GREAT MINDS every child is capable of greatness

How to Support Multilingual Learners in Engaging in Math Conversations in the Classroom

QUICK LINKS

$6\frac{2}{3}$	7
8	$1\frac{1}{2}$

One sunny September morning, I called my grade 4 class together to the carpet to start our math lesson for the day. I displayed the four numbers shown and asked students to identify which of the numbers are whole numbers. I provided a few seconds of wait time before calling on Jesus, one of my most vocal, outgoing students and a multilingual learner.

Me: Which numbers are whole numbers?

Jesus: I see that 6 2/3 and 8 are whole numbers.

Me: How did you identify 6 2/3 and 8 as whole numbers?

Jesus: Well, you can see the holes in them, right on the screen.

I was truly puzzled. While I spent time anticipating possible misconceptions before this—and every—math lesson, I did not anticipate this misconception rooted in language. Whole and hole are homophones, meaning they sound the same but are spelled differently and have different meanings. Most English homophones translate to two different words in other languages, which makes homophones and other multiple-meaning words a particular concern for multilingual learners (MLLs).

Previewing English homophones in the mathematics classroom is a predictable language support for MLLs. To support MLLs in the classroom to avoid experiences like Jesus had, *Eureka Math²*™ relied on language development research to outline and build in supports for this, and other, predictable language needs for MLLs engaging with the language-rich *Eureka Math²* lessons. These language needs include support for mathematical

discourse as well as support for the different tiers of terminology. The instructional design of *Eureka Math*[2] was developed with intentionality to address these language needs.

Research-Based Effective Supports for MLLs

Research on learning supports for MLLs includes general research about best practices in the classroom as well as more specialized research on how to support MLLs in math learning. The research on math learning focuses especially on supporting student discourse and learning terminology.

The *Universal Design for Learning framework* gives guidelines to improve and optimize teaching and learning for all people based on scientific insights into how humans learn. Included under the UDL guidelines are considerations for promoting mathematical learning and understanding across languages, especially for MLLs. The term language can be broken down into four aspects: the ability to read, write, speak, and listen to academic and conversational English (Aguirre and Bunch 2012). All four aspects of language must be considered when developing a language-rich classroom environment rooted in authentic mathematical discourse.

Research Focusing on Mathematical Discourse for MLLs

Research conducted by Judit Moschkovich provides educators with a strong backbone for how to support MLLs with mathematical discourse. Moschkovich's research is summarized by her five recommendations for connecting mathematical content to language through mathematical discourse (2008).

1. Focus on students' mathematical reasoning, not on the accuracy of their language.
2. Shift to practices that support mathematical discourse and move away from a simplified view of language.
3. Recognize the complexity of language in math classrooms and support student engagement.
4. Treat everyday language as experiences and resources, not obstacles.
5. Uncover the mathematics in what students say and do.

Facilitating authentic mathematical discourse starts with the belief that all students, including MLLs, can learn mathematics content at the same time they are learning the English used in math. A core aspect of this belief is the importance of creating a classroom environment where all students, including MLLs, feel comfortable taking risks, making mistakes, talking, and learning from one another. "Classroom environments that foster a sense of community that allows students to express their mathematical ideas—together with norms that expect students to communicate their mathematical thinking to their peers and teacher, both orally and in writing— positively affect participation and engagement among all students (Horn 2012; Webel, 2010)" (NCTM 2014). When students feel comfortable expressing their ideas, their affective filter is lowered. Affective filters are often thought of as "imaginary walls" that rise in the mind of a MLL when motivation, self-confidence, or anxiety are affecting communication in a second or third language (Krashen 1982).

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Research Focusing on how to Support MLLs with Terminology Acquisition

Beck, McKeown, and Kucan's Tiered Terminology Model

Tier 1 Terms: These words are part of conversational language. They appear primarily in context and can present a challenge to MLLs if contexts are unfamiliar to students for cultural reasons.

Tier 2 Terms: These words are high-utility terms found in several domains. Sometimes, these words are the academic verbs or prompts in student-facing directions (like *determine*). These verbs can pose a challenge to students because they are not usually part of conversational language. Sometimes, tier 2 words are homonyms.

