meet the needs of your students. If the Sprints at your current grade level are too challenging, go to the prior grade level and find others. The goal is for about 75 percent of students to have reached Set 3 when “Stop” is called.
- Create differentiated Sprints. Differentiated Sprints have different problems but the same answers, so all Sprints can be corrected at the same time. To create a differentiated Sprint, use the Sprint you want to administer and create another Sprint with the same answers, for example, a Sprint with problems that use the same skill at a different level of complexity or one that uses a

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different skill. For example, if the Sprint asks students to evaluate expressions when the value of x is $\frac{3}{4}$ and the expression is $2x - 1\frac{1}{2}$, the analogous Sprint might ask students to evaluate the expression when the value of x is 7 and the expression is $2x - 14$.

Number Correct: _____

Generating Equivalent Expressions—Round 1

Directions: Write each as an equivalent expression in standard form as quickly and as accurately as possible within the allotted time.

1.	$1 + 1$	
2.	$1 + 1 + 1$	
3.	$(1 + 1) + 1$	
4.	$(1 + 1) + (1 + 1)$	
5.	$(1 + 1) + (1 + 1 + 1)$	
6.	$x + x$	
7.	$x + x + x$	
8.	$(x + x) + x$	
9.	$(x + x) + (x + x)$	
10.	$(x + x) + (x + x + x)$	
11.	$(x + x + x) + (x + x + x)$	
12.	$2x + x$	
13.	$3x + x$	
14.	$4x + x$	
15.	$7x + x$	
16.	$7x + 2x$	
17.	$7x + 3x$	
18.	$10x - x$	
19.	$10x - 5x$	
20.	$10x - 10x$	
21.	$10x - 11x$	
22.	$10x - 12x$	

23.	$4x + 6x - 12x$	
24.	$4x - 6x + 4x$	
25.	$7x - 2x + 3$	
26.	$(4x + 3) + x$	
27.	$(4x + 3) + 2x$	
28.	$(4x + 3) + 3x$	
29.	$(4x + 3) + 5x$	
30.	$(4x + 3) + 6x$	
31.	$(11x + 2) - 2$	
32.	$(11x + 2) - 3$	
33.	$(11x + 2) - 4$	
34.	$(11x + 2) - 7$	
35.	$(3x - 9) + (3x + 5)$	
36.	$(11 - 5x) + (4x + 2)$	
37.	$(2x + 3y) + (4x + y)$	
38.	$(5x + 1.3y) + (2.9x - 0.6y)$	
39.	$(2.6x - 4.8y) + (6.5x - 1.1y)$	
40.	$\left(\frac{3}{4}x - \frac{1}{2}y\right) + \left(-\frac{7}{4}x - \frac{5}{2}y\right)$	
41.	$\left(-\frac{2}{5}x - \frac{7}{9}y\right) + \left(-\frac{7}{10}x - \frac{2}{3}y\right)$	
42.	$\left(\frac{1}{2}x - \frac{1}{4}y\right) + \left(-\frac{3}{5}x + \frac{5}{6}y\right)$	
43.	$\left(1.2x - \frac{3}{4}y\right) - \left(-\frac{3}{5}x + 2.25y\right)$	
44.	$(3.375x - 8.9y) - \left(-7\frac{5}{8}x - 5\frac{2}{5}y\right)$	

Number Correct: _____

Improvement: _____

Generating Equivalent Expressions—Round 2

Directions: Write each as an equivalent expression in standard form as quickly and as accurately as possible within the allotted time.

1.	$1 + 1 + 1$	
2.	$1 + 1 + 1 + 1$	
3.	$(1 + 1 + 1) + 1$	
4.	$(1 + 1 + 1) + (1 + 1)$	
5.	$(1 + 1 + 1) + (1 + 1 + 1)$	
6.	$x + x + x$	
7.	$x + x + x + x$	
8.	$(x + x + x) + x$	
9.	$(x + x + x) + (x + x)$	
10.	$(x + x + x) + (x + x + x)$	
11.	$(x + x + x + x) + (x + x)$	
12.	$x + 2x$	
13.	$x + 4x$	
14.	$x + 6x$	
15.	$x + 8x$	
16.	$7x + x$	
17.	$8x + 2x$	
18.	$2x - x$	
19.	$2x - 2x$	
20.	$2x - 3x$	
21.	$2x - 4x$	
22.	$2x - 8x$	