Tier 3 Terms: These words are domain-specific, meaning they are only encountered in math class. They can pose a challenge to MLLs because they are only exposed to domain-specific terms within math class. Examples of tier 3 terms are *hypotenuse* and *addend*.

In addition to supporting and fostering authentic mathematical discourse, language-rich classrooms must systematically develop the terminology needed to communicate mathematical concepts. This means that educators must consider multiple tiers of terminology support at any one time. Beck, McKeown, and Kucan (2013) organize terminology into a three-tiered model: tier 1 terms (conversational terms), tier 2 terms (academic terms), and tier 3 terms (domain-specific terms).

Because each tier of terminology is used differently in communicating in math class, each must be supported differently. However, in supporting each tier of terminology, instruction must center around honoring and acknowledging the funds of knowledge students bring to the class, instead of assuming that a student doesn't know the meaning of a term simply because they are a MLL. Adopting a funds of knowledge approach to terminology acquisition helps teachers move away from a simplified view of language and shift toward recognizing and supporting the complexity of language in mathematics (Moschkovich 2010).

Research in Action Supporting MLLs in *Eureka Math²*

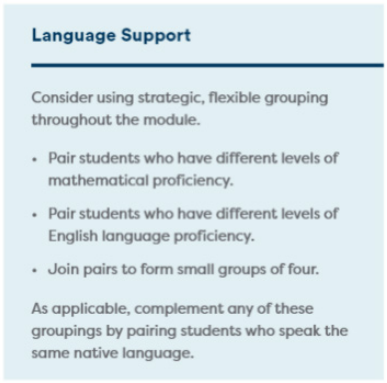
Eureka Math² supports MLLs through the instructional design, or how the plan for each lesson was created from the ground up. With the goal of supporting the clear, concise, and precise use of reading, writing, speaking, and listening in English, *Eureka Math²* lessons include the following embedded supports for students.

Embedded Supports	Where to Find in Eureka Math
» Activate prior knowledge (mathematics content, terminology, contexts)	» Launches, Fluencies, context videos, digital interactives
» Provide multiple entry points to the mathematics	» Notice and wonder questions, open-ended tasks, context videos, digital interactives, art connections, Math Pasts
» Use clear, concise student-facing language	» Readability guidelines, tier 2 academic verb support
» Provide strategic active processing time	» Instructional routines, 10/2 principle
» Illustrate multiple modes and formats	» Varied mathematical models, digital interactives, context videos, graphic organizers
» Provide opportunities for strategic review	» Fluencies, Practice program, Assessment program

In addition to the strong, built-in supports for all learners including MLLs outlined above, the teacher–writers of *Eureka Math²* also intentionally planned to support MLLs with mathematical discourse and the three tiers of terminology in every lesson. Language Support margin boxes provide these just-in-time, targeted instructional recommendations to support MLLs.

Supporting Mathematical Discourse in *Eureka Math²*

Authentically engaging in mathematical discourse can present a unique challenge for MLLs. They are constantly managing a large cognitive load by attempting to understand mathematics while also thinking—often in their native language—about how to communicate ideas and results in English. Additionally, everyday classroom interactions are heavily focused on listening and speaking rather than on reading and writing. To lighten the cognitive load of MLLs, *Eureka Math²* provides ample opportunities for students to engage in a balanced way with all four aspects of language—reading, writing, speaking, and listening—while engaging with mathematics. *Eureka Math²* supports teachers to create language-rich classrooms by modeling teacher–student discourse and by providing suggestions for supported student-to-student discourse. Since curricula in general have an abundance of receptive language experiences (reading and listening), *Eureka Math²* focuses specific supports on language production (speaking and writing) in mathematics.


Language Support

Consider using strategic, flexible grouping throughout the module.

- Pair students who have different levels of mathematical proficiency.
- Pair students who have different levels of English language proficiency.
- Join pairs to form small groups of four.

As applicable, complement any of these groupings by pairing students who speak the same native language.

The most all-encompassing Language Support margin box appears in the first lesson of every module in *Eureka Math²* prompting teachers to consider using strategic, flexible grouping in each activity of the entire module to support MLLs. These grouping suggestions invite teachers to leverage students' funds of knowledge and native language by assembling pairs of students in different ways. Each of these different ways of pairing students has different benefits for MLLs. Pairing students who have different levels of English language proficiency allows MLLs time for oral rehearsal before speaking or writing about mathematics. It also can provide a language model for MLLs new to the US. Pairing students who have the same native language can provide MLLs time to process in their native language, lowering their affective filter and allowing them to use their native language to solidify the math concept at hand.

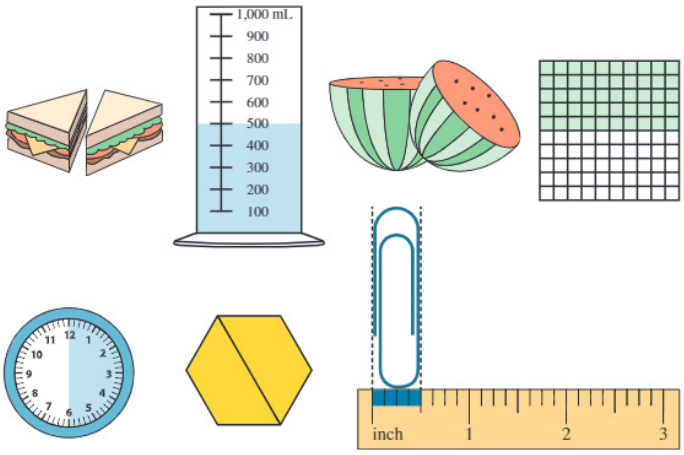
Eureka Math² uses common instructional routines across grade levels to support mathematical discourse for all students. These instructional routines, coupled with a positive, supportive classroom community, provide a predictable environment for students to participate in mathematical discourse. While speaking is at the center of each instructional routine, taken altogether, the instructional routines strategically integrate reading, writing, and listening. They provide equitable access to the math discourse community in the classroom by providing a structure for students to talk to one another. The instructional routines align with Stanford's language design principles of supporting sense-making, optimizing output, cultivating conversation, and maximizing linguistic and cognitive meta-awareness (Zwiers et al. 2017). Through supporting language, the instructional routines also align to CASEL's social and emotional learning core competencies of self-awareness, self-management, responsible decision-making, relationship skills, and social awareness. The chart below outlines the common routines used in *Eureka Math²*.

Instructional Routine	Purpose	Connections
1. Math Chat	To create an open-ended space for sharing mental math strategies and developing number sense, flexibility, efficiency, and accuracy	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, social awareness
2. Always Sometimes Never	To promote sense-making and mathematical discussion as students support a claim with examples and nonexamples	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills, social awareness
3. Which One Doesn't Belong?	To promote metacognition and mathematical discourse as students use precise language to compare different representations or examples	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, social awareness
4. Co-construction	To provide structured support for contextualizing and decontextualizing problems, which help students build abstract reasoning	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills, social awareness
5. Critique a Flawed Response	To promote effective communication techniques for critiquing others' work, correcting errors, and clarifying meaning	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills, social awareness
6. Take a Stand	To support students in making arguments and critiquing the reasoning of others	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills, social awareness
7. Five Framing Questions	To support students in analyzing a work sample or solution strategy by providing them with questions to frame their thinking Develops coherence across subject areas by using language that is parallel to routines in Wit & Wisdom® and PhD Science®	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills, social awareness
8. Stronger, Clearer Each Time	To provide a structured, interactive opportunity for students to revise and refine their writing language through rehearsal	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills
9. Numbered Heads	To help groups function effectively by encouraging students to build consensus and hold each student accountable for learning the material	Stanford's language design principles: support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness CASEL's social and emotional learning core competencies: self-awareness, self-management, responsible decision-making, relationship skills

In *Eureka Math*[2], directions for a routine are written in a lesson every time the routine is used. That way, the specific facilitation guidance is immediately available to the teacher as the lesson progresses. The following example shows the Math Chat routine written into a grade 5 lesson.

Display the pictures showing various halves and use the Math Chat routine to engage students in mathematical discourse.

Give students 2 minutes of silent think time to think about how the images are the same and different. Have students give a silent signal to indicate they are finished.



Have students discuss their thinking with a partner. Circulate and listen as they talk. Identify a few students to share their thinking. Purposefully choose work that allows for rich discussion about halves as two equal parts.