23.	$3x + 5x - 4x$	
24.	$8x - 6x + 4x$	
25.	$7x - 4x + 5$	
26.	$(9x - 1) + x$	
27.	$(9x - 1) + 2x$	
28.	$(9x - 1) + 3x$	
29.	$(9x - 1) + 5x$	
30.	$(9x - 1) + 6x$	
31.	$(-3x + 3) - 2$	
32.	$(-3x + 3) - 3$	
33.	$(-3x + 3) - 4$	
34.	$(-3x + 3) - 5$	
35.	$(5x - 2) + (2x + 5)$	
36.	$(8 - x) + (3x + 2)$	
37.	$(5x + y) + (x + y)$	
38.	$\left(\frac{5}{2}x + \frac{3}{2}y\right) + \left(\frac{11}{2}x - \frac{3}{4}y\right)$	
39.	$\left(\frac{1}{6}x - \frac{3}{8}y\right) + \left(\frac{2}{3}x - \frac{7}{4}y\right)$	
40.	$(9.7x - 3.8y) + (-2.8x + 4.5y)$	
41.	$(1.65x - 2.73y) + (-1.35x + 3.76y)$	
42.	$(6.51x - 4.39y) + (-7.46x + 8.11y)$	
43.	$\left(0.7x - \frac{2}{9}y\right) - \left(-\frac{7}{5}x + 2\frac{1}{3}y\right)$	
44.	$(8.4x - 2.25y) - \left(-2\frac{1}{2}x - 4\frac{3}{8}y\right)$	

Analyze a Middle School Sprint

Directions:

1. Partition the Sprint into four sets of problems.
2. Analyze each set of the Sprint, noting complexities and circling sets of related problems.

Number Correct: _____

Improvement: _____

Generating Equivalent Expressions—Round 2

Directions: Write each as an equivalent expression in standard form as quickly and accurately as possible within the allotted time.

1.	$1 + 1 + 1$	
2.	$1 + 1 + 1 + 1$	
3.	$(1 + 1 + 1) + 1$	
4.	$(1 + 1 + 1) + (1 + 1)$	
5.	$(1 + 1 + 1) + (1 + 1 + 1)$	
6.	$x + x + x$	
7.	$x + x + x + x$	
8.	$(x + x + x) + x$	
9.	$(x + x + x) + (x + x)$	
10.	$(x + x + x) + (x + x + x)$	
11.	$(x + x + x + x) + (x + x)$	
12.	$x + 2x$	
13.	$x + 4x$	
14.	$x + 6x$	
15.	$x + 8x$	
16.	$7x + x$	
17.	$8x + 2x$	
18.	$2x - x$	
19.	$2x - 2x$	
20.	$2x - 3x$	
21.	$2x - 4x$	
22.	$2x - 8x$	

23.	$3x + 5x - 4x$	
24.	$8x - 6x + 4x$	
25.	$7x - 4x + 5$	
26.	$(9x - 1) + x$	
27.	$(9x - 1) + 2x$	
28.	$(9x - 1) + 3x$	
29.	$(9x - 1) + 5x$	
30.	$(9x - 1) + 6x$	
31.	$(-3x + 3) - 2$	
32.	$(-3x + 3) - 3$	
33.	$(-3x + 3) - 4$	
34.	$(-3x + 3) - 5$	
35.	$(5x - 2) + (2x + 5)$	
36.	$(8 - x) + (3x + 2)$	
37.	$(5x + y) + (x + y)$	
38.	$\left(\frac{5}{2}x + \frac{3}{2}y\right) + \left(\frac{11}{2}x - \frac{3}{4}y\right)$	
39.	$\left(\frac{1}{6}x - \frac{3}{8}y\right) + \left(\frac{2}{3}x - \frac{7}{4}y\right)$	
40.	$(9.7x - 3.8y) + (-2.8x + 4.5y)$	
41.	$(1.65x - 2.73y) + (-1.35x + 3.76y)$	
42.	$(6.51x - 4.39y) + (-7.46x + 8.11x)$	
43.	$\left(0.7x - \frac{2}{9}y\right) - \left(-\frac{7}{5}x + 2\frac{1}{3}x\right)$	
44.	$(8.4x - 2.25y) - \left(-2\frac{1}{2}x - 4\frac{3}{8}y\right)$	

Development and Deliberate Practice of a Rapid Whiteboard Exchange (RWBE) and Whiteboard Exchange (WBE)**Skill to Practice:** _____

First, craft a series of problems that resemble those in Set 1. Be aware of the change in complexity from one problem to the next.