Then facilitate a class discussion. Invite students to share their thinking with the whole group. Record their reasoning.

As students discuss, highlight thinking that shows how each image represents halves.

The sandwich is cut into 2 equal parts, so each part is half of the whole sandwich.

Half of the clock is shaded. I can see that there are 2 equal parts.

Ask questions that invite students to make connections and encourage them to ask questions of their own.

Compare the halves. Are the halves the same shape? Explain why.

Yes. The rectangle with the grid has the same-size halves. Half of the rectangle is green and half is white.

I see two circular objects, but the clock is flat and the watermelon is solid.

No. I see different shapes. The sandwich is a square, the clock is a circle, and the yellow shape is a hexagon. Half of the sandwich is a triangle. Half of the rectangle is a rectangle.

What do you notice about how the halves are represented?

Some shapes, including the clock and the hexagon, show the whole and are just partitioned with a line into 2 parts. Others, including the watermelon and the sandwich, are broken into 2 parts.

The beaker of water is the whole, and it is half full of water. The other half is the empty space.

The paper clip is half of the inch. The width of 2 paper clips would be 1 inch.

What different measurement units are used?

The container measures the amount of liquid volume. It could be liters or milliliters.

The clock measures time. It may be hours or minutes.

The rectangle with the grid could measure area in square units.

The ruler measures length. It is measuring the length in inches.

What do you notice about the wholes?

They are all different sizes and shapes.

The size and shape of the halves depend on the size and shape of the whole.

Language Support

Consider supporting students in sharing thoughts and ideas. As students compare functions, provide sentence frames to help students precisely explain the transformations. For example:

The graph of _____ is a vertical stretch/compression (circle one) of the graph of $y = f(x)$. The [scale] factor is _____.

Talking Tool

Share Your Thinking	I know . . . . I did it this way because . . . . The answer is _____ because . . . . My drawing shows . . . .
Agree or Disagree	I agree because . . . . That is true because . . . . I disagree because . . . . That is not true because . . . . Do you agree or disagree with _____? Why?
I Can Ask Questions	Why did you . . . ? Can you explain . . . ? What can we do first? How is _____ related to _____?
Say It Again	I heard you say . . . . _____ said . . . . Another way to say that is . . . . What does that mean?

Put sticky notes here.

An educator simply facilitating an instructional routine likely does not provide enough guidance to support some MLLs in the productive language needed to prompt the blue student responses in the example above. Because of this, *Eureka Math²* periodically includes Language Support margin boxes that suggest sentence frames and sentence starters to support MLLs in student-to-student discussions, such as those used in instructional routines. These sentence frames and sentence starters could be specific to an activity, supporting students with communicating in the moment.

Language Support

Ask students to use the Say It Again section of the Talking Tool to revoice each of the rules from the Order of Operations anchor chart as they unfold in the lesson.

Language development research suggests that sentence frames and sentence starters that are general and that can be applied in more than one activity are more likely to be used by MLLs in new and novel situations. Because of this, *Eureka Math²* has developed the Talking Tool, which contains four sets of general sentence frames and sentence starters. The Talking Tool is referred to in Language Support margin boxes during times of sustained student-to-student discourse. For ease of use, the Talking Tool is provided as a classroom poster as well as directly to the student in the Learn book.

Supporting the Learning of Terminology in Eureka Math²

Terminology support in Eureka Math² is multifaceted and robust, encompassing support for all three tiers of terminology, as discussed in the research section above. The teacher–writers combed through every lesson with a lens of language and asked themselves the following questions: What are students expected to know and be able to do by the end of the lesson? What language is needed for what students need to know and be able to do? And how does that match with the way the lesson presents language? These questions led to lessons with support not just for domain-specific terms but also systematic support for academic and everyday terms.

Language Support

To support the term *additive identity*, have students identify where they add 0 when solving addition and subtraction equations.

$$f + 8 = 17 \qquad x - 9 = 15$$
$$f + 8 - 8 = 17 - 8 \qquad x - 9 + 9 = 15 + 9$$
$$f + 0 = 9 \qquad x + 0 = 24$$
$$f = 9 \qquad x = 24$$

additive identity — additive identity

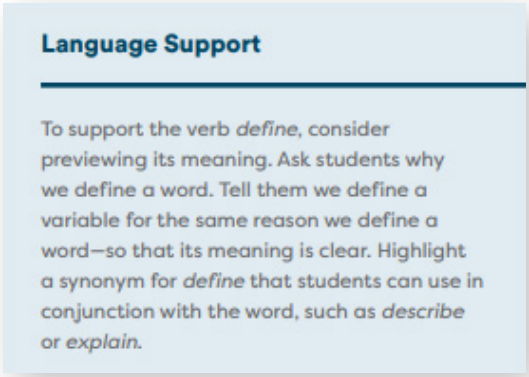
Language Support

To help students internalize the term *parallel*, have them hold their arms as shown in the photo. Have students say the term. Repeat the process, with students holding their arms vertically.

Explain that, like a pair of mittens or a pair of socks, parallel sides come in pairs.

Eureka Math² lessons have students experience a new mathematical concept before naming it with a precise mathematical term. Students see the concept come to life with a digital interactive, or manipulate counters in groups, or use an instructional routine to engage in mathematical discourse before the teacher gives that concept a name. This allows all learners, but especially MLLs, to attach that new term to a lived experience. In addition, *Eureka Math²* provides educative guidance for teachers, either within the lesson or a Language Support margin box, to support students with pairing the written term with a visual representation. This allows students to receive both written and spoken English language input, supporting MLLs with comprehension.

But often it's the domain-specific terms learned in previous lessons and grades that provide a language barrier to engaging with the mathematics of the current lesson for MLLs. They are simultaneously taking in English that contains logistical classroom instructions mixed with four or five domain-specific terms from yesterday's lesson, or last week's lesson, or last year's lesson. For many MLLs, this alone raises their affective filter. Therefore, Eureka Math² highlights what domain-specific terms from previous lessons are used in the current lesson, along with instructional recommendations for supporting those terms. These instructional recommendations focus on previewing the meaning of the terms before students are expected to interact with them in the mathematics of the lesson. This serves to lower the affective filter. Additionally, domain-specific terms from previous lessons are supported by pairing the written term with a visual representation, as shown above, but are also connected and woven together to create a network of related terms in the minds of students.



Eureka Math² lessons attend to terminology needs besides that of domain-specific terminology. In the instructional design process, curriculum writers at each grade level considered the academic verbs needed to engage with the mathematics by examining the function of the English language while students are learning the mathematics. Before students are asked to combine in grade 1, or classify in grade 4, or approximate in grade 8, Eureka Math² lessons preview the meaning of the academic verb, supporting the meaning of the term through a class discussion emphasizing the use of synonyms to that verb. In addition to lowering the affective filter for MLLs, the discussion helps to uncover the function of the language in the lesson. Each grade in *Eureka Math²* has a list of small, targeted academic verbs previewed before students are expected to use the language within the mathematics.

Language Support

To support the multiple meanings of the words *plane*, *map*, and *image*, consider facilitating a class discussion throughout the lesson that highlights the different uses and meanings of these words in mathematics and in the real world. Show pictures or examples of each different meaning, where applicable.

- Plane: A plane at the airport
- Plane: In math, a plane is a flat surface extending without end and having no thickness.
- Map: A map of the state of Michigan
- Map: A rigid motion maps point P to point P'.
- Image: An image in a magazine
- Image: Figure $A'B'C'D'$ is the image of figure $ABCD$ under a rigid motion.