Then, craft a series of problems that bridge Set 1 into Set 2, with the emphasis being on the Set 2 problems.

Finally, craft a series of problems that bridge Set 2 into Set 3. Remember, we only expect 25 percent of our students to complete Set 3 with accuracy, but this is still a skill we are trying to build and patterns we want students to recognize.

Reflection

In the space below, reflect on your experiences of the day and how you might use these instructional tools to boost fluency in your classroom. Be sure to address each of the purposes of fluency as they apply to the content of your grade level.

Directions for Administration of Sprints

Directions:

1. Read read the following text, and highlight statements and highlight the essential steps and information.

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers are like athletic coaches. A rousing routine pushes students to do their personal best. Celebrate every improvement so students recognize success, which is critical in Sprints.

One Sprint has two parts with closely related problems. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more correct problem.

With practice, the following routine takes about 9 minutes.

Sprint A

Place Sprint A facedown on students' desks. Instruct students not to look at the problems until you give the signal. (Some Sprints include words. If necessary, before starting the Sprint, quickly review the words so reading difficulty does not slow down students.)

T: You will have 60 seconds to do as many problems as you can. I do not expect you to finish all of them. Just do as many as you can, your personal best. (If some students are likely to finish before time is up, assign a number to count by on the back.)

T: Take your mark! Get set! THINK!

Students turn their papers over and work furiously to finish as many problems as they can in 60 seconds. Time precisely.

T: Stop! Draw a line under the last problem you finished. I will read just the answers. If you got a problem right, call out "Yes!" If you made a mistake, circle it. Ready?

T: (Energetically, rapid-fire call the first answer.)

S: Yes!

T: (Energetically, rapid-fire call the second answer.)

S: Yes!

Repeat to the end of Sprint A or until no student has a correct answer. If needed, read the count-by answers in the same way. Each number counted by on the back is considered correct.

T: Fantastic! Now, write the number you got correct at the top of your page. This is your personal goal for Sprint B.

T: How many of you got one right? (All hands should go up.)

T: Keep your hands up until I say the number that is one more than the number you got correct. So, if you got 14 correct, when I say 15, your hand goes down. Ready?

T: (Continue quickly.) How many got two correct? Three? Four? Five? (Continue until all hands are down.)

If the class needs more practice with Sprint A, continue with the following optional routine.

T: I'll give you one minute to do more problems on this half of the Sprint. If you finish, stand behind your chair.

As students work, the student who scored highest on Sprint A might pass out Sprint B.

T: Stop! I will read just the answers. If you got it right, call out "Yes!" If you made a mistake, circle it. Ready? (Reread the answers to the first half as students stand.)

Movement

To keep the energy and fun going, always do a stretch or movement in between Sprints A and B. For example, the class might do jumping jacks while skip-counting by 5 for about 1 minute. This activity leaves students feeling invigorated and ready to make every effort to complete more problems on Sprint B.

Sprint B

Pass Sprint B out quickly, facedown on students' desks. Instruct students not to look at the problems until you give the signal. (Repeat the procedure for Sprint A up through the show of hands for correct answers.)

T: Stand up if you got more correct on the second Sprint than on the first.

S: (Stand.)

T: Keep standing until I say the number that tells how many more you got right on Sprint B. If you got three more right on Sprint B than you did on Sprint A, when I say "three," you sit down. Ready? (Call out numbers starting with one. Students sit when their improvement number is called. Celebrate the most-improved students with a cheer.)

T: Well done! Now, take a moment to go back and correct your mistakes. Think about what patterns you noticed in today's Sprint.

T: How did noticing the patterns help you get better at solving the problems?

T: Discuss your thinking with your partner for 1 minute. Go!

Students may take Sprints home.

Sprint Index for Grades 4–8

Directions:

1. Read read the following text, and highlight Sprints you could use with your students.

Grade 4		
Module	Lesson	Topic
1	1	Multiply and Divide by 10
	3	Multiply by 3
	5	Multiply by 4
	8	Find the Midpoint
	10	Round to the Nearest 10,000
	16	Convert Meters and Centimeters to Centimeters
2	5	Convert to Kilograms and Grams
3	3	Squares and Unknown Factors
	7	Multiply Multiples of 10, 100, and 1,000
	13	Mental Multiplication
	19	Mental Division
	21	Division with Remainders
	27	Circle the Prime Number
	31	Divide
5	6	Multiply Whole Numbers Times Fractions
	21	Subtract Fractions
	22	Add Fractions
	30	Change Fractions to Mixed Numbers
	31	Change Fractions to Mixed Numbers
	33	Change Mixed Numbers to Fractions
	34	Change Mixed Numbers to Fractions
	39	Multiply Whole Numbers Times Fractions

6	1	Divide by 10
	4	Write Fractions and Decimals
	8	Write Fractions and Decimals
	16	Add Decimal Fractions
7	1	Convert to Dollars
	5	Convert Length Units

Grade 5		
Module	Lesson	Topic
1	1	Multiply by 10
	3	Multiply by 3
	5	Multiply Decimals by 10, 100, and 1,000
	7	Find the Midpoint
	9	Round to the Nearest One
	12	Add Decimals
	13	Subtract Decimals
	15	Multiply by Exponents
	16	Multiply and Divide by Exponents
2	2	Multiply by 10, 100, and 1,000
	7	Multiply by Multiples of 10 and 100
	11	Multiply Decimals
	15	Convert Inches to Feet and Inches
	16	Divide by Multiples of 10 and 100
	28	Divide Decimals by Multiples of 10

3	1	Write the Missing Factor
	2	Find the Missing Numerator or Denominator
	3	Find the Missing Numerator or Denominator
	5	Subtracting Fractions from a Whole Number
	7	Circle the Equivalent Fraction
	9	Add and Subtract Fractions with Like Units
	10	Add and Subtract Whole Numbers and Ones with Fraction Units
	12	Subtract Fractions with Unlike Units
	14	Make Larger Units
	15	Circle the Smaller Fraction
4	6	Divide Whole Numbers
	14	Multiply a Fraction and a Whole Number
	18	Multiply Fractions
	21	Multiply Decimals
	30	Divide Whole Numbers by Fractions and Fractions by Whole Numbers
	33	Divide Decimals
5	3	Multiply a Fraction and a Whole Number
	7	Multiply Fractions
	11	Multiply Decimals
	18	Divide Whole Numbers by Fractions and Fractions by Whole Numbers
	19	Multiply by Multiples of 10 and 100
	21	Divide by Multiples of 10 and 100

6	8	Multiply Decimals by 10, 100, and 1,000
	11	Round to the Nearest One
	12	Subtract Decimals
	19	Make Larger Units
	20	Subtracting Fractions from a Whole Number
	23	Change Mixed Numbers into Improper Fractions
	29	Multiply Decimals
	33	Divide Decimals

Grade 6		
Module	Lesson	Topic
3	10	Rational Numbers: Inequality Statements
4	2	Division of Fractions
	8	Division of Fractions II
	11	Greatest Common Factor
	17	Addition of Decimals I
	19	Subtraction of Decimals
	25	Division of Fractions
	28	Addition of Decimals II
	30	Subtraction of Decimals
5	1	Multiplication of Fractions I
	3	Multiplication of Decimals
	7	Addition of Decimals II
	9	Addition and Subtraction Equations

	11	Multiplication of Fractions II
	17	Addition and Subtraction Equations

Grade 7		
Module	Lesson	Topic
2	8	Integer Addition
	9	Integer Subtraction
	15	Integer Multiplication
	16	Integer Division
3	2	Generating Equivalent Expressions
4	1	Fractions, Decimals, and Percents
	3	Part, Whole, or Percent
	6	Percent More or Less
	10	Fractional Percents

Grade 8		
Module	Lesson	Topic
1	4	Applying Properties of Exponents to Generate Equivalent Expressions
	8	Applying Properties of Exponents to Generate Equivalent Expressions

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