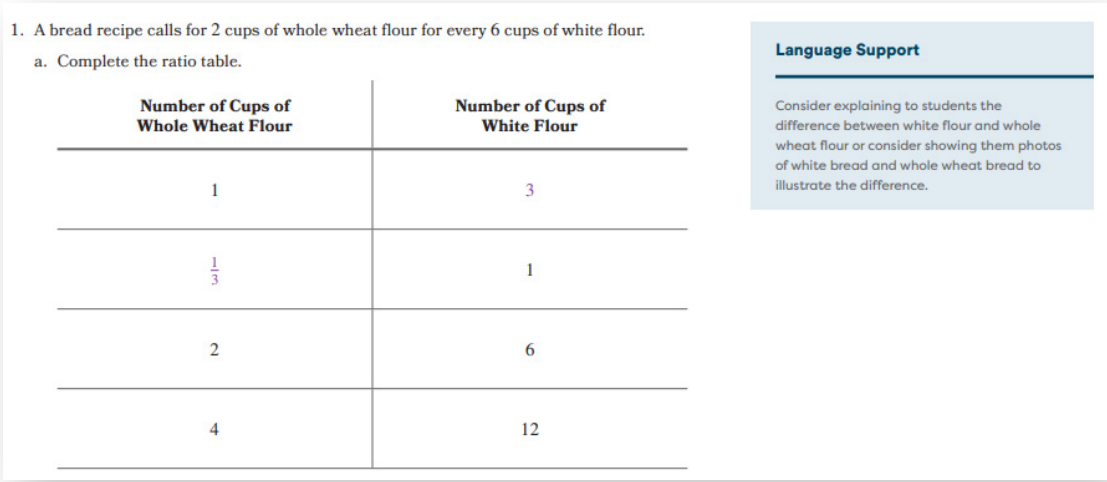
Students revisit the mathematical terms *plane*, *map*, and *image* again in grade 8 and in later courses. Students' understanding of these terms will evolve as they experience them in different contexts in mathematics.

The opening of this blog illustrated an example of other academic terms in use—multiple-meaning terms. Multiple-meaning terms encompass homophones like whole and hole, and homographs, like table and table, and other pronunciation-based needs, like the difference between estimate (as a noun as in "What is your estimate?") and estimate (as a verb as in "Estimate the sum."). The teacher–writers of Eureka Math² examined lessons for multiple meaning terms that could affect MLLs' understanding of the mathematics and authored Language Support margin boxes to preview the meaning of the term in the lesson. These previews include pairing the term with a visual like a picture, real items, or a video to highlight the different meanings of the term and emphasize the specific meaning used in the lesson.

Language Support

In addition to playing the video, consider supporting the context further by drawing on student experience. For example, show picture examples of each ride to facilitate a discussion about which ride might take up the greatest amount of total time based on the wait and ride times. Discuss and clarify possible assumptions needed to solve the problem, such as wait times remaining constant over the 1.5 hours or deciding whether to account for the time taken to get on and off a ride.

The instructional design of Eureka Math² is not limited to math discourse and terminology supports but also includes supports for conversational and everyday language needed to engage in the mathematics of the lessons. Launches and context videos serve to support the real-world contexts of the math problems students engage with. When Launches and/or context videos activate prior math knowledge instead of the real-world contexts used, Language Support margin boxes support real-world contexts with instructional recommendations to use a visual like a picture, real item, or a video to build background knowledge of the context.



1. A bread recipe calls for 2 cups of whole wheat flour for every 6 cups of white flour.

   a. Complete the ratio table.

Number of Cups of Whole Wheat Flour	Number of Cups of White Flour
1	3
$\frac{1}{3}$	1
2	6
4	12

Language Support

Consider explaining to students the difference between white flour and whole wheat flour or consider showing them photos of white bread and whole wheat bread to illustrate the difference.

Robust Curriculum Supports for MLLs Improve Equity in Math Instruction

All students deserve access to rigorous, knowledge-building mathematics instruction with appropriate supports. A variety of learners comprise any and every classroom in this country, and a high-quality curriculum builds in supports to address the learning needs of all students.

Curriculum writers must attend explicitly to the language needs of students, especially MLLs, so that language does not create a barrier to student engagement and learning. As Nora Ramirez and Sylvia Celedon-Pattichis (2012) state in their book, "Making texts more accessible means more than simplifying the language through which content manifests itself. ELs need access to mathematical discourse. This is a matter of social justice. If ELs are not given opportunities to engage and participate in experiences involving the use of appropriate mathematics discourse, they will continue to be at a disadvantage, and the so-called achievement gap between ELs and English-only speakers will be likely to remain a reality for these students" (204). All students deserve to be full participants in a math classroom, from building conceptual math understanding and knowledge to being able to converse with their peers about math and share their experiences and ask their questions. *Eureka Math*^{2} was intentionally designed to ensure that all students can be fully engaged learners in their classrooms.

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